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Metric Edition

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LEARNING SYSTEM TEX
Teacher's Guide

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Teacher's Guide
Level 6

# SRA MATHEMATICS LEARNING SYSTEM TEXT

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SRA

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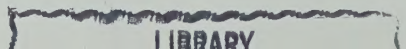

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Juanita Tolson, experienced public school and college educator in Washington, D.C., and J. Fred Weaver, author and professor at the University of Wisconsin, were asked to review SRA's existing basal mathematics program, tell us bluntly what was wrong with it, and recommend what should be in the next basal series. They did. We listened.

Jeremy Kilpatrick, Columbia University Teachers College, was asked to share his knowledge of problem solving in elementary mathematics and tell us what could be done to improve problem-solving materials. This was a hard job. He tried. We tried.

Classroom teachers Norma Jean Cheek, Joy Craig, Fran Engelbrecht, and Jane Hawley tried out new ideas before manuscript was prepared for field testing.

John C. Egsgard, well-known Canadian educator, helped us think through the best way to develop the geometry strand so that applications would be natural and valid. He brought to our attention the outstanding work done by the Ontario Institute of Curriculum Studies and unselfishly shared his own thoughts and efforts.

George T. Duncan, University of California at Davis, was asked to do a huge job: to construct the probability and statistics strand for the entire program. His approach was refreshingly simple and understandable. It's not every day that you can find a specialist in this area who can have empathy for the young child and his thinking process and yet handle the subject with expertise.

Jerome D. Kaplan guided the work of gifted educator-writers from Educational Analysis and Evaluation, Inc., who prepared some of the chapters.

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The information from nearly 700 teachers and 20 000 students in schools throughout the United States and Canada who participated in the 1972/73 verification study gave confidence that the revision of the first year's testing was effective.

Other teachers dug into their bag of tricks and contributed tried and tested classroom activities. Many thanks to James K. Bidwell, Philip Cox, Thomas S. Davis, Frances Greenberg, Muriel Greig, Bettye Hall, Donald Kamp, Evelyn Kozar, Kay Nebel, David O'Neil, Louise Petermann, Madolyn Reed, William Swart, and Judy Tate.

Literally hundreds of people in SRA have shared in the development of the SRA MATHEMATICS LEARNING SYSTEM. They all cared enough to do their very best. The editorial staff did even more. Special thanks to those exceptionally talented and dedicated people.

# Preface

*How was the* SRA MATHEMATICS LEARNING SYSTEM *developed?* Writing was almost the last step. We started by listening.

We visited schools of all kinds, from the inner city to remote rural areas. We sat in classrooms. Teachers, children, and parents told us about what they liked and didn't like about math programs.

We asked an independent research organization to interview supervisors and administrators. We talked to SRA Staff Associates about the needs they saw.

We reviewed all major basal math series. We studied standardized tests to see what children might be expected to know at various ages. Consultants evaluated existing SRA programs.

We also analyzed recommendations and reports from curriculum study groups, state and city adoption committees, and researchers in a variety of fields.

Then our authors, editors, and consultants worked together to prepare the rationale and learning objectives for the SRA MATHEMATICS LEARNING SYSTEM. Writing of the program did not begin until the entire scope and sequence had been defined by the learning objectives.

The manuscript was continuously reviewed, discussed, and revised by our development team. But it's arrogant for adults to sit in an office and predict what will work in the classroom. We needed answers to three questions:

- Will pupils attain the objectives?
- Will pupils develop positive attitudes toward the program?
- Will teachers find the program easy to use?

Our next step was to undertake two years of prepublication tryouts. We carefully selected classrooms across the United States and Canada to represent the broad range of pupil abilities, family backgrounds, and teaching styles. We visited, surveyed, listened, and tested. We rewrote and revised and retested before going to press with the program you see today.

We're confident that you'll find the SRA MATHEMATICS LEARNING SYSTEM effective, enjoyable, and easy to use. One of our tryout pupils wrote:

I liked the program and nothing was hard or dificalt. I think the book should come out to the world.

*What makes the* SRA MATHEMATICS LEARNING SYSTEM *a "system"?* The word *system* has many definitions. We call the SRA MATHEMATICS LEARNING SYSTEM a system because of the following five characteristics:

1. The entire program is based upon well-defined learning objectives.
2. Although the program can be enriched in many ways, the texts are complete in themselves. The teacher is not required to use any other materials.
3. There is a comprehensive evaluation program in each text.
4. Learning alternatives are provided for teachers and pupils who wish to use them.
5. The program provides information about the learners that will guide the teacher in altering or expanding a learning sequence.

*What kind of objectives are there?* Objectives are given for each chapter. Key to the program, however, are the year-end mastery objectives. These are the goals toward which instruction is directed.

Distinguished educator Ralph Tyler helped with the difficult task of defining learning objectives. He told us: "Remember the purpose of objectives. They are to guide, not dictate. Think of them as goals to be reached as a result of the teaching-learning process.

"Keep the number of objectives for any level under thirty, if possible. The teacher should be able to remember them all. A teacher who has to search through hundreds of objectives to figure out what to do cannot be free to teach anything more than bits and pieces.

"Avoid fashionable formulas for writing objectives. Fashions change. Avoid jargon. Keep the language simple. Objectives have to say something or their value is lost."

I liked the whole book because it told me things that I didn't even know about and it was very interesting.

*What are the texts like?* The SRA MATHEMATICS LEARNING SYSTEM focuses on the real world and develops many concepts from real-world situations.

Compared with other programs, this program spends a longer time on an idea and its related skills before introducing another topic. This gives skill competency a better chance to develop.

I like the S.R.A. math book because it's not just plane it you can gives you more time to think. 3x3

There are many invitations for pupils to *think* as well as to do. Not all questions are meant to be answered. Some questions have many answers; some have none.

I liked this section. They had fun math sentences. And they made you think.
Math is getting a lot eazer and its funner to; math really is fun

The language is informal. Purists may object to this departure from standard textbook English, but our tryout pupils responded enthusiastically to the style. Instead of stressing technical vocabulary and symbols, the program emphasizes skills and nonverbal understanding.

No pupil should have to moan, "Aw! It's the same old stuff," when he flips through his math book. The pages of the SRA MATHEMATICS LEARNING SYSTEM are varied. They're lively. They look like fun.

*How are the texts organized?* There are three major types of chapters.

*Exploratory* In an exploratory chapter pupils play with a big idea, think about it, and share their own ideas. The necessary vocabulary is introduced, along with some of the notation and operations related to the major idea.

*Instructional* In this type of chapter pupils begin the serious business of acquiring skills. Ideas are carefully sequenced into learning steps, and each learning step is accompanied by practice.

*Review* Here pupils must demonstrate understanding and skills. There is a review of the learning sequence, along with opportunities to explore applications.

No one text contains all three types of chapters for a single concept strand. For example, there are exploratory and instructional chapters on addition of whole numbers in level 1. The instructional chapters continue through level 4, and the review chapters start at level 5.

*How are pupils evaluated?* The evaluation program is built into each text. It allows a learner to check his own progress and determine his own strengths and weaknesses.

The first pages of an instructional chapter contain an informal survey to find out what the learner knows about the chapter to come. These pages indicate what the chapter is about, and they help to define the learning goal of the chapter.

The tests within an instructional chapter are called Progress Checks. A Progress Check identifies the knowledge that is a prerequisite for further work.

After learning from your math book I found this test very easy I found it easy because I learned from your book this is what made it easy.

A Progress Check can serve two functions:

1. If a pupil has gone through the preceding instructional pages, a Progress Check tells whether or not he has acquired the appropriate knowledge and skills. If he hasn't, he should try other kinds of instruction.
2. If a pupil seems to have prior knowledge of the chapter, as indicated by the survey at the start of the chapter, he may go directly to the Progress Check. It will determine if the preceding instructional pages can be skipped. If he's not successful on the test, he simply goes back to pages he skipped.

At the end of every chapter a Checkout lets the pupil and teacher know whether the learning objectives of the chapter have been reached.

*What learning alternatives are there?* There's a limit to the number of pages in a textbook. Beyond a certain length the book becomes too expensive and hard to use. So only a few pages can be devoted to extras for pupils who need more help or who would benefit from additional activities. To make sure these important extras are available, we've supplied a wealth of reinforcement and extension activities in the Teacher's Guides.

*The* SRA MATHEMATICS LEARNING SYSTEM *was built on some simple convictions.* Mathematics is relevant and vital. It's useful and interesting. Everyone should relax and enjoy it.

I like math a lot more than I did last year.

Some of the greatest learning opportunities are found in everyday things and are discovered when people talk together.

I liked the chapter because I think we learned the most and because we had a lot of discussions.

Mathematics doesn't have to be formal and abstract to be good mathematics.

I think this math is very good because I know what I'm doing

Everyone—students and teachers—should succeed in mathematics.

I liked it very much I never learned this kind of measures but I think it is good that you wrote it in the book because soon America will learn ~~to learn~~ a new kind of measure and you already are teaching it to us.

A textbook can't transmit the joy of learning as well as an enthusiastic teacher, but it can help.

# Notes & Things

We have asked a lot of questions. We have listened to the answers. One set of questions had to do with the organization of a teacher's guide.

Teachers told us that they wanted the guide pages numbered the same as the pupil pages. That was not an easy request to handle, since there are teacher resource pages at the end of each chapter and some pages of introduction to the next chapter. But the request made sense, so we put the alphabet to work.

The pupil page number repeats at the end of a chapter like the tune of a broken record; but attached to the number is a letter, which changes to accommodate those extra guide pages. When we finally get to the first pupil page of the next chapter, the guide page number will once again match the pupil page number. This whole thing sounds terrible. But flip through the book. Keep an open mind. It won't be so bad once you get used to it.

As we listened to teachers we found that there were many possible classroom organizations. They varied from large, heterogenous grade-level groups to small, homogeneous groups composed of pupils of different ages. It was impossible to tailor a guide to fit all kinds of organizations. But it was possible to let you know the special features of each page so that you can quickly decide how you want to use that page.

Each pupil page appears, slightly reduced in size, on the guide page. And the answers are right in place. You will find comments beside each pupil page. They will be in categories like these:

**lesson** The pages indicated here are related but need not all be presented on the same day. They provide the continuity of experience necessary to get an idea established.

**goal** The words needed to turn this goal into a behavioral or performance objective have not been printed, although they could have been. The goal simply helps you pinpoint the learning task for the page.

**memo** These words will tell you if a discussion is needed to get the learners started in the right direction. Or they may warn you about a potential problem or provide an explanation of why something is done the way it is. Or they may be simply a suggestion to make your work easier.

**things** All materials that you'll need for the page activities are listed here.

**warm-up** You can guess where this label came from. If the mathematics idea is a new one or maybe a hard one, a suggestion is made that will help get the learners ready to learn.

**page 1** The comments about the page itself may be introduced with traffic-light colors and will mean about the same things.

Caution—everyone needs most of this information, but the use of the page will vary according to the ability of individual pupils.

Stop—think about which pupils should do this activity. Your independent learners will have no trouble, but your pupils at the other end of the performance scale may benefit much more by doing other, more appropriate activities.

As you flip through the pages you will see a few sentences in *italic type.* These sentences are intended to give you some ideas about the words you might use to talk about the page. (How many times have we all thought, "I understand, but I don't know what words to use to explain it"?) In talking, probably nobody would use the exact words that are given. They are not meant to be a script. They are offered as a guide, nothing more.

You will find the new concept-development words in SMALL CAPITAL LETTERS, and some other functional words that deserve emphasis will be in **this kind** of type.

Now flip through the book again. Notice the copy below the pupil pages. There are 3 types of activities. Each type has its special symbol—one of the international traffic signals.

The old familiar red *stop* sign will signal an activity for the youngster who simply needs more work before he goes on to a new learning goal.

The yellow *divided highway* sign signals an activity for those youngsters who can accept a challenge beyond the learning expectations of the page.

The new, blue *rest area ahead* sign signals an activity you can use with any group. This type of activity departs from the standard math work and gives everyone a chance to have a break and hopefully some fun.

Here you will find the ideas that will extend the lessons and suggestions that offer specific help for those children who need it. Each activity was carefully selected so that you could have something special to personalize each child's learning.

You'll find a Resource Section at the end of each chapter. This provides alternate forms of all progress checks and the checkout and still more activities. Not every Resource Section is the same, but you will find references to additional learning aids that may be in your school and are just right to use. At the end of some chapters you will find a description of these learning aids. At the end of the guide you'll find a bibliography that lists children's books and film medium references. These features were planned to help you with your job of teaching and the children's job of learning.

A glossary of mathematical terms used in the text appears at the end of each pupil book. It is also reproduced at the end of each of the Teacher's Guides. Why not take time to look at it now, so you can get an overview of how the SRA Mathematics Learning System uses math words?

## Canadian SI Edition of the SRA Mathematics Learning System

The Canadian SI pupil texts of the SRA Mathematics Learning System have been revised to be fully in accord with the specifications of the Canadian Metric Commission, the Canadian Government Specifications Board, and with teaching requirements of Canadian schools. In revising the Teacher's Guides, we have been governed by considerations of economy. In order to keep costs to schools at a reasonable level, the revision of the Teacher's Guides has been limited to what is necessary for teachers.

Where significant changes have been made on pages of the pupil's book, the Teacher's Guide has been revised accordingly. Where only minor changes have been made in the pupil's book, the Teacher's Guide has not been changed. You will find occasional references in the Teacher's Guide which are not applicable to schools in Canada. While the pupil's texts are fully metric, in the Teacher's Guides there are refences to non-metric units of measure or to working in both systems. In the pupil's book, all numbers of five or more digits are separated by spaces between millions, thousands, and so on, instead of the traditional commas; this change has not been made on pages of the Teacher's Guide that needed no other revision. Similarly, in the pupil's book there is always a zero before the decimal point where there is no other number; again, this change has not been made on otherwise unchanged pages of the Teacher's Guide. In the text, questions have been revised to avoid reference to non-metric units, but when the answers have remained numerically the same, the Teacher's Guide has not always been altered. Pages that have been revised to assist the teacher and avoid ambiguity will be easily recognized; these pages are printed in black and red.

# Special Features of
# Mathematics Learning System — Canadian SI Edition

- A *complete* instructional system which includes clear objectives, built-in evaluation, and alternate instructional and assessment procedures
- *Extensive field testing* before final publication to ensure a program that works
- A *real-world approach* that involves students in practical, everyday mathematics
- *Three types of chapters* to develop each content strand:
  - (i) *Exploratory* chapters in which major concepts are introduced
  - (ii) *Instructional* chapters in which specific learning and skill building takes place
  - (iii) *Review* chapters in which the student's mastery of skills is asssessed and challenged
- *Complete SI* (International System of Measures) metrication, Levels 1 through 6
- Well-defined *mastery objectives* for each level and specific objectives at the beginning of each chapter
- *Flexible learning sequence* by which a teacher can meet the needs of individual students by selecting sequences of chapters that differ from the order in the text
- *Evaluation within the text* by means of
  - (i) *Survey questions* leading into each chapter
  - (ii) *Progress checks* - periodic trouble shooting within each chapter
  - (iii) *Checkouts* - measurement of student achievement at the end of each chapter
  - (iv) *Alternate checkouts* for each chapter in the Teacher's Guide
- *Practice Sheets* for each level that contain additional materials for skill building and evaluation
- *Teacher's Guides* giving comments and suggestions for every page of the text. Resources at the end of each chapter provide
  - (i) alternate evaluation for Progress Checks
  - (ii) activities
  - (iii) additional learning aids, including specific card references to many related SRA educational materials, as well as to those of other suppliers
  - (iv) in some chapters, games and and exercises, with permission to reproduce the pages for class use
- *Level placement tests* by which pupil readiness for a given level can be determined, or by which year-end mastery can be assessed
- *Chapter placement tests* by which enrichment or remedial needs can be determined

# the curriculum

In order to maintain a balance of concept development and drill, each SRA MATHEMATICS LEARNING SYSTEM chapter is built around one big idea.

The three kinds of chapters discussed in the preface—exploratory, instructional, and review—serve to organize major mathematical ideas into continuous strands. The big-idea chapters in strand organization give you more control over the learning sequence. In cooperative planning with other teachers, you can safely change the order of chapters across several levels in order to emphasize a particular content strand at a given level.

The following chart shows the chapter organization for levels 3 through 7. Think of this chart as a road map. You will use only the part you need to get you where you want to go. The strands of content are listed down the side. The levels are listed across the top. The numeral in each rectangular shape tells the chapter number and the location of the rectangle itself signals what part of the strand's content is featured in each chapter.

If you want to emphasize the whole numbers for example, the chart will show you the sequence of the strand. If you wish to use a measurement chapter in some order other than that in the book, it's O.K. The chart indicates that certain chapters can be used much earlier than that designated by the printed sequence. Most important, the chart lets you see the flow of content from one level to another.

| | |
|---|---|
| **whole numbers** | concept<br>operations + − × ÷ |
| **fractions** | concept<br>operations + − × ÷<br>decimal notation<br>operations + − × ÷ |
| **measurement** | length<br>mass<br>capacity<br>time<br>money |
| **geometry** | 3-d, 2-d shapes<br>1-dimensional measurement |
| **statistics** | collecting/recording<br>interpreting information |
| **probability** | |
| **rational and real numbers** | concept<br>operation |
| **graphs and functions** | concept<br>application |

**In planning your year's work with the SRA Mathematics Learning System, be sure to consult your local or provincial curriculum guidelines.**

level 4
level 5
level 6
level 7
level 8
1 8 15
2 10 R
3 5
9
1 13
6 R
2
5
1
2
1
6 11
4 7 10
8
7
10
4 5
2
3
2
3
4
12
9
13
R
12
7 14
3 12
3 11
9
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7 10
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6 10
12
13
11
8
11
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1 3 7
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4 5 6 8
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4 5 13
8 9 14

# OBJECTIVES

This program was built upon learning objectives. Each objective clearly states what pupil behavior is to be observed, the conditions under which the pupil is to perform, and the criteria for acceptable performance.

These objectives by definition are limited to observable behavior; therefore they are limited to the cognitive domain. This is a severe limitation, for the field-test information reveals that the affective domain also has been penetrated. A greater pupil involvement in things relating to mathematics has been shown in the complete range of abilities. Attitudes have been changed. But writing objectives for the affective domain is a new endeavor, and frankly we have not yet mastered the art. The following, therefore, are learning objectives in the cognitive domain. Each objective states the performance expected at the completion of this level.

### whole-number notation

1. Given any 10-digit number, the learner can round that number to a given place.
2. Given any number with 12 or less digits, the learner can tell the value of each digit.

### whole-number operations

1. Given any 2-digit number and any 3-or-more digit number, the learner can estimate and find the quotient and the remainder (if any).
2. Given a computation situation, the learner can round appropriately to estimate the answer.

### sentences

1. Given a simple word problem with only one arithmetic operation and extraneous information, the learner can solve it and write a math sentence to summarize and verify his computation.

### fractional-number concepts

1. Given two or more common or decimal fractions expressed to thousandths, the learner can order them.

### fractional-number notation

1. Given orally a 3-digit decimal, the learner can write it and tell the value of each digit.
2. Given a decimal, the learner can express it as a common fraction.

### fractional-number operations

1. Given two decimals, the learner can find their sum or their difference.
2. Given two frequently used mixed numbers, the learner can find their product.
3. Given a division sentence with a fraction as a divisor, the learner can use the common-denominator method to find the quotient.
4. Given two decimals less than 1000 expressed in tenths, hundredths, or thousandths, the learner can find their product.

### geometry concepts

1. Given a movable model such as a tracing, the learner can match and determine congruent figures.
2. Given a figure on graph paper, the learner can make a congruent figure.
3. Given a set of figures, some of which have been flipped or turned, the learner can find the congruent figures by tracing and then matching.

### measurement

1. Given a metric unit of measure with a prefix, the learner can tell whether the unit is larger or smaller than other related units.
2. Given regions of polygons on geopaper, the learner can find the area by counting.

### measurement of events

1. Given a chart or a table used in everyday situations, the learner can interpret the data.
2. Given the description of a probability experiment with equally likely outcomes, the learner can express the probability with a fraction.

# 1 NUMBER RELATIONSHIPS AND OPERATIONS WITH + AND −

**before this chapter the learner has —**

1. Rounded 6-digit numbers to a given place
2. Estimated and found the sum or difference of any two 4-digit numbers
3. Estimated and found the sum of columns of three or four numbers

**in chapter 1 the learner is —**

1. Reviewing and extending his rounding and estimation skills
2. Reviewing and checking his skill in estimating and finding the sum or difference of any two 4-digit numbers
3. Reviewing and checking his skill in estimating and finding the sum of columns of three or four numbers
4. Using an estimate to check the reasonableness of a computed answer
5. Finding renaming patterns in addition and subtraction

**in later chapters the learner will —**

1. Extend his rounding skills to 10-digit numbers
2. Apply his addition and subtraction skills to multiplication and division computation
3. Apply his addition and subtraction skills to working with decimals

Estimation is perhaps the mathematical skill most frequently used in the everyday world and it is one of the major thrusts of this program. Estimation is a powerful problem-solving tool. It is a means to determine approximately what the answer to a computation will be, before the computing is done; and it is a means to check the reasonableness of an answer after the computation is done.

The intent of this chapter is to start the development of an attitude rather than a skill. It is hoped that each pupil will come to understand and say to himself, "A good estimate is many times all that's necessary in the real world; **but** when I compute, I must be accurate." The emphasis on computational skills then falls into a meaningful context – the skills are more important than before.

You will know a great deal about each pupil's addition and subtraction skills by the time you finish with this chapter. Estimation provides the thrust for this review but the comprehensive diagnosis of addition and subtraction skills is the feature that will help you the most. You may find some pupils who need to go as far back as the addition and subtraction facts. You may find others who will demonstrate complete mastery of these operations. There are many options for individual progress in this chapter. The self-study extension pages, for example, will give you the time to work with those who need your personal help. Those pupils who show they know their skills can also help you as peer tutors. Sharing talents and responsibilities is a great way to start the study in a new level. Everyone will benefit.

For the extra activities you will want to have these things available:

10 same-size boxes (milk cartons)
index cards
markers
2 small boxes

**goal** Think about and explore ideas through a picture clue

**page 1** Each chapter opens with a full-page photograph. The page can do much more than identify the chapter number and chapter title. This page, for example, will let you get acquainted with your pupils. Ask questions—lots of them.

Does your community publish its own newspaper? More than one? Are any out-of-town newspapers sold in your community? How are newspapers different? Which newspaper appears to be the most popular? Which families read a newspaper every day? Is the newspaper delivered to the home? Can the newspaper be purchased at a newsstand? How much does the daily paper cost? Does the Sunday paper cost more? Why?

What is a subscription? Would a newspaper subscription for 1 year cost more or less than a newspaper bought every day at a newsstand? Is delivering newspapers to homes a good way to make money? Does anyone have a paper route? There may be some relevant experiences to share.

The newspaper is the theme that introduces the chapter. Everyone should be ready to go directly to page 2.

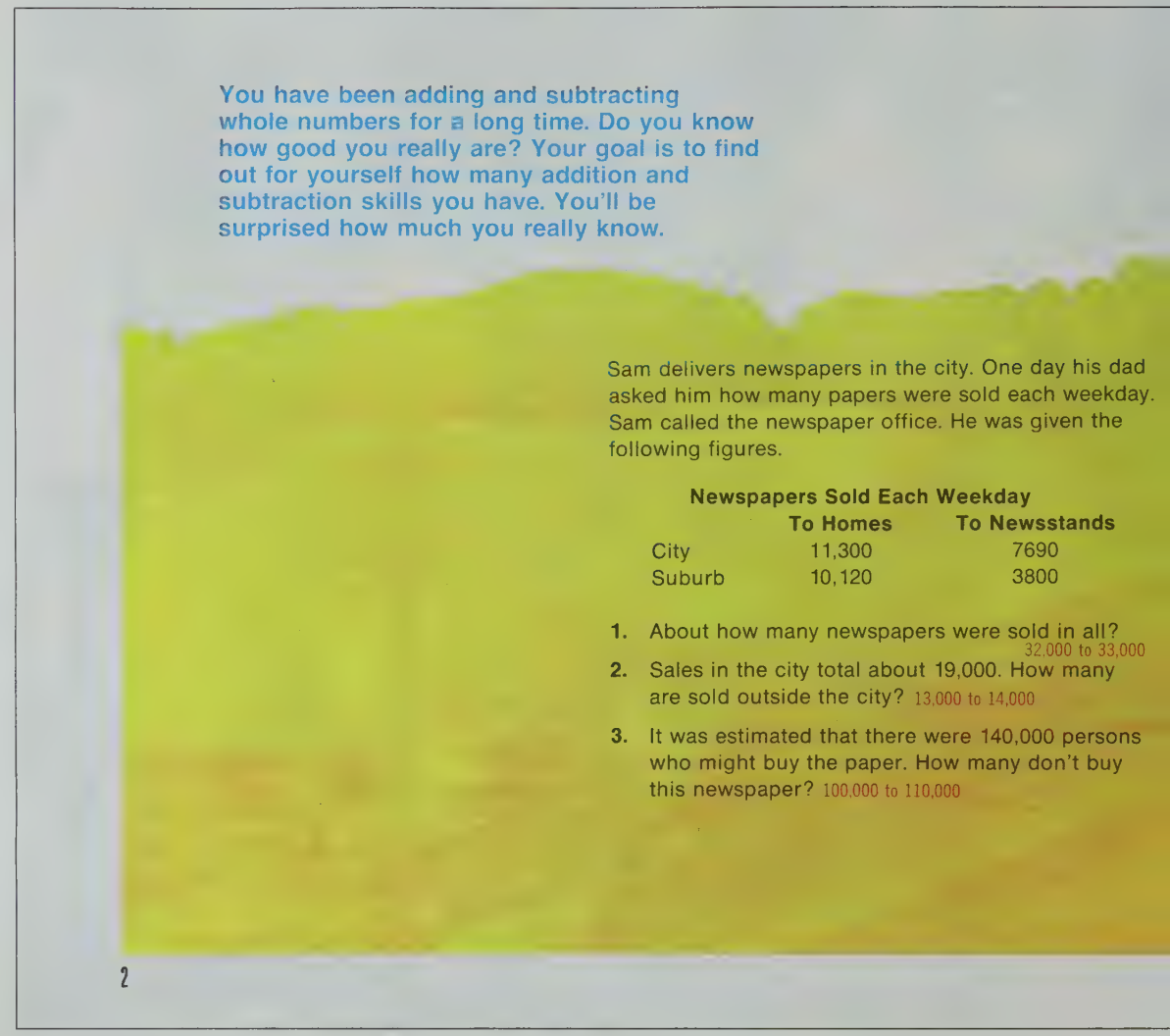
**lesson** Pages 2, 3

**goal** **Survey**—ability to round numbers and estimate sums and differences

**page 2** Answers will vary, depending on the place-value position to which the pupil rounded. A range for estimates is given in the answer key. The word *suburb* may be new to some youngsters.

A discussion of the following questions will provide a more complete picture of your pupils' ability to round numbers.

- Does the circulation information mean that there are as many families in the city as in the suburbs? Are these exact numbers?
- Might the city sales actually be as great as 12,000? (Not likely. The newspaper would give the most "optimistic" figure.)
- Might the actual number of sales in the city be as low as 11,000? (Not likely. It appears the figures are rounded to multiples of 10 or 100.)
- Is the last problem realistic? Would every person ever buy a newspaper? Why? (Consider the average family of four members. How does this affect the numbers? Ask about families that do not buy a paper at all.)
- Might there be two different newspapers to choose from in this city? (There could be.)
- If most of the families bought one newspaper and if there were two newspapers to choose from, which newspaper would have the larger circulation? (They could have about the same circulation. One newspaper may have only a few more customers than the other, but it's doubtful.)


You have been adding and subtracting whole numbers for a long time. Do you know how good you really are? Your goal is to find out for yourself how many addition and subtraction skills you have. You'll be surprised how much you really know.
Sam delivers newspapers in the city. One day his dad asked him how many papers were sold each weekday. Sam called the newspaper office. He was given the following figures.
Newspapers Sold Each Weekday
To Homes
To Newsstands
City 11,300 7690
Suburb 10,120 3800
1. About how many newspapers were sold in all? 32,000 to 33,000
2. Sales in the city total about 19,000. How many are sold outside the city? 13,000 to 14,000
3. It was estimated that there were 140,000 persons who might buy the paper. How many don't buy this newspaper? 100,000 to 110,000
2

1. People who get their papers at home usually pay the newsboy once a month. When Sam goes collecting each month, he also gets tipped. He thinks that he gets more tips on weekends than on weekdays.

| Monday's Tips | | Saturday's Tips | | |
|---|---|---|---|---|
| 24¢ he rounds to | 20¢ | 57¢ he rounds to | 60¢ | Did he get tipped more |
| 38¢ he rounds to | 40¢ | 89¢ he rounds to | 90¢ | on Saturday or |
| 92¢ he rounds to | 90¢ | 31¢ he rounds to | 30¢ | on Monday? |
| 154¢ or $1.54 | ? 150¢ or $1.50 | 177¢ or $1.77 | ? 180¢ or $1.80 | |

2. Use Sam's rounding method to estimate the tips for the other days of the week. (Sums are for problem 3.)

| Tues. | | Wed. | | Thurs. | | Fri. | | Sun. | |
|---|---|---|---|---|---|---|---|---|---|
| 59¢ | 60¢ | 57¢ | 60¢ | 62¢ | 60¢ | 89¢ | 90¢ | 65¢ | 70¢ |
| 12¢ | 10¢ | 31¢ | 30¢ | 79¢ | 80¢ | 43¢ | 40¢ | 55¢ | 60¢ |
| 100¢ | 100¢ | 88¢ | 90¢ | 16¢ | 20¢ | 90¢ | 90¢ | 41¢ | 40¢ |
| 171¢ or $1.71 | 170¢ or $1.70 | | | 157¢ or $1.57 | 160¢ or $1.60 | 222¢ or $2.22 | 220¢ or $2.20 | 28¢ | 30¢ |
| | | | | | | | | 189¢ or $1.89 | 200¢ or $2.00 |

3. Add the tips for each day of the week. See problems 1, 2

4. What day did he collect the most money in tips? How much more than on Monday? than on Thursday? than on Sunday? Friday; 68¢; 65¢; 33¢

5. Did Sam get tipped more on Friday, Saturday, and Sunday than he did on Monday through Thursday? Yes

3

**goal** **Survey**—ability to estimate and find sums with three and four 2-digit addends

**page 3** Interviews with newsboys indicate people have a tendency to give loose change as tips. There apparently is a tendency to give larger tips when the weather is bad, and right before the Christmas holidays. We hear that January, however, is a lean month. Check to see whether you have a pupil who delivers newspapers. What are his experiences?

Focus on why Sam rounds the amount of money he gets before he checks out his hunch. You may want to get into the notation for money (150¢ or $1.50). If you do, keep the discussion informal because money will be used as an entry into the study of decimals in a later chapter. Discuss Sunday's 65¢ and 55¢ tips. How can these numbers be handled in estimating? You should get some good answers.

Results from these problems will help you identify pupils who are ready to skip pages and proceed to the Progress Check on page 7. Those who need additional help with rounding and estimating skills should continue on to page 4.

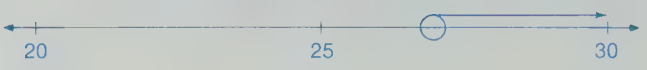
**lesson** Pages 4, 5, 6, 7

**goal** Examining rounding numbers on a number line

**page 4** The abilities of your students will determine whether the page can be completed independently. Problem 6 is important. The concept of **halfway** is essential when developing skill in rounding numbers. Ask what the word **halfway** means. Pupils may be better able to answer if you use an example—what number is halfway between 70 and 80? between 600 and 700?

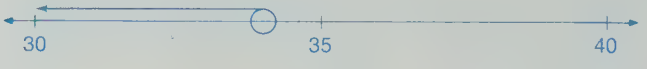
You have to round a number before you estimate.
But how do you round?

**1.** Round 27 to nearest *ten*.




THINK 27 is between 20 and 30.
27 is closer to 30 than to 20.
So round to 30.

**2.** Round 34 to nearest *ten*.

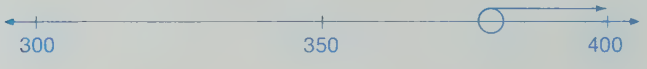



THINK 34 is between 30 and 40.
34 is closer to 30 than to 40.
So round to ?. 30

**3.** Round 55 to nearest *ten*.

THINK 55 is between 50 and 60.
55 is halfway.
So round to 60.

**4.** Round 380 to nearest *hundred*.




THINK 380 is between 300 and 400.
380 is closer to 400 than to 300.
So round to ?. 400

**5.** Round 145 to nearest *hundred*.

THINK 145 is between 100 and 200.
145 is closer to 100 than to 200.
So round to 100.

**6.** Round 550 to nearest *hundred*.

THINK 550 is between 500 and 600.
550 is halfway.
So round to ?. 600

1. Sam was rounding numbers to the nearest ten.
This number line shows multiples of 10. It will help you understand why he rounded up or down.

**a** 24 was rounded to 20 because 24 is closer to 20 than to 30.

**b** 38 was rounded to 40 because 38 is closer to 40 than to 30.

**c** 92 was rounded to 90 because 92 is closer to 90 than to 100.

**d** Is 59 closer to 50 or 60?

**e** Is 12 closer to 10 or 20?

**f** Is 31 closer to 30 or 40?

**g** Is 16 closer to 10 or 20?

**h** Is 65 closer to 60 or 70?
STOP! What was the agreement? Halfway numbers should be rounded up to the next ten.

**i** Why doesn't 20 or 50 or 90 need to be rounded?
They are already multiples of 10

2. Rounding is a skill needed to estimate answers.
Estimates help you make sure your answer is reasonable.

**a**
$$\begin{array}{rcl} 58 & \text{rounds to} & 60 \\ +\,20 & \text{---------} & +\,20 \\ ? & & 80 \end{array}$$
is a reasonable answer. Will the answer be more than 80 or less than 80? Why? What is the answer? 78
58 rounded up

**b**
$$\begin{array}{rcl} 53 & \text{rounds to} & 50 \\ +\,40 & \text{---------} & +\,40 \\ ? & & 90 \end{array}$$
is a reasonable answer. Will the answer be more than 90 or less than 90? Why? What is the answer? 93
53 rounded down

**c** $34 + 90 = ?$ Round and then estimate. 30 + 90 = 120
Will your answer be more than your estimate? Yes
Will it be less? No

5

**goal** Practice in rounding to the nearest 10; examining the relationship of estimated sums to computed sums

**memo** The ability to round numbers and estimate answers is developed for all four number operations and used throughout this program as a check on the reasonableness of a computed answer. The emphasis on pages 5, 6, and 7 is on estimating sums for this purpose.

**page 5** Check on pupils' understanding of **multiples of 10**. What signals a multiple of 10? (Zero in the ones place.) What signals a multiple of 100? (Zero tens, zero ones.)

Talk about the word **reasonable**. An estimate should be fairly close to the computed answer, provided the numbers have been rounded properly. Everyone should be happy to know that there is no such thing as a **wrong** estimate; however, some estimates are a lot better than others.

Knowing whether a computed answer will be more than or less than the estimate may not be obvious to all pupils. Number-line diagrams will help them understand how such a prediction can be made.

**goal** Practice in rounding to the nearest 10 when estimating sums

**memo** This is the first of many pages to have rows of computation problems. Recopying problems when you have the ability to work mentally is one of the first lessons in "how to hate math." Use the folded-paper technique to save everyone valuable learning time and help avoid errors made in recopying problems. The pupil puts a sheet of paper directly below a row of problems and then records the answers only. After completing one row, he folds the answers under, places the clean surface under the next row, records the answers for that row, and so on until the page is done.

Problems that require several steps should be recopied on paper. But urge your students to use the folded-paper technique whenever possible.

**page 6** Each problem extends the pupil's ability to round each number to be added and then estimate the sum. Estimation is a mental skill; therefore, only answers should be recorded. You might even consider asking that the estimate be given orally. The computed answer should be between two estimated multiples of 10 but closer to one of the multiples.

Pupils can recognize both the strengths and weaknesses of estimation by examining problems **1h**, **2b**, and **3f** as a group. Why doesn't rounding to the nearest multiple work well here?

**1.** Think before you answer.

**a** Is 27 + 20 closer to 40 or 50?

**b** Is 62 + 50 closer to 110 or 120?

**c** Is 60 + 22 closer to 80 or 90?

**d** Is 40 + 77 closer to 110 or 120?

**e** Is 46 + 30 closer to 70 or 80?

**f** Is 86 + 30 closer to 110 or 120?

**g** Is 90 + 17 closer to 100 or 110?

**h** Is 65 + 70 closer to 130 or 140?
Either, but we agreed to round up to 140.

**Remember?**
**That symbol > means**
**"is greater than"**

**2.** Complete each of the following.
Name the multiple of 10 that is nearest to the correct answer.

**a** 42 + 10 > ? 50 **b** 25 + 20 > ? 40

**c** 82 + 20 > ? 100 **d** 59 + 30 < ? 90

**e** 60 + 18 < ? 80 **f** 49 + 70 < ? 120

**3.** Both numbers will need rounding on these.
But it is not a hard job.

**a** Is 21 + 57 closer to 70 or 80?

**b** Is 33 + 61 closer to 90 or 100?

**c** Is 28 + 33 closer to 60 or 70?

**d** Is 48 + 69 closer to 110 or 120?

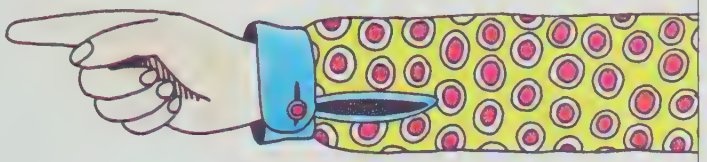
**e** Is 44 + 26 closer to 60 or 70?

**f** Is 35 + 45 closer to 80 or 90?

Estimate the answers for each.

| | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| **4.** | 35 + 40 = 80 | 60 + 27 = 90 | 70 + 18 = 90 | 57 + 30 = 90 | 81 + 50 = 130 | 92 + 63 = 150 | 49 + 74 = 120 |
| **5.** | 57 + 41 = 100 | 82 + 28 = 110 | 46 + 31 = 80 | 84 + 25 = 110 | 72 + 17 = 90 | 69 + 52 = 120 | 95 + 59 = 160 |

Are estimates
wild guesses
or can they predict
a reasonable answer?



1. Estimate the answers first. Then compute the exact answers. (Estimate in parentheses)

| a | b | c | d | e |
|---|---|---|---|---|
| 48 | 38 | 72 | 23 | 44 |
| +27 | +35 | +59 | +36 | +89 |
| (80) 75 | (80) 73 | (130) 131 | (60) 59 | (130) 133 |

How close to the exact answer was your estimate?

2. Copy and complete. Give the estimate to the nearest ten. Then compute.

| | Estimate | Exact |
|---|---|---|
| 46 + 37 | a 90 | b 83 |
| 49 + 38 | c 90 | d 87 |
| 53 + 44 | e 90 | f 97 |

| | Estimate | Exact |
|---|---|---|
| 56 + 47 | g 110 | h 103 |
| 62 + 53 | i 110 | j 115 |
| 66 + 57 | k 130 | l 123 |

## PROGRESS CHECK

Give the estimate to the nearest ten. Then compute. Skill: Estimating sums: adding two 2-digit numbers with renaming

| | Estimate | Exact |
|---|---|---|
| 13 + 26 | (1.) 40 | (2.) 39 |
| 19 + 37 | (3.) 60 | (4.) 56 |
| 21 + 42 | (5.) 60 | (6.) 63 |

| | Estimate | Exact |
|---|---|---|
| 27 + 46 | (7.) 80 | (8.) 73 |
| 32 + 52 | (9.) 80 | (10.) 84 |
| 36 + 57 | (11.) 100 | (12.) 93 |

7

See activity 1, page 27a.

See activity 2, page 27b.

**goal** Using estimates to check the reasonableness of a computed answer: **Progress Check**—estimating and computing sums for 2-digit addends

**page 7** There is no such thing as a **wrong** estimate, but it is possible to make a useless one. Would an estimate such as the following help determine the **reasonableness** of the exact answer?

$$\begin{array}{r} 46 \\ +37 \end{array} \quad \text{rounded to} \quad \begin{array}{r} 40 \\ +30 \\ \hline 70 \end{array}$$

The rules for rounding as developed on the preceding pages were not followed in this example. Pupils who are not able to estimate sums within the limits set in a problem should practice this skill, using a number line, before continuing.

You may want to hold the Progress Check for another day, depending on the results from the two practice problems. The Progress Check should be completed independently. Use the following guidelines to help you meet individual needs:

- Perfect score—proceed directly to page 9.
- Renaming errors in addition—go to page 8.
- Errors in rounding and estimation—provide further practice in rounding, using a number line. Try a quick oral drill to check the pupil's ability to add multiples of ten—30 + 40, 50 + 70, and so on. These problems will also indicate those who have not mastered the addition facts.

**lesson** Page 8

**goal** Review of renaming in addition

**memo** Use this page only with pupils who are experiencing difficulty with renaming in addition.

**page 8** Examine the work of the four boys together. Could estimating have helped the boys catch their mistakes? Which answer is the only **reasonable** one? Verbalizing the error made in answer to problems 1, 2, and 3 is difficult for many adults—not to mention youngsters. The errors shown are sometimes made by pupils who have not learned the algorithm. They feel that this is the correct way to compute the answer and must be led to see their error. Problem 4 should help to correct the situation.

Have the pupils complete row 5 independently. Use row 6 only with those students who need further practice. You may want to use peer tutors with these youngsters.

Four boys were given the problem $39 + 57 = \underline{?}$ as homework. Their answers were all different. Estimate the answer to see which is probably correct. 40 + 60 = 100; Sam is probably correct.

| Ralph | Simon | Steven | Sam |
|---|---|---|---|
| 39 | 39 | 139 | 39 |
| +57 | +57 | +57 | +57 |
| 816 | 86 | 186 | 96 |

1. What was Ralph's mistake? Wrote 16 instead of renaming
   What must he remember to do? Rename when needed
2. What help does Simon need? Help in remembering to add renamed ten with others
3. Where did Steven make his mistake? Renamed 10 ones as 1 hundred
4. Sam's answer was correct.
   He computed the answer in two steps.

| Problem | Thought Step 1 | Thought Step 2 |
|---|---|---|
| | tens \| ones | tens \| ones |
| 39 | 3 9 | 3 9 |
| +57 → | +5 7 → | +5 7 |
| | 6 | 96 |

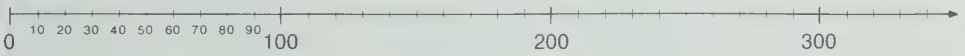
Step 1. Sam knew that $9 + 7 = 16$. He knew that 16 was 1 ten and 6 ones. He wrote the 6 in the ones column and the 1 at the top of the tens column.

Step 2. He added 1 ten, 3 tens, and 5 tens.

Copy these problems. If you estimate your answers before you compute, you will not make the kinds of errors that Ralph and Steven did. Try to think as Sam did.

| | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| 5. | 29 | 17 | 75 | 53 | 46 | 68 | 82 |
| | +38 | +67 | +15 | +29 | +36 | +26 | +15 |
| | 67 | 84 | 90 | 82 | 82 | 94 | 97 |
| 6. | 82 | 75 | 84 | 33 | 56 | 69 | 97 |
| | +19 | +36 | +77 | +99 | +34 | +48 | +55 |
| | 101 | 111 | 161 | 132 | 90 | 117 | 152 |

We can round hundreds, too.
Use the number line to help you answer the question.




**1.** Is 110 closer to 100 or 200?
Is 220 closer to 200 or 300?
Estimate the sum of 110 + 220.

$$\begin{array}{rl} 110 & \text{rounds to } 100 \\ +\,220 & \text{rounds to } +\,200 \\ \hline & 300 \text{ ? estimate} \end{array}$$

Will the sum be > or < the estimate?

**2.** Is 240 closer to 200 or 300?
Is 170 closer to 100 or 200?
Estimate the sum of 240 and 170.

$$\begin{array}{rl} 240 & \text{rounds to } 200 \\ +\,170 & \text{rounds to } +\,200 \\ \hline & 400 \text{ ? estimate} \end{array}$$

Will the sum be > or < the estimate?
Accept either > or < if reason is given

**3.** Answer these.

**a** Is 260 + 200 closer to 400 or 500?
**b** Is 310 + 420 closer to 700 or 800?
**c** Is 420 + 260 closer to 600 or 700?
**d** Is 495 + 160 closer to 600 or 700?
**e** Is 332 + 521 closer to 800 or 900?
**f** Is 167 + 221 closer to 300 or 400?

**4.** Estimate the sums.

**a** 500 + 400 900 **b** 750 + 150 1000 **c** 220 + 740 900 **d** 180 + 190 400 **e** 290 + 570 900

9

**lesson** Pages 9, 10, 11

**goal** Extension of rounding and estimating skills to hundreds

**page 9** Review naming the multiples of 100 just so everyone has them firmly in mind. Once again, **halfway** is an important concept. What number is halfway between 300 and 400? Try several more examples to check understanding.

Your pupils' performance in the review will tell you whether they can proceed independently or whether they need your guidance. Success with problem 3 is prerequisite to completing problem 4.

**goal** Application of rounding and estimating skills to two 3-digit addends

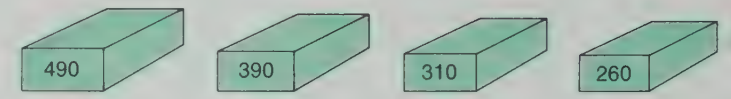
**page 10** Problem 1 should provoke some interesting discussion—especially parts **g** and **h**.

Encourage your pupils to use the folded-paper technique when computing answers for problems 2 and 3. Emphasize that estimates should always be made mentally and only the estimated sum recorded.

You will want to check for two skills:
- Ability to round and estimate
- Ability to add with renaming

Then use these guidelines to help you meet individual needs:
- No errors computing exact answers—proceed to page 12.
- Errors computing exact answers—go to page 11 for help with renaming.
- Errors in rounding and estimating—more work on a number line identifying **halfway.**

**1.** Jane promised she would get about 600 advertising flyers delivered to the store. She went to pick them up at the printer's and found they had been put into boxes. The quantity was on the label of each box. Which boxes should she take to the store? 390 and 260 or 310 and 260




**a** 490 + 390 ? 600 (>)

**b** 490 + 310 ? 600 (>)

**c** 490 + 260 ? 600 (>)

**d** 390 + 310 ? 600 (>)

**e** 390 + 260 ? 600 (<)

**f** 310 + 260 ? 600

**g** Did she take more than or less than 600? Did she have a good second choice about which boxes she could take? What other choices did she have? More, unless she took f. Yes. Any other combination except f

**h** If she had wanted exactly 600, what would she have had to do? Take out part of a box until she had exactly 600, or add 30 more to combination f

**i** If she had taken all the boxes, about how many hundred would she have? About 15 hundred

Round to the nearest hundred.
Then estimate the answers.
(Estimate in parentheses)

**2.**

| a | b | c |
|---|---|---|
| 420 + 330 | 640 + 190 | 160 + 480 |
| (700) 750 | (800) 830 | (700) 640 |

| d | e | f |
|---|---|---|
| 280 + 260 | 461 + 942 | 722 + 275 |
| (600) 540 | (1400) 1403 | (1000) 997 |

**3.**

| a | b | c |
|---|---|---|
| 620 + 290 | 430 + 540 | 670 + 720 |
| (900) 910 | (900) 970 | (1400) 1390 |

| d | e | f |
|---|---|---|
| 560 + 790 | 870 + 240 | 891 + 511 |
| (1400) 1350 | (1100) 1110 | (1400) 1402 |

**4.** Find the exact answers for either problem 2 or problem 3.

10

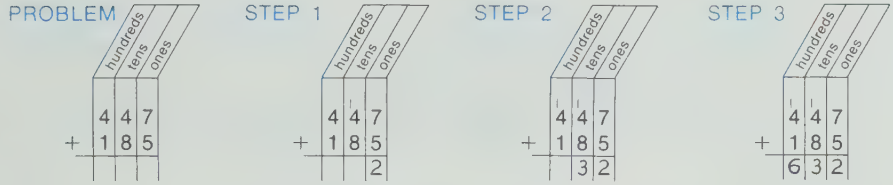
Several mistakes were made on one of the class's homework problems.
The problem was 447 + 185 = ?
What is your estimate of the answer?

600

| Cecile | Rozanne | Simone | Babs |
|---|---|---|---|
| 447 | 447 | 447 | 447 |
| +185 | +185 | +185 | +185 |
| 51312 | 1532 | 522 | 632 |

1. What was Cecile's mistake?
   Didn't rename 12 ones and 13 tens
2. What should she have done?
   Correctly renamed ones, tens, and hundreds
3. Where did Rozanne go wrong?
   Renamed 10 tens as 1 thousand
4. Who was right—Simone or Babs?

She worked the problem in three steps.




STEP 1 7 + 5 = 12. Rename 12 as 1 ten and 2 ones.
Write the 2 in the ones column and
the 1 at the top of the tens column.

STEP 2 Add the tens. 1 ten + 4 tens + 8 tens = 13 tens.
Rename 13 tens as 1 hundred and 3 tens.
Write the 3 in the tens column, and
the 1 at the top of the hundreds column.

STEP 3 Add the 1 hundred, the 4 hundreds, and the 1 hundred.

Copy and compute. Try to think as Babs did.

| 5. | 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|---|
| 148 | 289 | 639 | 298 | 628 | 262 |
| +364 | +516 | +148 | +736 | +492 | +879 |
| 512 | 805 | 787 | 1034 | 1120 | 1141 |

11

**goal** Review of two 3-digit addends requiring renaming

**memo** Use this page only with pupils who are experiencing difficulty with renaming in addition.

**page 11** The same technique was used before on page 8. Use this page as you did the other. Emphasize how an estimate could have helped catch an unreasonable answer. Straighten out those errors. Watch for the pupil who thinks there is no error. He may be making the same mistake. Investigate whether any pupils have a different way of computing the answer.

After problems 5 through 10 are done, have pupils exchange and correct each other's work. Let them discuss the errors they found. Assign a peer tutor to anyone who had too many errors. Flash-card drill with the addition facts may be necessary for some individuals.

**lesson** Pages 12, 13

**goal** Diagnosis of ability in and help with addition requiring renaming

**memo** You will find many pages similar to this one. The page has a short series of problems designed to identify skill competency, the answers to those problems, a chart directing the pupil what to do next, and special help for those in trouble.

Because the answers to the diagnostic problems are in plain sight, you may want to discuss the notion of cheating. If the pupils know that you do trust them and if they understand that simply copying the answers is an extremely short-term gain, then these pages can be a help to everyone.

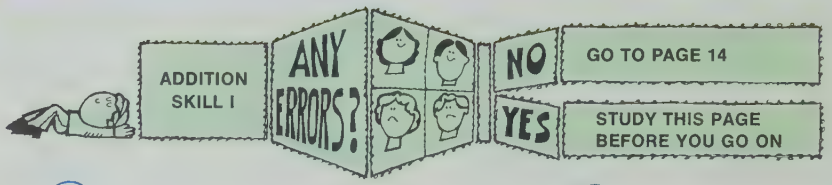
**page 12** The procedure to follow is explained right on the page. The diagnostic check directs those who make errors to the additional help given on the page. The others are directed to page 14. The pedagogy is to review and check out prerequisite skills as quickly as possible—then on to the new concepts and skills for this level.

## Time out to check your skills in addition

DO THESE THREE PROBLEMS

| a | b | c |
|---|---|---|
| 436<br>+218<br>654 | 34<br>17<br>+25<br>76 | 7<br>6<br>5<br>+3<br>21 |

Check your answers with the key on the next page.




**1** 436 + 218 = 4 (carry 1 + 4 = 14). 6 + 8 is 14.

436 + 218 = 54

436 + 218 = 654

**2** 34 + 17 + 25 = 6 (carry 1 + 6 = 16). 4 + 7 is 11. 11 + 5 is 16.

34 + 17 + 25 = 76

Now for the tens—
1 + 3 is 4.
4 + 1 is 5.
5 + 2 is 7.

**3** 7 + 6 + 5 + 3 = 21

Add two numbers at a time.
7 + 6 is 13.
13 + 5 is 18.
18 + 3 is 21. Write 21.

**4** 21 + 57 + 43 + 68 = 189

Add the ones column first.
1 + 7 is 8.
8 + 3 is 11.
11 + 8 is 19. Write 9 in the ones column.
Remember to add that 1 ten as you add the tens column.

12

# PROGRESS CHECK

ADD.
(Remember to estimate to prevent careless mistakes.)

Skill: Adding more than two 1-digit numbers

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| 3<br>4<br>+7<br>14 | 9<br>2<br>2<br>+4<br>17 | 3<br>6<br>2<br>+5<br>16 | 4<br>7<br>5<br>+9<br>25 |

Skill: Adding three 2-digit numbers

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| 21<br>42<br>+19<br>82 | 13<br>28<br>+40<br>81 | 59<br>17<br>+18<br>94 | 24<br>15<br>+27<br>66 |

Skill: Adding four 2-digit numbers

| 9. | 10. | 11. | 12. |
|---|---|---|---|
| 40<br>13<br>19<br>+17<br>89 | 24<br>19<br>18<br>+15<br>76 | 13<br>24<br>15<br>+28<br>80 | 36<br>15<br>19<br>+24<br>94 |

Skill: Adding two 2-digit numbers

| 13. | 14. | 15. | 16. |
|---|---|---|---|
| 32<br>+39<br>71 | 57<br>+18<br>75 | 73<br>+18<br>91 | 27<br>+46<br>73 |

Skill: Adding two 3-digit numbers

| 17. | 18. | 19. | 20. |
|---|---|---|---|
| 353<br>+218<br>571 | 427<br>+133<br>560 | 269<br>+458<br>727 | 732<br>+189<br>921 |

Go on to the next page if you have time.

13

See activity 3, page 27b.

See activity 4, page 27b.

**goal** **Progress Check**—skill in addition requiring renaming

**page 13** Every problem in each row requires exactly the same skill. Each row tests a different skill. But you will still find that there are three main reasons for errors:

- Addition facts not mastered
- Lack of understanding of renaming
- Inability to remember partial sums when adding columns

Provide tutors for pupils who need to work on addition facts. A reward system works wonders with these youngsters—points earned for time to play a special game, or whatever.

Pupils who are still having trouble with renaming after the review on the preceding pages need your help. Check for understanding of place value. Manipulatives such as bundles of 10 are not out of the question.

Encourage the pupil who has trouble remembering the partial sums in a column of numbers to write them to one side. It is important to develop a technique to find the correct answer even if it's not the same procedure that everyone else uses. You may want to put your answer key in a place where your pupils can use it. Learning to be critical of one's own work is valuable.

**lesson** Pages 14, 15

**goal** Diagnosis of ability in and help with advanced addition skills

**page 14** The problems on this page are rigged. They involve renaming at every place-value position. This computation should be easier than it would be if renaming were not consistently required.

Do these two addition problems.

| | | |
|---|---|---|
| | 3684 | 478 |
| | +2498 | 251 |
| **a** | 6182 | +639 |
| **b** | | 1368 |

Check your answers with the key on the next page.

**1.**

| | | | | |
|---|---|---|---|---|
| 3684 | 4 + 8 is 12. | 3684 | 3684 | 3684 |
| +2498 | (1)(2) | +2498 | +2498 | +2498 |
| 2 | | 82 | 182 | 6182 |

**2.**

| | | | |
|---|---|---|---|
| 478 | 8 + 1 is 9. | 478 | 478 |
| 251 | 9 + 9 is 18. | 251 | 251 |
| +639 | (1)(8) | +639 | +639 |
| 8 | | 68 | 1368 |

14

# PROGRESS CHECK

Add. (Estimates help prevent mistakes.) Skill: Adding two 3-digit numbers

| | | | | |
|---|---|---|---|---|
| 1. | 378 + 224 = 602 | 2. | 483 + 134 = 617 | 3. | 271 + 548 = 819 | 4. | 333 + 267 = 600 |
| Skill: Adding two 4-digit numbers 5. | 4173 + 3394 = 7567 | 6. | 6752 + 1586 = 8338 | 7. | 4520 + 2899 = 7419 | 8. | 1537 + 3416 = 4953 |
| 9. | 5487 + 2596 = 8083 | 10. | 3278 + 4956 = 8234 | 11. | 2107 + 3886 = 5993 | 12. | 4548 + 3667 = 8215 |
| Skill: Adding three 3-digit numbers 13. | 369 + 254 + 103 = 726 | 14. | 273 + 224 + 318 = 815 | 15. | 194 + 630 + 128 = 952 | 16. | 239 + 182 + 457 = 878 |
| 17. | 303 + 247 + 198 = 748 | 18. | 172 + 449 + 259 = 880 | 19. | 250 + 348 + 365 = 963 | 20. | 231 + 249 + 270 = 750 |

Go on to the next page if you have time.

15

See activity 4, variation 3, page 27b.

See activity 5, page 27b.

**goal** **Progress Check**—ability in advanced addition

**page 15** Once again, all problems in one row require the same skill. Check especially for pupils who have difficulty only with problems that do not involve renaming in every place-value position. Their papers may look like this:

| | | |
|---|---|---|
| 271 + 548 = 829 | 6752 + 1586 = 8348 | 1537 + 3416 = 5053 |

rather than this:

| | | |
|---|---|---|
| 271 + 548 = 819 | 6752 + 1586 = 8338 | 1537 + 3416 = 4953 |

These students are operating by rote. They learned **how** to show renaming and include the step whether it is needed or not. They still need to learn **when** to rename and **why**.

**lesson** Page 16

**goal** Providing practice requiring independent thinking

**page 16** The page is self-directive. Every student should have the opportunity to experience the fun side of mathematics. Provisions are made for pupils of all levels of ability. Use the page now or whenever time permits. Pages 25 and 26 provide similar opportunities.

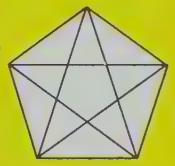
# Just for Fun

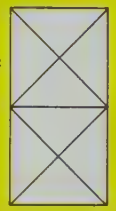
1. Arrange the digits 1 through 9 in such a way that when added the answer is 100. Use each digit only once.

```
 15
 36
+47
 98
÷ 2
100
```

2. Put a sheet of paper over the three shapes. Find out which you can trace without lifting your pencil and without going over any part more than once.

a

b 

c 

3. Most codes use letters or numbers to stand for other letters or numbers. This code is based on addition.

| ALPHABET | A | B | C | D | E | F | . . . | Z |
|---|---|---|---|---|---|---|---|---|
| CODE | 2 | 3 | ? | 8 | 13 | ? | . . . | 317,811 |

5 21

Here's a clue: A + C = 7.
Now decode this supersecret dispatch.

STATE DEPARTMENT
10,946 – 13 – 5 – 6765 – 13 – 17,711 *
610 – 89 – 10,946 – 10,946 – 89 – 1597 – 987 * 2 *
10,946 – 28,657 – 5 – 5 – 13 – 10,946 – 10,946!

SECRET MISSION A SUCCESS!

16

**1.** Sam delivers newspapers to 37 families in one building. There are 72 families living in the building. About how many families do not get papers from Sam?

$$\begin{array}{rl} 72 & \text{rounds to } 70 \\ -37 & \text{rounds to } -40 \\ & ? \quad 30 \end{array}$$

**2.** Sam delivers papers to several apartment buildings. Look at the table below. How many families in each building do not get papers from Sam?

| | Bldg. A | Bldg. B | Bldg. C | Bldg. D | Bldg. E |
|---|---|---|---|---|---|
| Families in all | 64 | 57 | 89 | 72 | 47 |
| Families who get papers from Sam | 43 | 19 | 32 | 49 | 29 |
| | (20) 21 | (40) 38 | (60) 57 | (20) 23 | (20) 18 |

**3.** Sam delivers papers to 172 families in buildings A, B, C, D, and E. The total number of families living in those buildings is 329. About how many families do not get papers from Sam?

$$\begin{array}{rl} 329 & \text{rounds to } 300 \\ -172 & \text{rounds to } -200 \\ & 100 \end{array}$$

**4.** Round to hundreds and estimate the difference.

| a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|
| 614 | 490 | 569 | 719 | 964 | 423 | 870 |
| −290 | −379 | −110 | −180 | −275 | −331 | −442 |
| 300 | 100 | 500 | 500 | 700 | 100 | 500 |

**5.** Copy and complete. Name the multiple of 10 that is nearest the exact answer. If estimation used, accept false statements.

**a** $47 - 30 > n$ 20 **b** $82 - 30 < n$ 50 **c** $66 - 20 < n$ 50 **d** $79 - 40 > n$ 40

**e** $99 - 40 < n$ 60 **f** $27 - 10 > n$ 20 **g** $84 - 60 < n$ 20 **h** $68 - 40 < n$ 30



**lesson** Pages 17, 18, 19

**goal** Application of rounding and estimating skills to the subtraction of 2-digit numbers

**page 17** A great page for discussion, with the focus on mentally rounding and estimating differences. This is how we use estimation most often in real life. Listen for clues that more work on the number line is necessary.

Watch out for those "greater than" and "less than" symbols in problem 5.

**goal** Practice in rounding to the nearest ten and estimating differences

**page 18** Each problem extends the pupil's ability to apply his skill in rounding to estimating differences. By now you know which pupils can proceed independently and which ones need more guidance.

Reinforce the idea that the estimate indicates the reasonableness of a computed answer.

These guidelines will help you meet individual needs:

- No errors in computing exact answers —proceed directly to page 20.
- Errors in computing exact answers— go to page 19 for help with renaming.

**1** You will have to round only one number to answer these.

**a** Is 47 − 20 closer to 30 or 20?
**b** Is 92 − 20 closer to 70 or 60?
**c** Is 70 − 46 closer to 30 or 20?
**d** Is 60 − 19 closer to 50 or 40?
**e** Is 80 − 57 closer to 30 or 20?
**f** Is 50 − 36 closer to 20 or 10?
**g** Is 140 − 77 closer to 70 or 60?
**h** Is 140 − 54 closer to 90 or 80?

**2** You will round both numbers to answer these.

**a** Is 54 − 32 closer to 30 or 20?
**b** Is 36 − 16 closer to 20 or 30?
**c** Is 73 − 48 closer to 30 or 20?
**d** Is 40 − 16 closer to 20 or 30?
**e** Is 97 − 42 closer to 50 or 60?
**f** Is 71 − 46 closer to 30 or 20?
**g** Is 122 − 78 closer to 50 or 40?
**h** Is 157 − 89 closer to 60 or 70?

Estimate the answer for each. (Estimate in parentheses)

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| **3** | 53 − 30 (20) 23 | 47 − 30 (20) 17 | 28 − 10 (20) 18 | 79 − 20 (60) 59 | 75 − 30 (50) 45 | 27 − 14 (20) 13 |
| **4** | 79 − 37 (40) 42 | 64 − 21 (40) 43 | 35 − 13 (30) 22 | 77 − 64 (20) 13 | 70 − 14 (60) 56 | 30 − 21 (10) 9 |
| **5** | 80 − 48 (30) 32 | 40 − 27 (10) 13 | 90 − 33 (60) 57 | 44 − 26 (10) 18 | 75 − 17 (60) 58 | 47 − 14 (40) 33 |

Find the exact answers to the exercises in any row above.
How close were your estimates to the exact answers?
All are < 9 from exact answer

18

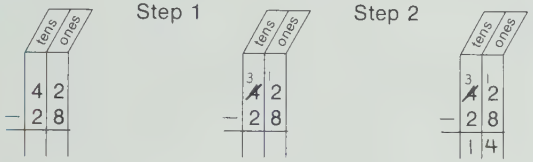
Jerry sells papers on the corner. One day he sold 42. The next day he sold only 28. He subtracted to find the difference. He made some mistakes before he finished.

**First try**

| | | | | | |
|---|---|---|---|---|---|
| 42 | He estimated | 42 | rounded to | 40 |
| −28 | the answer to | −28 | rounded to | −30 |
| 24 | check his work. | | | 10 |

The difference between 24 and his estimate of 10 was large enough to convince him that he had probably made a mistake.

**Second try**



Copy and subtract to find the exact answer. Rename as Jerry did.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | 42 | **2.** | 65 | **3.** | 74 | **4.** | 43 | **5.** | 81 | **6.** | 60 |
| | −27 | | −46 | | −39 | | −25 | | −38 | | −23 |
| | 15 | | 19 | | 35 | | 18 | | 43 | | 37 |

**7.** Write four subtraction problems that do not require renaming.
Accept problems in which top digit > subtracted digit.

**8.** Write four addition problems that do not require renaming.
Accept problems in which sum of each column < 10.

**9.** Write four subtraction problems that do require renaming.
Accept problems in which top digit < subtracted digit.

**10.** Write four addition problems that do require renaming.
Accept problems in which sum of one or more columns ≥ 10.

**11.** What is the relationship between the sizes of the two numbers in the ones place that makes renaming necessary in subtraction? in addition? Sum of two numbers ≥ 10
Top number < number subtracted

19

**goal** Review of renaming in subtraction

**memo** Use this page only with pupils who are experiencing difficulty with renaming in subtraction.

**page 19** Talk about the top of the page together. Review **when** and **how** to rename for subtraction. When problems 1 through 6 are completed, look for pupils who need individual help. Begin by checking out the subtraction facts, and then the algorithm for renaming.

Problems 7 through 11 should be completed independently.

Notice that the pupils are asked only to write problems, not to compute the answers. Writing problems to fit the requirements given may be quite demanding of some pupils. Do **not** force them to verbalize how they chose the numbers for the problems. They may not be ready to make this generalization. Another opportunity to figure out what's happening will be given on pages 21 and 22.

Exchange papers to determine whether each pupil did manage to find a set of four problems for each requirement. The best way to check, of course, is to find the answers and be aware of the thought processes required in computation.

**lesson** Pages 20, 21, 22

**goal** Application of rounding and estimating skills to subtraction of 3-digit numbers

**page 20** The criteria for a useful estimate is given at the top of the page. Reemphasize there are no wrong estimates, **but** an estimate must be close to be useful. Problem 3 reinforces the idea of using an estimate to check the reasonableness of the computed answer.

Talk about problem 4. Look for patterns to identify **when** renaming is necessary.

The next set of numbers to be subtracted will have three digits. If there are more digits, are there more patterns? fewer patterns? Will the patterns that signal renaming be the same or different? Find out. No No Same, except for one more column of digits

151 rounded to 150
−143 rounded to −140

–How "good" is this estimate? Within 10 of the answer
–Is it within ten greater or less than the accurate answer? Yes
–Would this estimate help prevent a silly mistake? Yes

**1** Subtract.

**a** 700 − 300 = ? 400 **b** 360 − 140 = ? 220 **c** 620 − 480 = ? 140

**d** 720 − 270 = ? 450 **e** 530 − 420 = ? 110 **f** 700 − 410 = ? 290

**2** Estimate the answers. (Estimate in parentheses)

| | | | | |
|---|---|---|---|---|
| **a** 437 − 217 (220) 220 | **b** 892 − 226 (660) 666 | **c** 659 − 382 (280) 277 | **d** 718 − 109 (610) 609 | **e** 435 − 213 (230) 222 |
| **f** 364 − 192 (170) 172 | **g** 736 − 525 (210) 211 | **h** 274 − 181 (90) 93 | **i** 465 − 177 (290) 288 | **j** 657 − 384 (280) 273 |

**3** Find exact answers to any four exercises. See above.
How "good" were your estimates?

**4** Was renaming sometimes needed in the ones place? Yes
Was renaming sometimes needed in the tens place? Yes
Was renaming sometimes needed in the tens place AND the ones place? Yes

20

Find the answers to these next problems. Your completed work should serve as a reference for the questions on this page.

```
  579
 −232
    ?   347
```
Did you rename ones, tens, or hundreds? No

```
  347
 +232
    ?   579
```

```
  682
 −154
    ?   528
```
Did you rename ones, tens, or hundreds? Yes – ones and tens

```
  528
 +154
    ?   682
```

```
  575
 −379
    ?   196
```
Did you rename ones, tens, or hundreds? Yes – ones, tens and hundreds

```
  196
 +379
    ?   575
```

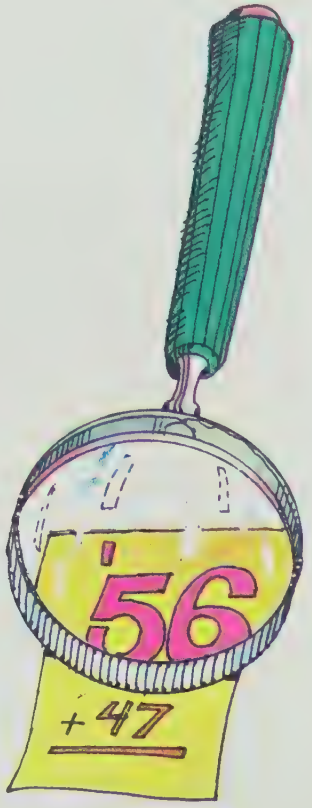
```
  600
 −436
    ?   164
```
Did you rename ones, tens, or hundreds? Yes – ones, tens and hundreds

```
  164
 +436
    ?   600
```

You may have to rename pairs of numbers no matter what their value. Would this be true if you had two 4-digit numbers to add or subtract? Yes
How about renaming with two 5-digit numbers? Yes
With two 6-digit numbers? Yes

Let's use the magnifying-glass routine to look just at the renaming. If you add any two 2-digit numbers, what is the largest number you will rename? Prove your answer.

```
  ¹
  99
+ 99
 198
```
18 is greatest number of ones to be renamed (9 + 9).
19 is greatest number of tens to be renamed (9 + 9 + 1).




21

**goal** Examining renaming patterns in addition and subtraction

**memo** The concepts presented on pages 21 and 22 are quite sophisticated. The main idea is to combine place-value concepts with number operations in order to simplify computation. When operating with only two numbers, no matter how small or how large, renaming in addition and subtraction follows the same pattern—the greatest quantity that can be renamed is:

10 ones to 1 ten or 1 ten to 10 ones
10 tens to 1 hundred or 1 hundred to 10 tens
10 hundreds to 1 thousand or 1 thousand to 10 hundreds
and so on.

**page 21** What signals the need to rename in subtraction? (The youngsters say, "I don't have enough to subtract.") In addition? ("The column added up to 10 or more.") Don't be surprised when someone points out the doing-undoing relationship of addition and subtraction. When renaming is necessary in subtraction, is it also necessary in the related addition problem? This may be a good time to talk about how addition is used to check subtraction.

**goal** Examining renaming patterns in addition and subtraction (*continued*)

**page 22** The focus is on **when** renaming in subtraction is necessary and **how many** to rename. Both these ideas are emphasized by examining the patterns.

The problem 366 − 67 displayed on the page is a classic example of a subtraction that looks so easy BUT . . . . Each renaming step is shown. Once you get started, that renaming never ends. First you rename a ten; then you have to rename a hundred also. You may want to put the problems 366 − 66 and 366 − 65 on the board to make the point of the lesson more obvious. Follow up with the problems 366 − 57, 366 − 56 and 366 − 55; the clues in the ones place apply to the tens place too.

Check specifically for pupils who have trouble with problem **5d**. Two approaches may be used to help:

$\begin{array}{r} {\scriptstyle 8\,9\,1}\\ \not{9}\not{0}0 \\ -\,737 \end{array}$ 90 tens renamed as 89 tens 10 ones

**or**

$\begin{array}{r} {\scriptstyle 9}\\ {\scriptstyle 8\not{9}\,1}\\ \not{9}\not{0}0 \\ -\,737 \end{array}$ 9 hundreds renamed as 8 hundreds 10 tens; 10 tens renamed as 9 tens 10 ones

The pupil's ability will determine the method he can best handle.

**1.** What single-digit numbers subtracted from 50 make renaming necessary? Numbers > 0

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| − 0 | − 1 | − 2 | − 3 | − 4 | − 5 | − 6 | − 7 | − 8 | − 9 |

**2.** What single-digit numbers subtracted from 51 make renaming necessary? Numbers > 1

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 |
| − 0 | − 1 | − 2 | − 3 | − 4 | − 5 | − 6 | − 7 | − 8 | − 9 |

**3.** What single-digit numbers subtracted from 52 make renaming necessary? > 2
from 53? from 54? from 55? > 5
> 3 > 4

**4.** In the example above, you must rename twice. How many tens must be renamed as ones? How many hundreds must be renamed as tens? Make up two more subtraction problems in which you have to rename twice. Then find the answers. 1 ten 1 hundred

Subtract.

**5.**

| a | b | c | d | e |
|---|---|---|---|---|
| 968 | 454 | 867 | 900 | 5280 |
| −756 | −129 | −768 | −737 | − 591 |
| 212 | 325 | 99 | 163 | 4689 |

Time out to check your skills in subtraction.

Do these two problems.

| | | | | |
|---|---|---|---|---|
| | 755 | | 631 |
| | −297 | | − 48 |
| **a** | 458 | **b** | 583 |

Check your answers with the key on the next page.

**1.**

$$\begin{array}{r} {\scriptstyle 4\ 1}\\ 7\not{8}5\\ -297\\ \hline 8 \end{array}$$

15 − 7 is 8.
But 4 − 9 cannot be done here.

*Rename* 7 hundreds becomes 6.
4 tens becomes 14.

$$\begin{array}{r} {\scriptstyle 6\ 14\ 1}\\ \not{7}\not{8}5\\ -297\\ \hline 8 \end{array}$$

$$\begin{array}{r} {\scriptstyle 6\ 14\ 1}\\ \not{7}\not{8}5\\ -297\\ \hline 58 \end{array}$$

Now work on tens.
14 − 9 is 5.
Write 5.

$$\begin{array}{r} {\scriptstyle 6\ 14\ 1}\\ \not{7}\not{8}5\\ -297\\ \hline 458 \end{array}$$

Finish with hundreds.
6 − 2 is 4.
Write 4.

**2.**

$$\begin{array}{r} {\scriptstyle 5\ 12\ 1}\\ \not{6}\not{3}1\\ -\ 48\\ \hline 83 \end{array}$$

Compute ones, then tens.
11 − 8 is 3. Write 3.
12 − 4 is 8. Write 8.

$$\begin{array}{r} {\scriptstyle 5\ 12\ 1}\\ \not{6}\not{3}1\\ -\ 48\\ \hline 583 \end{array}$$

And finish the hundreds.
Write 5.



**lesson** Pages 23, 24

**goal** Diagnosis of ability in and help with subtraction requiring renaming

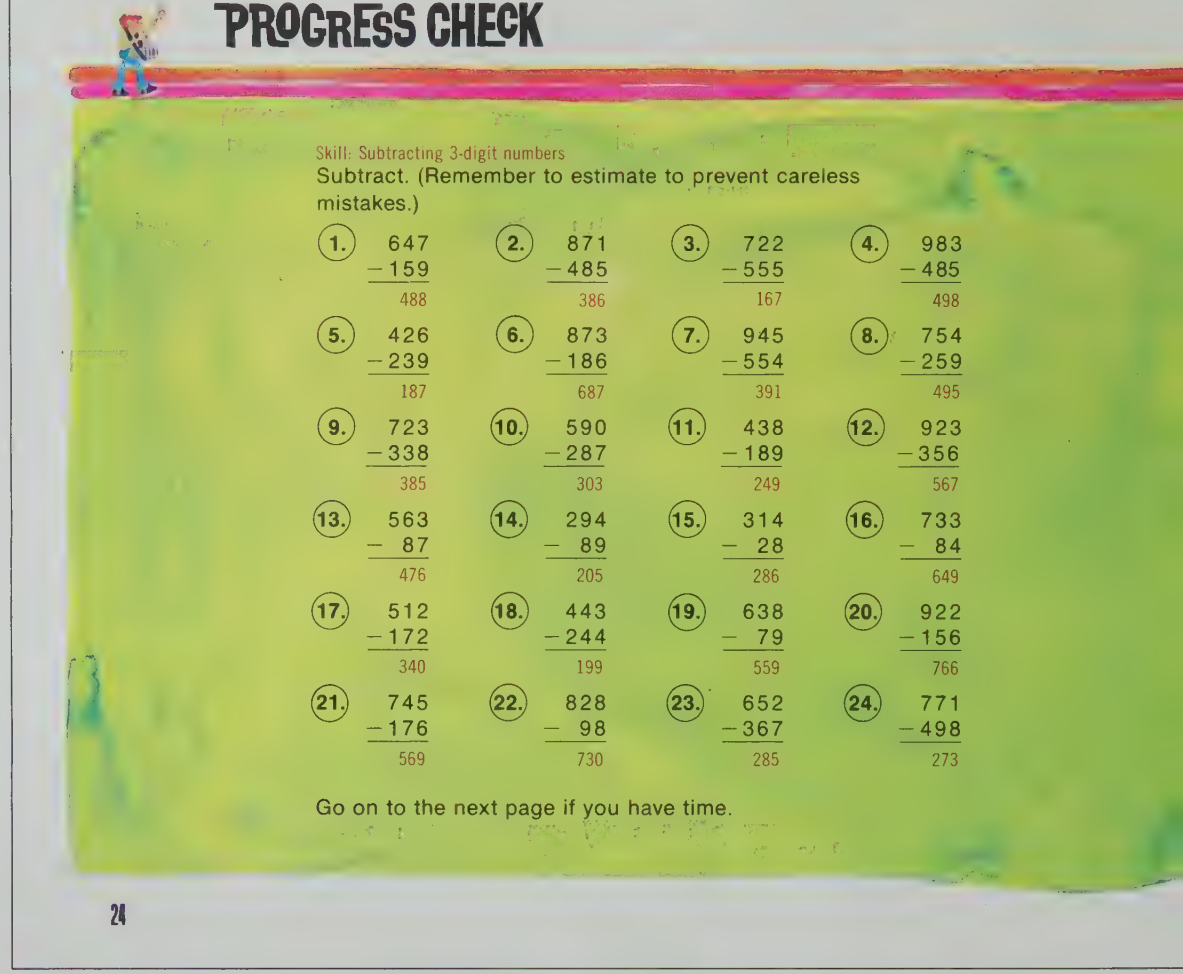
**page 23** Your pupils should be familiar with this type of page by now. Be ready to provide additional help or practice for those pupils who have trouble with renaming.

**goal** **Progress Check**—subtraction of 3-digit numbers requiring renaming

**page 24** More problems are provided than are necessary to check skill in subtraction. Consider using only part of the problems for the Progress Check. The remaining problems can be used for additional practice or retest after providing individual help.

Check errors for indications that the pupil has not yet mastered the subtraction facts. Peer tutors can help here.

Those who are still making errors in renaming need your attention.



# PROGRESS CHECK

Skill: Subtracting 3-digit numbers

Subtract. (Remember to estimate to prevent careless mistakes.)

| | | | |
|---|---|---|---|
| 1. 647 − 159 = 488 | 2. 871 − 485 = 386 | 3. 722 − 555 = 167 | 4. 983 − 485 = 498 |
| 5. 426 − 239 = 187 | 6. 873 − 186 = 687 | 7. 945 − 554 = 391 | 8. 754 − 259 = 495 |
| 9. 723 − 338 = 385 | 10. 590 − 287 = 303 | 11. 438 − 189 = 249 | 12. 923 − 356 = 567 |
| 13. 563 − 87 = 476 | 14. 294 − 89 = 205 | 15. 314 − 28 = 286 | 16. 733 − 84 = 649 |
| 17. 512 − 172 = 340 | 18. 443 − 244 = 199 | 19. 638 − 79 = 559 | 20. 922 − 156 = 766 |
| 21. 745 − 176 = 569 | 22. 828 − 98 = 730 | 23. 652 − 367 = 285 | 24. 771 − 498 = 273 |

Go on to the next page if you have time.

24

See activity 6, page 27b.

See activity 7, page 27c.

# Just for fun

Answers will vary. One example is shown:

1. **a** Pick any three different digits. Write them in order.
   **b** Reverse the order of the digits.
   **c** Subtract the smaller number from the larger number.
   **d** If there are only two digits in the answer, put a zero in front of them to form a 3-digit number.
   **e** Reverse the order of the digits in your answer.
   **f** Add these two numbers (the numbers in steps **c** and **e**).
   **g** Your answer will be 1089.

Example only

$$\begin{array}{r} 432 \\ -\ 234 \\ \hline 198 \end{array}$$

Skip for this example.

891

1089

Try 3-digit numbers and see if you can get an answer other than 1089.

Following steps a through g, you will always get 1089.

Try 4-digit numbers. Is the answer 10,890? Yes

2. Here's another code just for fun. This code is based on a pattern of subtraction.

| ALPHABET | A | B | C | D | E | F | G | H | I | J | . . . |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CODE | 260 | 250 | ? | ? | ? | ? | ? | ? | ? | ? | . . . |
| | | | 240 | 230 | 220 | 210 | 200 | 190 | 180 | 170 | |

Here's a clue: A − C = 20.
Now decode this spy message.

80 − 220 − 130 − 230 * 70 − 190 − 220 *
110 − 260 − 240 − 160 − 260 − 200 − 220 *
70 − 120 * 20 − 120 − 60 − 90 *
240 − 120 − 130 − 70 − 260 − 240 − 70

SEND THE PACKAGE TO YOUR CONTACT.



**lesson** Pages 25, 26

**goal** Providing activities requiring independent thinking

**page 25** Math can be fun! Right? So let your pupils enjoy it whenever time permits. Don't be fooled—there's lots of practice involved.

**goal** Examining a mental-arithmetic shortcut

**page 26** A challenge for sharpies.

## Computation Sidetrack

**Some people are very good at mental arithmetic.
They use all sorts of shortcuts.
Here's one. Maybe you'll want to use this idea.**

**Think about subtracting 9.** Another name for 9 is 10 − 1.
It's easier to subtract 10 and then add 1 to the answer.

47 − 9 = ? Think 47 − 10 = 37
37 + 1 = 38
So 47 − 9 = 38

56 − 9 = ? Think 56 − 10 = 46
46 + 1 = 47
So 56 − 9 = 47

**It works for subtracting 19 too.**

63 − 19 = ? Think 63 − 20 = 43
43 + 1 = 44
so 63 − 19 = 44

76 − 19 = ? Think 76 − 20 = 56
56 + 1 = 57
So 76 − 19 = 57

People who subtract nines this way usually add nines in much the same way.

47 + 9 = ? Think 47 + 10 = 57
57 − 1 = 56
So 47 + 9 = 56

83 + 9 = ? Think 83 + 10 = 93
93 − 1 = 92
So 83 + 9 = 92

Would this work for adding 19? 29? 39? 59? Yes

Could the idea be used for subtracting or adding 8? Yes
What changes would have to be made in the "Think" steps?
For subtraction: Subtract next higher multiple of 10; add 2.
For addition: Add next higher multiple of 10; subtract 2.

26

# CHECKOUT

Skill: Rounding to nearest ten

**1.** Round each number to the nearest ten.

**a** 41 40 **b** 56 60 **c** 79 80

**d** 35 40 **e** 64 60 **f** 16 20

Skill: Rounding to nearest hundred

**2.** Round each number to the nearest hundred.

**a** 125 100 **b** 156 200 **c** 179 200

**d** 935 900 **e** 550 600 **f** 779 800

Skill: Estimating sums and differences

**3.** Estimate. Watch out! You will check out both addition and subtraction.

**a** Is 570 + 230 closer to 700 or 800?

**b** Is 310 − 170 closer to 200 or 100?

**c** Is 460 − 215 closer to 200 or 300?

**d** Is 265 + 492 closer to 700 or 800?

Skill: Adding and subtracting 3-digit numbers

**4.** Compute. You will check out both addition and subtraction.

| | | | | | |
|---|---|---|---|---|---|
| **a** | 590 | **b** | 460 | **c** | 525 |
| | +230 | | −310 | | +387 |
| | 820 | | 150 | | 912 |
| **d** | 635 | **e** | 407 | **f** | 423 |
| | −595 | | +649 | | −367 |
| | 40 | | 1056 | | 56 |

27

STOP See activity 8, page 27c.

See activity 9, page 27c.

**lesson** Page 27

**goal** **Checkout**—rounding; estimating and computing sums and differences

**page 27** Youngsters who still have difficulty at this point will require individual help. You are but one person. Don't overlook the assistance of peer tutors after you have diagnosed the specific practice each pupil needs.

Praise those who have been having trouble and have finally caught on.

# RESOURCES

## another form of evaluation

**for progress check**—page 7

Give the estimate to the nearest ten. Then compute.

| | Estimate | Exact |
|---|---|---|
| 31 + 62 | 1. 90 | 2. 93 |
| 24 + 39 | 3. 60 | 4. 63 |
| 18 + 47 | 5. 70 | 6. 65 |
| 56 + 12 | 7. 70 | 8. 68 |
| 35 + 48 | 9. 90 | 10. 83 |
| 23 + 52 | 11. 70 | 12. 75 |

**for progress check**—page 13

Add.

| | | | |
|---|---|---|---|
| 1. 5<br>2<br>+6<br>13 | 2. 3<br>5<br>4<br>+7<br>19 | 3. 8<br>2<br>3<br>+2<br>15 | 4. 6<br>9<br>4<br>+7<br>26 |
| 5. 43<br>12<br>+37<br>92 | 6. 28<br>30<br>+14<br>72 | 7. 11<br>65<br>+19<br>95 | 8. 36<br>28<br>+24<br>88 |
| 9. 35<br>21<br>12<br>+16<br>84 | 10. 24<br>18<br>20<br>+19<br>81 | 11. 17<br>13<br>18<br>+35<br>83 | 12. 14<br>32<br>18<br>+26<br>90 |
| 13. 57<br>+38<br>95 | 14. 29<br>+64<br>93 | 15. 46<br>+25<br>71 | 16. 17<br>+39<br>56 |
| 17. 324<br>+456<br>780 | 18. 136<br>+727<br>863 | 19. 256<br>+686<br>942 | 20. 473<br>+158<br>631 |

**for progress check**—page 15

Add.

| | | | |
|---|---|---|---|
| 1. 243<br>+685<br>928 | 2. 165<br>+439<br>604 | 3. 397<br>+424<br>821 | 4. 254<br>+346<br>600 |
| 5. 6514<br>+1283<br>7797 | 6. 3492<br>+2547<br>6039 | 7. 5480<br>+3434<br>8914 | 8. 2853<br>+4273<br>7126 |
| 9. 1865<br>+7651<br>9516 | 10. 4748<br>+2655<br>7403 | 11. 3097<br>+5964<br>9061 | 12. 6756<br>+2498<br>9254 |
| 13. 428<br>294<br>+135<br>857 | 14. 276<br>205<br>+394<br>875 | 15. 135<br>248<br>+176<br>559 | 16. 210<br>389<br>+143<br>742 |
| 17. 612<br>156<br>+148<br>916 | 18. 426<br>174<br>+350<br>950 | 19. 381<br>326<br>+149<br>856 | 20. 507<br>185<br>+264<br>956 |

**for progress check**—page 24

Subtract.

| | | | |
|---|---|---|---|
| 1. 546<br>−278<br>268 | 2. 723<br>−459<br>264 | 3. 876<br>−277<br>599 | 4. 614<br>−368<br>246 |
| 5. 988<br>−293<br>695 | 6. 647<br>−268<br>379 | 7. 436<br>−189<br>247 | 8. 553<br>−364<br>189 |
| 9. 860<br>−496<br>364 | 10. 791<br>−294<br>497 | 11. 525<br>−178<br>347 | 12. 842<br>−396<br>446 |
| 13. 236<br>− 87<br>149 | 14. 864<br>− 97<br>767 | 15. 683<br>− 75<br>608 | 16. 456<br>− 98<br>358 |
| 17. 841<br>−251<br>590 | 18. 327<br>−189<br>138 | 19. 564<br>−367<br>197 | 20. 248<br>− 89<br>159 |
| 21. 956<br>−479<br>477 | 22. 427<br>− 97<br>330 | 23. 635<br>−468<br>167 | 24. 462<br>−273<br>189 |

**for checkout**—page 27

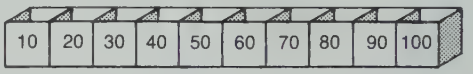
1. Round each number to the nearest ten.
   a) 53 50 b) 68 70 c) 45 50
   d) 39 40 e) 72 70 f) 17 20
2. Round each number to the nearest hundred.
   a) 286 300 b) 350 400 c) 423 400
   d) 845 800 e) 762 800 f) 638 600
3. Estimate. Watch out! You will check out both addition and subtraction.
   a) Is 360 + 220 closer to 500 or 600?
   b) Is 730 − 490 closer to 200 or 300?
   c) Is 546 + 263 closer to 700 or 800?
   d) Is 896 − 329 closer to 500 or 600?
4. Compute. You will check out both addition and subtraction.

| | | |
|---|---|---|
| a) 650<br>+280<br>930 | b) 590<br>−230<br>360 | c) 436<br>+387<br>823 |
| d) 943<br>−863<br>80 | e) 358<br>+407<br>765 | f) 827<br>−498<br>329 |

## activities

**1. things** 10 same-size boxes (milk cartons); slips of paper

Show a different multiple of 10 on one face of each box.




On another face show a multiple of 100 (100 through 1000). On a third face show a multiple of 1000 (1000 through 10,000); on the fourth, a multiple of 10,000 (10,000 through 100,000).

For practice in rounding to the nearest ten, turn the boxes to show the multiples of 10. Place them side by side in order. Write 2-digit numerals on the slips of paper, one per slip. The pupil is to select a slip, round the number to the nearest ten, and then place it in the appropriate box.

Change the set of numbers and turn the boxes to practice rounding to hundreds, thousands, ten thousands—whatever type of rounding practice is needed.

To practice estimating sums or differences write an appropriate problem on each slip of paper. Mix up the slips. The pupil selects a slip, rounds appropriately, mentally estimates the answer, and then places the problem in the box labeled with the estimated answer. For example:

$$\begin{array}{r} 37 \\ +29 \end{array} \quad \text{rounds to} \quad \begin{array}{r} 40 \\ +30 \end{array}$$

The problem is placed in the 70 box. Turn the boxes to accommodate the type of problem used for practice.

**2. things** game boards; index cards; markers; container

Have each player prepare his own game board consisting of a 5-by-5 array of squares. A multiple of 10 is then written in each square as follows:

- 1st column—less than 100
- 2d column—between 100 and 200
- 3d column—between 200 and 300
- 4th column—between 300 and 400
- 5th column—between 400 and 500

Write a whole number between 5 and 500 on each index card. Mix the cards in the container. To begin play, a card is drawn and shown to the players. Each player rounds the number to the nearest ten and covers that multiple of 10 with a marker if it is shown on his game board. The first player to cover five numbers in a row, column, or diagonal wins.

Variation: Practice rounding to hundreds by using multiples of 100 on the game boards and 3- and 4-digit numbers on the index cards.

**3. things** spirit master

Prepare a spirit master of grids as shown. The numbers are added across each row and down each column. The sums found are then added down and across. Both ways, the sum of the sums should be the same. This sum is written in the bottom right-hand corner.

| | | |
|---|---|---|
| 12 | 58 | (70) |
| 29 | 65 | (94) |
| (41) | (123) | (164) |

Omit the numbers on some of the grids so that each youngster can select his own and then challenge a friend.

**4. things** small cards; 2 small boxes

Pair players. Each player prepares a set of digit cards consisting of 2 cards for each digit from 0 through 9. He then puts his set of cards into his box.

Play begins with each player drawing 2 cards from his box and arranging a 2-digit number. The first player adds these two 2-digit numbers. The sum is his score for the round. The cards are returned to their respective boxes. The second player adds the next numbers drawn and arranged. Play continues to alternate. Each time the sum is added to the player's previous score.

Players must predetermine rules for the following:

- Total number of points needed to win
- Penalty for an incorrect sum

Some possible penalties:

- Player must subtract the correct sum from his total score.
- Player earns no points for that round.
- Player earns no points for that round; opponent adds the correct sum to his score.

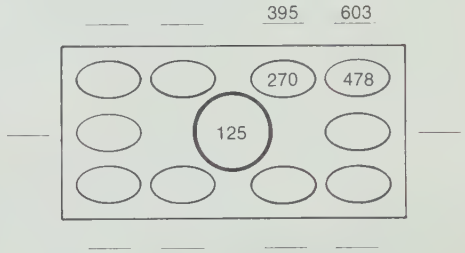
Variations:

1. Increase the number of cards for each digit to three. Each player draws 3 cards and arranges a 3-digit number.
2. Increase the number of cards for each digit to four. Each player draws 4 cards and arranges two 2-digit numbers. Players find the sum of four 2-digit numbers.
3. Increase the number of cards for each digit to four. Each player draws 4 cards and arranges a 4-digit number.

**5. things** spirit master

Prepare a spirit master of targets as shown. Omit the numbers. Since each target provides ten practice problems, you may want to cut the duplicated sheets apart and use only one target on any given day.

Have the pupils select a number to be written in the bull's-eye and a number for each oval on the target. A line is provided near each oval to record the sum of the number in the bull's-eye and in the oval.

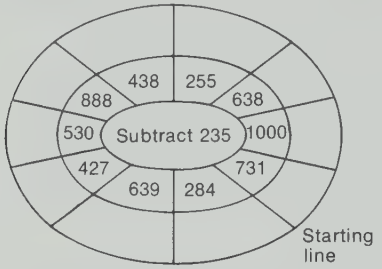



**6. things** spirit master (See page 27d.)

Prepare a spirit master of the racetrack shown. Omit the operation and numbers. These will be written in by the pupil to provide the type of practice needed.

For the example shown, 235 is subtracted from each number written in the **inner** track. The difference is recorded in the **outer** track.

If working for speed, you may want to impose a time limit. One point is earned for each correct difference found within the allotted time. Highest score wins.




**7.** Individual activity. (Provide the pupil with the following directions.)

Can you solve this Math-O-Gram? Replace each letter with one of the digits 0 through 9 to form a correctly computed addition problem. Begin by replacing the K with a 3.

```
    A
  MERRY
+  XMAS
 TURKEY
```

Rules:
1. All the *R*'s are the same digit, all the *E*'s are the same digit, and so on.
2. Two different letters may not be replaced by the same digit.

Solution:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| U | T | A | K | R | Y | X | E | S | M |

References containing more Math-O-Grams:

Brooke, Maxey. *One Hundred and Fifty Puzzles in Crypt-Arithmetic*. New York: Dover, 1972.

Dudeney, Henry E. *Five Hundred Thirty-Six Puzzles and Curious Problems*. New York: Scribner, 1967.

Hunter, James A., and Madachy, Joseph S. *Mathematical Diversions*. New York: Van Nostrand Reinhold, 1963.

Longley-Cook, L. H. *Work This One Out*. 2d ed. New York: Fawcett World.

**8. things** spirit master

Prepare a spirit master as shown. Include only the numbers not shown in parentheses. Make at least one more frame without numbers. Challenge the youngsters to select their own numbers for this frame.

| − | | | + |
|---|---|---|---|
| (39) | 87 | 48 | (135) |
| 18 | 53 | (71) | (124) |
| (157) | (322) | 165 | 487 |
| (115) | 276 | (391) | 667 |
| 71 | (500) | 429 | (929) |

The sum of the numbers in the two center columns is recorded under the plus sign. The difference is recorded under the minus sign.

**9.** Individual activity. (Provide the pupil with the following directions.)

Can you find the pattern? If so, name the next three numbers.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, (55), (89), (144)

How did you decide what these numbers should be? (Sum of the preceding two numbers)

These numbers are called Fibonacci numbers. Your job—find the answers to the following questions:
1. Why are these numbers called Fibonacci numbers?
2. How are they important?

# additional learning aids

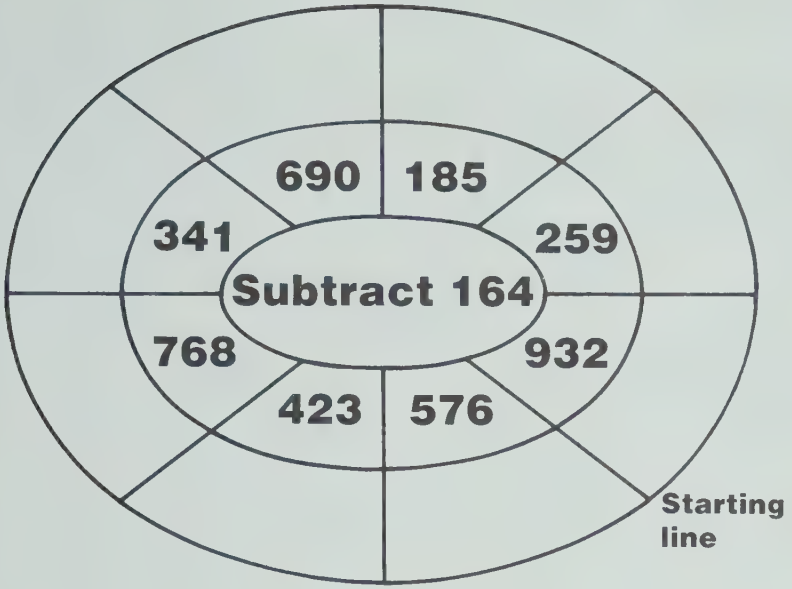
**operation**—chapter objectives 2, 3, 4, 5

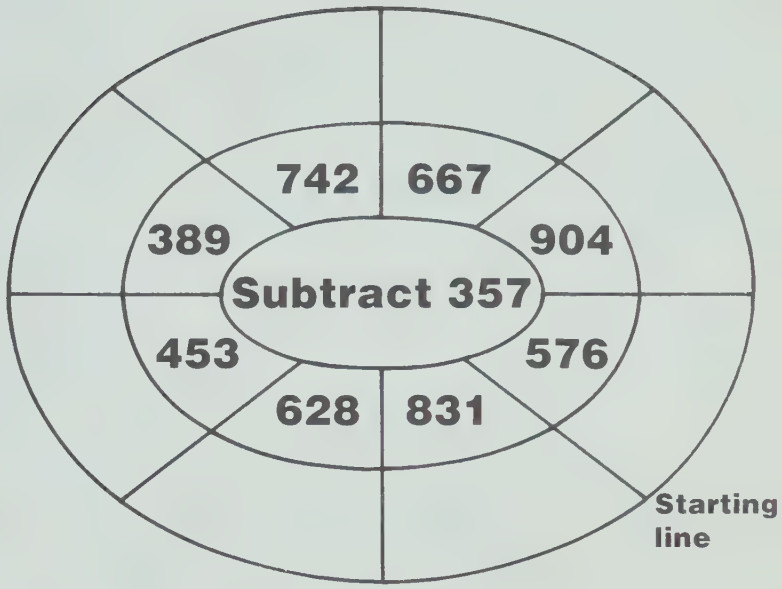
### SRA products

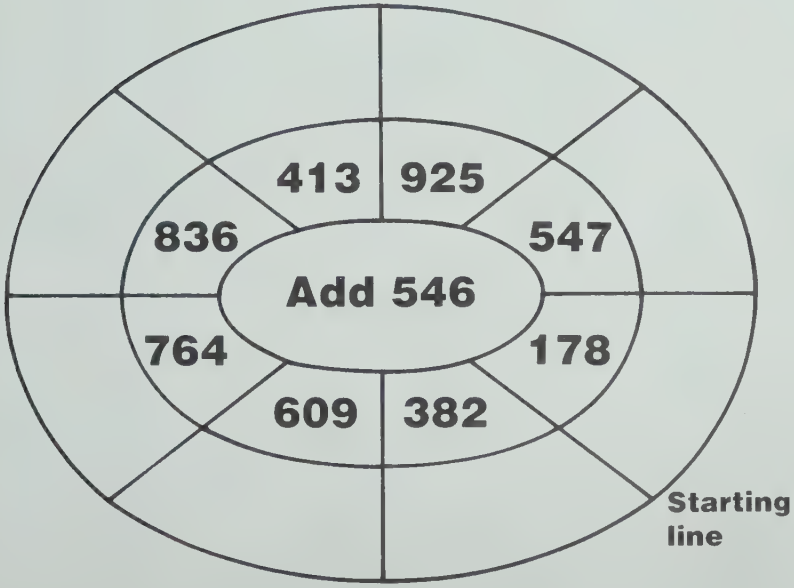
*Mathematics Learning System*,
*Activity Masters, level B*, SRA (1974)
Spirit masters: W-2, 4, 10, 11, 12, 17, 18, 35, 36
*Computapes*, SRA (1972)
Module 2, Lessons: AS 24, 35, 36
*Computational Skills Development Kit*, SRA (1965)
Addition cards: 1–17
Subtraction cards: 1–15
*Cross-Number Puzzles* (*Whole Numbers*), SRA (1966)
Addition cards: 1–21
Subtraction cards: 1–16
*Diagnosis*™*: an instructional aid—Mathematics Level B*, SRA (1972)
Probes: M-1, 2
*Math Applications Kit*, SRA (1971)
Appeteasers card: 5
Science cards: 38, 41, 42, 49
*Mathematics Involvement Program*, SRA (1971)
Card: 26
*Skill Modes in Mathematics*, SRA (1974)
Level I, Molecules: A, B
Level II, Molecule: A
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 2, 3, 5, 7, 8, 9, 24

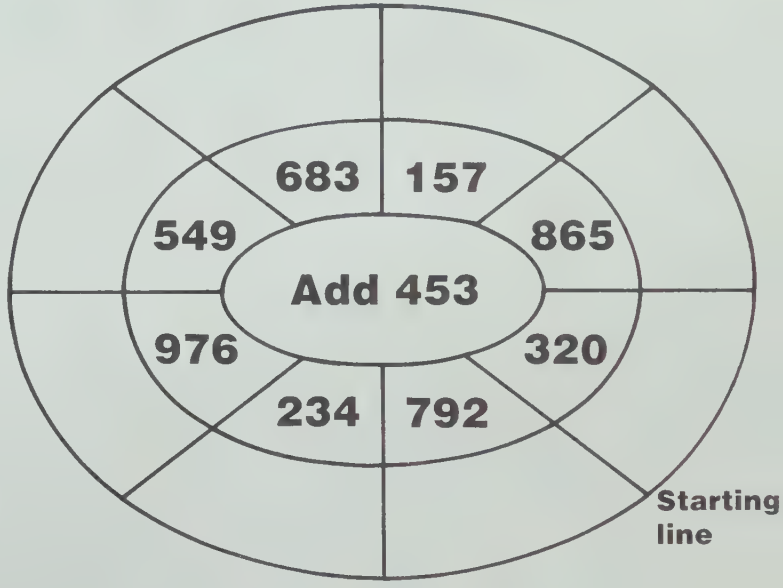
**other learning aids** (described on page 72j)—Chip Trading, Dial-A-Matic* Adding Machine, I Win (sets 1 and 2), Japanese Abacus, Orbiting the Earth (addition and subtraction), Sequence, Triscore, Veri-Tech Senior (addition and subtraction books)

*Registered trademark of Sigma Scientific, Inc.

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# 2 NUMBER RELATIONSHIPS AND OPERATIONS WITH × AND ÷

**before this chapter the learner has —**

1. Found the product for a 2-digit and a 3-digit factor
2. Divided, using a 1-digit or 2-digit divisor

**in chapter 2 the learner is —**

1. Checking his skills in multiplication and division
2. Practicing whole-number skills in all operations
3. Identifying number relations

**in later chapters the learner will —**

1. Apply his skills to computing with decimals
2. Master dividing by any 2-digit divisor
3. Apply his skills to solving word problems

# Notes & Things

Number relationships are used to review all the basic operations and focus on math sentences that describe the action. The last half of the unit is organized into a self-study learning format—a now-or-never comprehensive diagnosis of multiplication and division skills and opportunity for prescribed practice. The skills reviewed are necessary for success throughout the remainder of the year.

Once again you will find that there are many opportunities for individual progress and success. You will also be able to identify the range of pupils' strengths and weaknesses.

The technique used for reviewing each algorithm may come as a surprise. The kindest thing we can say about the technique used is that it is the "brute force" approach. Pupils who have not mastered the operations by now have probably had the attention of at least three teachers in earlier grades who tried to get the pupils to *understand* the operations. Lack of skill at this level indicates that past efforts just haven't worked. In this chapter the review of the algorithm is cut to the most essential operational steps, minimum reading, and maximum practice. There may even be too much practice. Select problems that are most appropriate for the individual. Above all, please do not assign all of any drill page at the risk of killing off personal pupil motivation. Be kind. Know that some of the pages would take forever to complete if all the problems had to be done.

Consider using peer tutors during the self-study sections of the unit. Having a buddy who genuinely wants to help can do more for the "slow" learner than any other technique available.

There is only one word of warning. You may find some pupils who have an intuitive understanding of the relationship between two sets of numbers and yet simply cannot clearly state the specific relationship. Be especially sensitive to these youngsters. Let their actions substitute for words.

This review chapter should be much more fun than the review chapter children have sometimes experienced. There are lots of secret codes to break and all computation is related to problems in the real world. Ideally, each pupil will come to appreciate that there is a reason for doing all these computational practices.

For the extra activities you will want to have these things available:

for each pupil: tongue depressors
10 same-size boxes (milk cartons)

**goal** Think about and explore ideas through a picture clue

**page 28** Heaven knows what objects are shown in the photograph but whatever they are, they are all the same. Because they are the same size and shape, they are probably manufactured. The notion of mass production is not new to anyone but a discussion of the cost per item and the related selling price may be new or at least an interesting place to start to think about multiplication and division. Use questions such as these:

- If you need only one part of a machine, would it cost more to make it by hand or by another machine? Why?
- If you need 10,000 parts, would it cost more to make the parts by hand or by another machine? Why? If you need 10,000,000 parts?
- How does a manufacturing plant decide how much to sell each part for?
- Would the selling price probably be the same if the plant made 10,000 parts as it would if it made 10,000,000 parts? Why or why not?

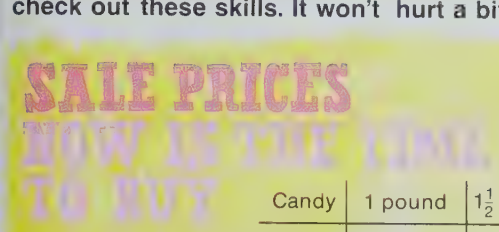
The first question will let you know who knows how volume relates to costs, and that last question will reveal who knows about the principle of supply and demand.

Establish a manufacturing cost for those unknown objects in the photograph and check mental arithmetic skills.

- If 1000 items were made at a cost of $1 each, how much would they cost all told?
- At that cost per item, how many parts would have been made if total costs were $500?

Change the cost (or the selling price) and ask more questions.

**You know what you know about addition and subtraction. What about your skills in multiplication and division? Your goal is to check out these skills. It won't hurt a bit!**




| Candy | 1 pound | $1\frac{1}{2}$ pounds | 2 pounds | 3 pounds | 4 pounds | 10 pounds |
|---|---|---|---|---|---|---|
| Price | $.44 | $.66 | ? $.88 | ? $1.32 | ? $1.76 | ? $4.40 |

The amount you pay depends on how much you buy. If you buy 2 pounds, will this math sentence fit the situation? Yes
Let $m$ stand for the money you pay. $m = 2 \times \$.44$

What sentence would fit if you bought 3 pounds? 4 pounds? $m = 3 \cdot \$.44$; $m = 4 \cdot \$.44$

**1.** Go back and complete the chart.

The candy had to be packaged before the sale. There were 16 pieces in each pound bag.

The number of bags depends on the number of pieces. You have 48 pieces. Let $b$ stand for the number of bags. Will this sentence fit the situation? $b = 48 \div 16$ Yes

| Number of pieces | 16 | 48 | 80 | 128 | 176 | 320 |
|---|---|---|---|---|---|---|
| Number of bags | 1 | 3 | ? 5 | ? 8 | ? 11 | ? 20 |

**2.** Copy and complete the chart.



**lesson** Page 29

**goal** **Survey**—multiplication and division skills

**page 29** Problem 1 will identify the pupil's ability in multiplying a 2-digit factor by a 1-digit factor and by 10. Problem 2 identifies his ability in dividing 2- and 3-digit numbers by a 2-digit divisor. These are prerequisite skills for an intensive review and check on more advanced skills throughout the chapter. Check anyone who has trouble here on mastery of multiplication facts. Additional practice is provided on the next several pages.

**lesson** Pages 30, 31

**goal** Finding number patterns formed by number relationships

**memo** The codes found on page 30 are used to decode the messages on page 31.

**page 30** The idea of codes and how they work will probably need to be discussed. Are there other ways of writing codes besides using numbers? (Scrambled words, special meanings for specific words ) Consider whether a lock combination is a code. Talk about situations in which a code might be used.

The key words in the introduction at the top of the page are **depends on**. Make sure everyone understands the procedure before beginning. Much computation is involved in completing these codes. Some energy can be conserved by writing the alphabet down the left-hand edge of a piece of lined paper, then folding a column for each code. You will need to encourage some of the pupils; and if there are some real slowpokes, you may want them to work together and split the job.

The messages on page 31 cannot be decoded unless the codes are completed accurately. Have volunteers read one of their codes so there is agreement before going on to the decoding job.

## Here is a warm-up

Have you ever sent a message to a friend in code? Codes are used in serious business too. There are some people who earn their living breaking codes and decoding messages. The alphabet is frequently coded into number patterns. The message depends on finding the number patterns and breaking the code.

1. Here's an easy code. What is the number pattern? Add 2 to the preceding number.
Complete the code for the whole alphabet.

| ALPHABET | A | B | C | D | E | F | G | H | I | J | K | L | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CODE I | 1 | 3 | 5 | 7 | 9 | ?11 | ?13 | ?15 | ?17 | ?19 | ?21 | ?23 | ... * |

2. Try this one. What is the number pattern? Increase number added by 1.
Finish the code for the whole alphabet.

| ALPHABET | A | B | C | D | E | F | G | H | I | J | K | L | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CODE II | 1 | 2 | 4 | 7 | 11 | ?16 | ?22 | ?29 | ?37 | ?46 | ?56 | ?67 | ... ** |

3. Here's another. What is the number pattern? Subtract 2 from the preceding number.
Finish the code for the whole alphabet.

| ALPHABET | A | B | C | D | E | F | G | H | I | J | K | L | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CODE III | 52 | 50 | 48 | ?46 | ?44 | ?42 | ?40 | ?38 | ?36 | ?34 | ?32 | ?30 | ... *** |

4. This one is a real challenge. (Hint: Two operations are used.)
Try to complete this code for the whole alphabet. Multiply first number by 2; subtract 1 for next number.

| ALPHABET | A | B | C | D | E | F | G | H | I | J | K | L | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CODE IV | 2 | 4 | 3 | 6 | 5 | 10 | 9 | ?18 | ?17 | ?34 | ?33 | ?66 | ... **** |

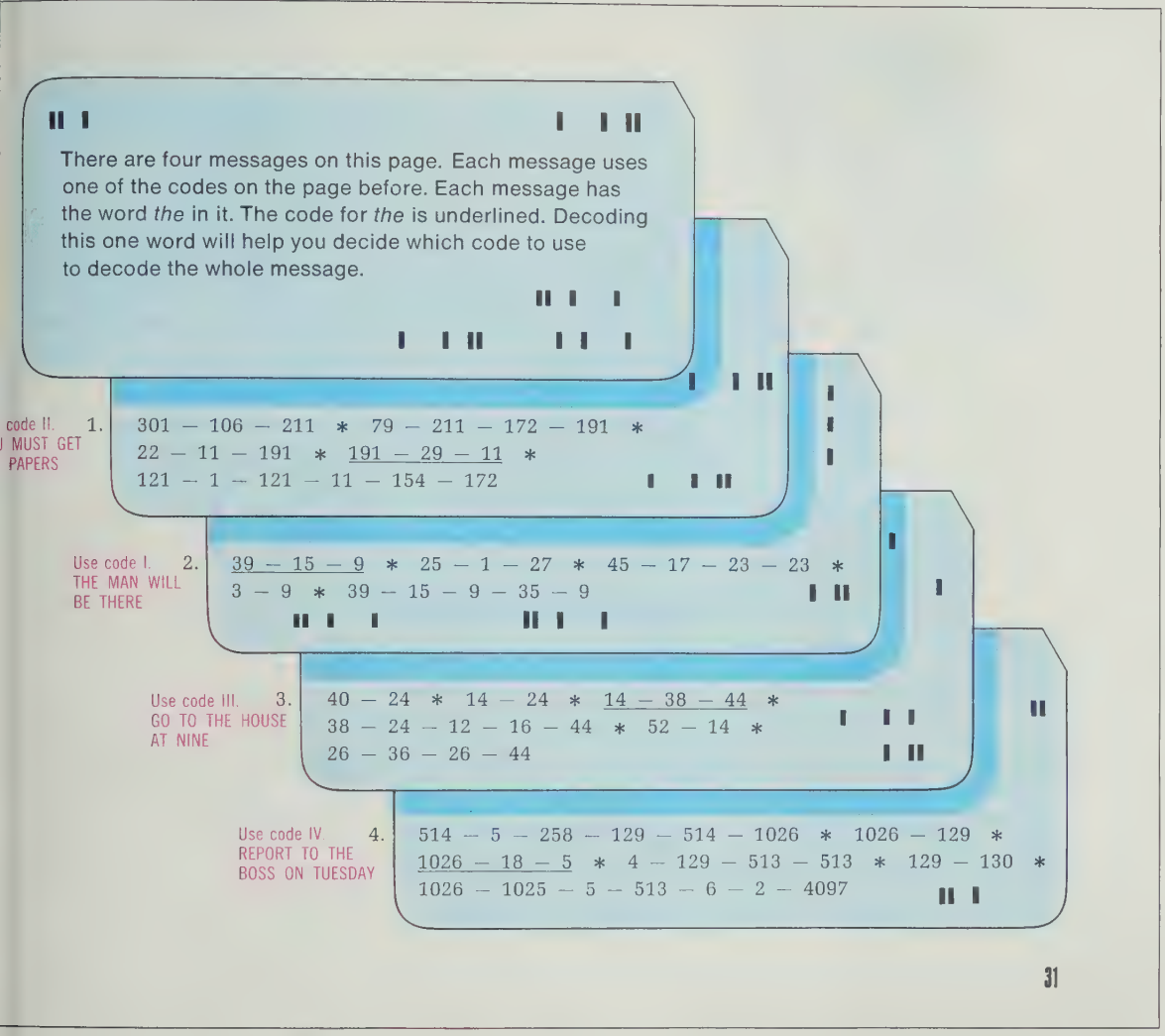
30

* M N O P Q R S T U V W X Y Z
25 27 29 31 33 35 37 39 41 43 45 47 49 51

** M N O P Q R S T U V W X Y Z
79 92 106 121 137 154 172 191 211 232 254 277 301 326

*** M N O P Q R S T U V W X Y Z
28 26 24 22 20 18 16 14 12 10 8 6 4 2

**** M N O P Q R S T U V W X Y Z
65 130 129 258 257 514 513 1026 1025 2050 2049 4098 4097 8194


There are four messages on this page. Each message uses one of the codes on the page before. Each message has the word *the* in it. The code for *the* is underlined. Decoding this one word will help you decide which code to use to decode the whole message.
code II. I MUST GET PAPERS
1. 301 — 106 — 211 * 79 — 211 — 172 — 191 *
22 — 11 — 191 * 191 — 29 — 11 *
121 — 1 — 121 — 11 — 154 — 172
Use code I. THE MAN WILL BE THERE
2. 39 — 15 — 9 * 25 — 1 — 27 * 45 — 17 — 23 — 23 *
3 — 9 * 39 — 15 — 9 — 35 — 9
Use code III. GO TO THE HOUSE AT NINE
3. 40 — 24 * 14 — 24 * 14 — 38 — 44 *
38 — 24 — 12 — 16 — 44 * 52 — 14 *
26 — 36 — 26 — 44
Use code IV. REPORT TO THE BOSS ON TUESDAY
4. 514 — 5 — 258 — 129 — 514 — 1026 * 1026 — 129 *
1026 — 18 — 5 * 4 — 129 — 513 — 513 * 129 — 130 *
1026 — 1025 — 5 — 513 — 6 — 2 — 4097
31

**goal** Decoding messages

**page 31** Talk about the key word. This word will help match the proper code to each message. Then everyone on his own.

**lesson** Pages 32, 33

**goal** Examining number relationships that involve addition

**memo** Pages 32 and 33 develop readiness for the mathematical concept of a function that is developed in the next level of this program. The word "function" is not used at this level and probably should not be. Function notation is not introduced. A table is used on page 33 simply to organize information in such a way that the relationship of one set of numbers to the other set is most obvious. The focus is on recognizing this relationship.

**page 32** It is O.K. to talk about what's happening, but the actual computations can be completed independently. There may be a variety of opinions on who had the best deal. Consider what would happen if Jack saved only amounts less than 25¢.

1

Jack has a good deal. Every time he puts some money into his bank, his father puts in a quarter. He showed this on a chart like this.

| IF I SAVE | DAD ADDS |
|---|---|
| 10¢ | 25¢ |
| 25¢ | 25¢ |
| 75¢ | 25¢ |
| 1.30 | 25¢ |
| ANY AMOUNT | 25¢ |

The amount Jack's dad adds depends only on Jack saving some money. Jack saves 10¢. His dad adds 25¢.
Total saved is ?. 35¢

2 Complete this chart.

| Jack saves | His dad adds $.25 | Total saved | |
|---|---|---|---|
| $.10 | $.25 | a | $ .35 |
| $.25 | $.25 | b | $ .50 |
| $.75 | $.25 | c | $1.00 |
| $1.30 | $.25 | d | $1.55 |
| e $4.50 | $.25 | $4.75 | |

3

Jay has a good deal too. Every time he puts some money in his bank, his father puts in the same amount. Jay made this chart.

| IF I SAVE | DAD ADDS |
|---|---|
| 10¢ | 10¢ |
| 25¢ | 25¢ |
| 75¢ | 75¢ |
| 1.30 | 1.30 |
| ANY AMOUNT | MATCHING AMOUNT |

The amount Jay's dad adds depends on how much money Jay saves.

Who has the best deal?
Jack or Jay?

Unless the amount saved is < $.25
Example: Jack 5¢ + 25¢ = 30¢ ← best
Jay 5¢ + 5¢ = 10¢

Jack 30¢ + 25¢ = 55¢
Jay 30¢ + 30¢ = 60¢ ← best

32

Look at the relationship between the two sets of numbers.
The number in column B depends on the number in column A.
But how?

### Think

| A | B | |
|---|---|---|
| 2 | 5 | What is the relationship between A and B? |
| 3 | 6 | What has been added to 2 to get 5? 3 |
| 5 | 8 | What has been added to 3 to get 6? 3 |
| 7 | ? 10 | Does that same number added to 5 give 8? Yes |
| 9 | ? 12 | Add that same number and get the second. |
| | | Add that same number and get the second. |

Would this sentence tell what's happening? Yes
*The first number plus another number equals the second number.*

$a + \blacksquare = b$ $\quad 2 + 3 = 5$

Your job is to find the relationship between A and B.
Then complete the table. (Hint: Think addition.)

1

| A | B (A + 10 = B) |
|---|---|
| 10 | 20 |
| 15 | 25 |
| 20 | 30 |
| 21 | ? 31 |
| 35 | ? 45 |

2

| A | B (A + 101 = B) |
|---|---|
| 3 | 104 |
| 6 | 107 |
| 8 | 109 |
| 12 | ? 113 |
| 27 | ? 128 |

3

| A | B (A + 134 = B) |
|---|---|
| 21 | 155 |
| 50 | 184 |
| 79 | 213 |
| 82 | ? 216 |
| 98 | ? 232 |

4

| A | B (A + 100 = B) |
|---|---|
| 135 | 235 |
| 147 | 247 |
| 168 | 268 |
| 205 | ? 305 |
| 494 | ? 594 |

5

| A | B (A + 157 = B) |
|---|---|
| 20 | 177 |
| 34 | 191 |
| 86 | 243 |
| 159 | ? 316 |
| 268 | ? 425 |

6

| A | B (A + 999 = B) |
|---|---|
| 1 | 1000 |
| 13 | 1012 |
| 67 | 1066 |
| 101 | ? 1100 |
| 599 | ? 1598 |

33

**goal** Practice in identifying number relationships that involve addition

**page 33** A method for determining the number relationship between A and B is presented at the top of the page. Some pupils may prefer to think, What do I do to the number in column A to get the number in column B? The math sentence $a + \square = b$ is the key to the tables. The emphasis now is not on writing math sentences for each problem but rather on identifying the relationship. Once the pupil finds the relationship between the three pairs of numbers given in any one table, he should be able to provide the missing numbers for the remainder of that table.

**lesson** Pages 34, 35

**goal** Examining number relationships that involve subtraction

**page 34** You'll want to share ideas about this page. These problem situations have a new twist—number relationships that involve subtraction. This may be a curve ball for some of your pupils. The tables should be completed independently.

Have the youngsters share the sentences written for problem 2. Are only the words different? Or are the ideas different too? Have they found more than one true sentence?

**1**

Mr. Tanzini runs a bakery. He bakes nut bread once a week. He delivers 23 loaves every week to a local restaurant. He sells the rest. How many does he have left to sell? It depends on how many loaves he bakes.

Complete the chart.

| | Loaves baked | Loaves delivered | Loaves to sell |
|---|---|---|---|
| Week 1 | 46 | 23 | 23 |
| Week 2 | 52 | 23 | **a** 29 |
| Week 3 | 95 | 23 | **b** 72 |
| Week 4 | 47 | 23 | **c** 24 |
| Week 5 | 61 | 23 | **d** 38 |
| Week 6 | 100 | 23 | **e** 77 |

Would this sentence tell what has happened? Yes

*Number of loaves baked minus those delivered equals number to sell.*

$b - d = s$ $46 - 23 = 23$

**2**

Jan's mother lets her babysit to earn spending money. But she has to put $.45 in her bank each time she babysits. The money she can spend depends on how much she earned.

Complete the chart.

| | Money earned | Money in bank | Money to spend |
|---|---|---|---|
| Week 1 | $1.20 | $.45 | $.75 |
| Week 2 | $2.50 | $.45 | **a** $2.05 |
| Week 3 | $1.75 | $.45 | **b** $1.30 |
| Week 4 | $3.25 | $.45 | **c** $2.80 |
| Week 5 | $5.00 | $.45 | **d** $4.55 |
| Week 6 | $3.10 | $.45 | **e** $2.65 |

**f** Write a sentence that tells what has happened.

Money earned minus money in bank equals money to spend. $e - b = s$

Accept other appropriate sentences.

34

Look for the relationship between these two sets of numbers.

| A | B |
|---|---|
| 5 | 1 |
| 7 | 3 |
| 10 | 6 |
| 12 | ? 8 |
| 25 | ? 21 |

**Think**

What is the relationship between A and B? 5 − 4 = 1, A − 4 = B

Is the same relationship true? Yes

Is the same relationship true again? Yes

Then you know the missing number.

You know this one too.

Complete each table.
Ask yourself: What is the relationship between A and B?

**1**

| A | B | (A − 10 = B) |
|---|---|---|
| 15 | 5 | |
| 37 | 27 | |
| 58 | 48 | |
| 110 | ? | 100 |
| 201 | ? | 191 |
| 300 | ? | 290 |

**2**

| A | B | (A − 21 = B) |
|---|---|---|
| 72 | 51 | |
| 85 | 64 | |
| 91 | 70 | |
| 94 | ? | 73 |
| 103 | ? | 82 |
| 200 | ? | 179 |

**3**

| A | B | (A − 125 = B) |
|---|---|---|
| 204 | 79 | |
| 356 | 231 | |
| 400 | 275 | |
| 562 | ? | 437 |
| 604 | ? | 479 |
| 875 | ? | 750 |

**4**

| A | B | (A − 35 = B) |
|---|---|---|
| 235 | 200 | |
| 236 | 201 | |
| 240 | 205 | |
| 500 | ? | 465 |
| 614 | ? | 579 |
| 1000 | ? | 965 |

**5**

| A | B | (A − 109 = B) |
|---|---|---|
| 109 | 0 | |
| 198 | 89 | |
| 229 | 120 | |
| 625 | ? | 516 |
| 700 | ? | 591 |
| 1108 | ? | 999 |

**6**

| A | B | (A − 999 = B) |
|---|---|---|
| 999 | 0 | |
| 1000 | 1 | |
| 1999 | 1000 | |
| 3009 | ? | 2010 |
| 4909 | ? | 3910 |
| 5000 | ? | 4001 |

35

**goal** Practice in identifying number relationships that involve subtraction

**page 35** Treat this page as a parallel to page 33. Youngsters who had trouble there will probably need help here too. Sharpies who have caught on should proceed independently.

**lesson** Pages 36, 37

**goal** Examining number relationships that involve multiplication

**memo** Pages 36 and 37 require a degree of skill in multiplication. Pupils are required to multiply a 3-digit factor by a 2-digit factor. Lack of mastery will quickly become evident. Focus on the number relationships for the present. Ability in multiplication will be diagnosed and help provided on pages 38 through 41.

**page 36** Again problem situations are used to build a foundation for a new type of number relationship—those that involve multiplication. Treat this page as you did pages 32 and 34. All those zeros in the factors may cause some pupils trouble.

**1.**
Mr. Simons is a salesman. He rents the cars he drives. The Davis Rent-A-Car Company charges $.20 per mile. How much does Mr. Simons pay? That depends on how many miles he drives.

Mr. Simons usually travels the same roads. Here is a chart that shows 5 days of driving. Complete the chart.

| He drives | 40 miles | 60 miles | 120 miles | 200 miles | 500 miles |
|---|---|---|---|---|---|
| He pays | $8.00 | $12.00 | ? $24.00 | ? $40.00 | ? $100.00 |

Is it true that the more he drives, the more he pays? Yes
The cost depends on cost per mile times the number of miles.
Will this math sentence fit the story if he drives 400 miles? Yes

$$c = \$.20 \times 400$$

What does $c$ stand for? Cost
How much would he pay if he drove 400 miles? $80.00

Someone said the amount was $800. Is that answer reasonable? What did that person forget? Where the decimal should go
No

**2.**
Mr. Simons tried another car-rental company for a while. They charged $14 a day plus $.12 per mile.

He prepared another chart. You complete it.

| He drives | 40 miles | 60 miles | 120 miles | 200 miles | 500 miles |
|---|---|---|---|---|---|
| He pays | $14 plus $4.80 | $14 plus ? $7.20 | ? $14 plus $14.40 | ? $14 plus $24.00 | ? $14 plus $60.00 |

**3.**
He heard of still another agency that charged $20 a day with no charge for mileage. Would he save money here? Compared to Davis Rent-A-Car: Yes—if he drives over 100 miles a
Compared to other agency: Yes—if he drives over 50 miles a day.

36

Look for the relationship between these two sets of numbers.

| A | B |
|---|---|
| 1 | 5 |
| 5 | 25 |
| 6 | 30 |
| 8 | ? 40 |
| 10 | ? 50 |

**Think**

What is the relationship between A and B? $1 \times 5 = 5$, $A \times 5 = B$

Is the same relationship true? Yes

Is it true again? Yes

Then you know the missing number.

You know this one too.

Complete each table.

Ask yourself: What is the relationship between A and B?

**1**

| A | B (A · 10 = B) |
|---|---|
| 1 | 10 |
| 10 | 100 |
| 20 | 200 |
| 70 | ? 700 |
| 100 | ? 1000 |
| 1000 | ? 10,000 |

**2**

| A | B (A · 9 = B) |
|---|---|
| 2 | 18 |
| 6 | 54 |
| 8 | 72 |
| 9 | ? 81 |
| 12 | ? 108 |
| 15 | ? 135 |

**3**

| A | B (A · 12 = B) |
|---|---|
| 1 | 12 |
| 10 | 120 |
| 12 | 144 |
| 18 | ? 216 |
| 36 | ? 432 |
| 73 | ? 876 |

**4**

| A | B (A · 23 = B) |
|---|---|
| 1 | 23 |
| 2 | 46 |
| 3 | 69 |
| 7 | ? 161 |
| 15 | ? 345 |
| 19 | ? 437 |

**5**

| A | B (A · 65 = B) |
|---|---|
| 1 | 65 |
| 2 | 130 |
| 5 | 325 |
| 12 | ? 780 |
| 156 | ? 10,140 |
| 279 | ? 18,135 |

**6**

| A | B (A · 37 = B) |
|---|---|
| 2 | 74 |
| 4 | 148 |
| 10 | 370 |
| 56 | ? 2072 |
| 123 | ? 4551 |
| 370 | ? 13,690 |

37

**goal** Practice in identifying number relationships that involve multiplication

**page 37** Your experiences with pages 33 and 35 should tell you how best to handle this page with your pupils.

Encourage your capable pupils to write a math sentence for each table, but don't let anyone become discouraged by this task. And for the curious—yes, number relationships could involve division also. But enough for now. Division will come later.

**lesson** Pages 38, 39

**goal** Diagnosis of ability and help in multiplying two 2-digit factors.

**memo** The Progress Check on page 39 parallels the diagnostic check at the top of page 38. Pupils who are successful here have already shown their ability with these skills and are directed in the chart to page 40 to check on the next skill.

**page 38** The format of the page should be familiar from chapter 1. It may or may not require further discussion, depending on your pupils.

Be ready to help youngsters who make an error in the diagnostic check. Help with the algorithm is given on the page. Watch for two causes for trouble:

- Addition facts not mastered
- "Mislearning" of the renaming algorithm either in multiplication or in addition

For example:

$$\begin{array}{r} {\scriptstyle 5}\,49 \\ \times\ 65 \\ \hline 254 \\ 2454 \\ \hline 2608 \end{array}$$

254 – wrote tens

2454 – did not rename

2608 – did not rename in addition

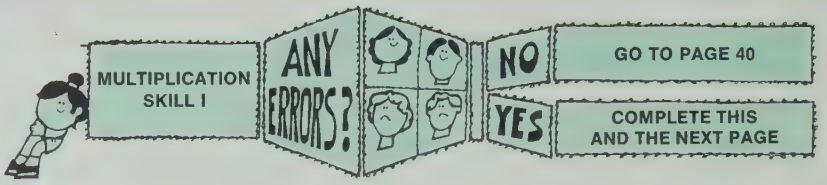
You may want to select some problems from page 39 for additional practice.

## Time out to check your skills in multiplication.

DO THESE THREE PROBLEMS

| a | b | c |
|---|---|---|
| 53 × 40 = 2120 | 31 × 23 = 713 | 49 × 65 = 3185 |

Check your answers with the key on the next page.




**1.** $53 \times 40$

$53 \times 40 = 0$ (ones)

$4 \times 53$ is 212.

Write 212.

$53 \times 40 = 2120$

Can you figure out what happened in this shortcut?

**2.** $31 \times 23$

$3 \times 31$ is 93. Write 93.

$20 \times 31$ is 620. Write 620.

$93 + 620$ is 713. Write 713.

$$\begin{array}{r} 31 \\ \times 23 \\ \hline 93 \\ 620 \\ \hline 713 \end{array}$$

**3.** $49 \times 65$

$5 \times 49$ is 245. Write 245.

$60 \times 49$ is 2940. Write 2940.

$245 + 2940$ is 3185. Write 3185.

$$\begin{array}{r} 49 \\ \times 65 \\ \hline 245 \\ 2940 \\ \hline 3185 \end{array}$$

38

# PROGRESS CHECK

Skill: Multiplying by multiples of ten

| | | | |
|---|---|---|---|
| 1. 38 × 40 = 1520 | 2. 46 × 20 = 920 | 3. 72 × 60 = 4320 | 4. 29 × 40 = 1160 |

Skill: Multiplying – no renaming

| | | | |
|---|---|---|---|
| 5. 21 × 33 = 693 | 6. 42 × 12 = 504 | 7. 23 × 31 = 713 | 8. 22 × 42 = 924 |
| 9. 63 × 11 = 693 | 10. 12 × 14 = 168 | 11. 33 × 23 = 759 | 12. 22 × 22 = 484 |

Skill: Multiplying to get a 3-digit product – renaming

| | | | |
|---|---|---|---|
| 13. 34 × 23 = 782 | 14. 28 × 34 = 952 | 15. 19 × 43 = 817 | 16. 46 × 12 = 552 |

Skill: Multiplying to get a 4-digit product

| | | | |
|---|---|---|---|
| 17. 78 × 23 = 1794 | 18. 49 × 45 = 2205 | 19. 61 × 38 = 2318 | 20. 39 × 67 = 2613 |
| 21. 64 × 54 = 3456 | 22. 75 × 48 = 3600 | 23. 89 × 78 = 6942 | 24. 96 × 87 = 8352 |

Check your answers on page 53.
Go to the next page. Do the first two problems.

39

See activity 1, page 53b.

See activity 2, page 53b.

**goal** **Progress Check**—multiplication of two 2-digit factors requiring and not requiring renaming

**page 39** There are far too many problems here for a Progress Check. Be selective. The skills are identified on the answer key. The following chart may help also.

| Problems | Correlate to diagnostic problem |
|---|---|
| 1–4 | **a** |
| 5–12 | **b** |
| 13–24 | **c** |

Save the remaining problems for additional practice or for retest if additional help is signaled.

**lesson** Pages 40, 41, 42

**goal** Diagnosis of ability and help in multiplying a 3-digit factor

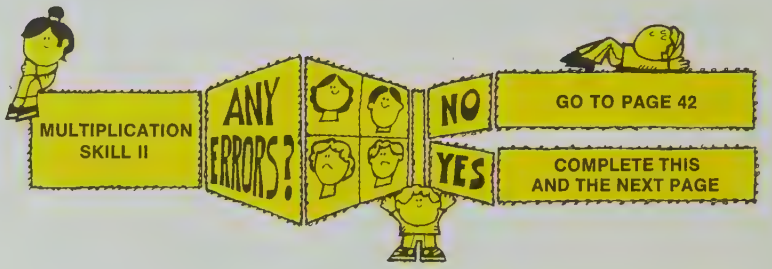
**page 40** The procedure is old stuff by now. Watch especially for pupils whose only problem is handling zeros in multiplication.

Select problems from page 41 if additional practice is needed.

Do these two multiplication problems.

| | | |
|---|---|---|
| | 376 | 654 |
| | × 43 | ×300 |
| **a** | 16,168 | **b** 196,200 |

Check your answers with the key on the next page.




**1.**

| | | |
|---|---|---|
| 376 | 376 | 376 |
| × 43 | × 43 | × 43 |
| 1128 | 1128 | 1128 |
| | 15040 | 15040 |
| | | 16,168 |
| 3 × 376 is 1128. Write 1128. | 40 × 376 is 15,040. Write 15,040. | 1128 + 15,040 is 16,168. Write 16,168. |

**2.**

| | | |
|---|---|---|
| 654 | 654 | 3 × 654 is 1962. |
| ×300 | ×300 | So 300 × 654 is 196,200. |
| 00 | 196,200 | Write 196,200. |

40

# PROGRESS CHECK

Skill: Multiplying a 3-digit by a 2-digit number

| | | | |
|---|---|---|---|
| 1. 443 × 24 = 10,632 | 2. 517 × 53 = 27,401 | 3. 383 × 45 = 17,235 | 4. 245 × 37 = 9065 |
| 5. 729 × 25 = 18,225 | 6. 638 × 36 = 22,968 | 7. 218 × 49 = 10,682 | 8. 935 × 68 = 63,580 |
| 9. 369 × 89 = 32,841 | 10. 726 × 94 = 68,244 | 11. 338 × 26 = 8788 | 12. 474 × 83 = 39,342 |
| 13. 573 × 89 = 50,997 | 14. 694 × 78 = 54,132 | 15. 783 × 86 = 67,338 | 16. 967 × 29 = 28,043 |
| 17. 658 × 86 = 56,588 | 18. 378 × 77 = 29,106 | 19. 947 × 59 = 55,873 | 20. 775 × 84 = 65,100 |

Skill: Multiplying by a multiple of 100

| | | | |
|---|---|---|---|
| 21. 793 × 400 = 317,200 | 22. 429 × 300 = 128,700 | 23. 382 × 600 = 229,200 | 24. 166 × 700 = 116,200 |

Check your answers on page 53.
Go on the next page.



**goal** **Progress Check** – multiplying a 3-digit factor by a 2-digit factor or a multiple of 100

**page 41** Once again, be selective. The purpose is to check ability, not endurance.

| Problems | Correlate to diagnostic problem |
|---|---|
| 1–20 | **a** |
| 21–24 | **b** |

See activity 3, page 53b.

See activity 4, page 53b.

**goal** Experiencing the fun side of mathematics

**page 42** Warn everyone that the first problem is a real pencil-eating monster.

You may want to make the answers available somewhere in the room. Then the youngsters can work on these problems as time permits and check their own results.

# Just for Fun

1. Suppose you were offered a job and were promised 1 cent for the first day's work, double that, or 2 cents, for the second day, 4 for the third, and so on, getting double the preceding day's pay for every day in the month. Would you take the job? Yes
   Now figure out how much you would have made had you taken the job for the month of March.
   $21,474,836.47 if you worked each day; $83,886.07 if there were 4 weekends you didn't work.

2. Multiply these numbers.
   $24 \times 4 \times 10 \times 5 \times 0 \times 1 = \underline{?}$ 0

3. Are you ready for another code? The message depends on what each symbol stands for. This code is based on a simple pattern of multiplication.

| ALPHABET | A | B | C | D | E | F | G | H | I | J | K | . . . |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CODE | 2 | ? | 8 | ? | ? | ? | ? | ? | ? | ? | ? | . . . |
| | | 4 | | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | |

Here's a clue: $A \times B = 8$.
Now decode the intercepted diplomatic letter.

| | |
|---|---|
| L | 4096 |
| M | 8192 |
| N | 16,384 |
| O | 32,768 |
| P | 65,536 |
| Q | 131,072 |
| R | 262,144 |
| S | 524,288 |
| T | 1,048,576 |
| U | 2,097,152 |
| V | 4,194,304 |
| W | 8,388,608 |
| X | 16,777,216 |
| Y | 33,554,432 |
| Z | 67,108,864 |

**Office of the Ambassador**

65,536 – 4096 – 32 – 2 – 524,288 – 32 *

524,288 – 32 – 16,384 – 16 *

524,288 – 32 – 8 – 262,144 – 32 – 1,048,576 *

64 – 512 – 4096 – 32

PLEASE SEND SECRET FILE.

42

**lesson** Pages 43, 44

**goal** Examining number relationships that involve division

**page 43** Notice that the words **depends on** keep popping up every time one of these tables appears. By now the words should be a clue to every learner that the same number relationship will be true for each set of numbers within a table.

Having already completed pages 32, 34, and 35, you've developed a procedure that works for your pupils. Use it one more time.

**1**

The supermarket is doing a test. The manager thinks that items sell faster if they are on a certain shelf. The manager experiments in this way: There are 4 shelves. He puts the same number of items that are alike on each shelf. The number of items on each shelf depends on how many he has in all.

Complete this chart.

| | Total number | Number of shelves | Number on each shelf |
|---|---|---|---|
| Packages of hard rolls | 44 | 4 | 11 |
| Hamburger buns | 76 | 4 | **a** 19 |
| Hotdog buns | 64 | 4 | **b** 16 |
| Loaves of white bread | 96 | 4 | **c** 24 |
| Loaves of rye bread | 48 | 4 | **d** 12 |
| Loaves of wheat bread | 60 | 4 | **e** 15 |

Does this sentence tell what's happening? Yes

*The total divided by the number of shelves equals the number on each shelf.* $t \div s = n$ $44 \div 4 = 11$

**2**

The club made artificial flowers to sell. They put them in boxes that held 12. How many boxes could they sell? That depends on the number they made in all.

Complete this chart.

| | Total number made | Number in each box | Number of boxes |
|---|---|---|---|
| January | 12 | 12 | 1 |
| February | 60 | 12 | **a** 5 |
| March | 36 | 12 | **b** 3 |
| April | 144 | 12 | **c** 12 |
| May | 96 | 12 | **d** 8 |

*e Write a sentence that tells what's happening.

*(Total made divided by number in each box equals the number of boxes.) $t \div b = n$
Accept other appropriate sentences

**goal** Practice in identifying number relationships that involve division

**memo** Division skills are less likely to have been mastered than any of the other skills. Please move anyone who becomes frustrated because of the required division skills to the diagnostic check on page 45.

**page 44** Many pupils will recognize the page as an extension of pages 33, 35, and 37. They now can identify number relationships involving all four number operations. Follow any procedure that was successful on these similar preceding pages.

Look for the relationship between these two sets of numbers.

| A | B |
|---|---|
| 100 | 10 |
| 150 | 15 |
| 170 | 17 |
| 200 | ? 20 |
| 350 | ? 35 |

**Think**

What is the relationship between A and B? $10 \div 10 = 10$, $A \div 10 = B$

Is the same relationship true? Yes

Is the same relationship true again? Yes

Then you know the missing number.

You know this one too.

There is a relationship between A and B in each of these tables. Find out what it is and complete the table.

**1**

| A | B ($A \div 25 = B$) |
|---|---|
| 25 | 1 |
| 50 | 2 |
| 150 | 6 |
| 275 | ? 11 |
| 325 | ? 13 |
| 500 | ? 20 |

**2**

| A | B ($A \div 13 = B$) |
|---|---|
| 39 | 3 |
| 91 | 7 |
| 273 | 21 |
| 728 | ? 56 |
| 1053 | ? 81 |
| 1300 | ? 100 |

**3**

| A | B ($A \div 46 = B$) |
|---|---|
| 92 | 2 |
| 138 | 3 |
| 276 | 6 |
| 460 | ? 10 |
| 598 | ? 13 |
| 1242 | ? 27 |

**4**

| A | B ($A \div 11 = B$) |
|---|---|
| 22 | 2 |
| 66 | 6 |
| 121 | 11 |
| 473 | ? 43 |
| 957 | ? 87 |
| 1100 | ? 100 |

**5**

| A | B ($A \div 35 = B$) |
|---|---|
| 140 | 4 |
| 245 | 7 |
| 280 | 8 |
| 350 | ? 10 |
| 420 | ? 12 |
| 945 | ? 27 |

**6**

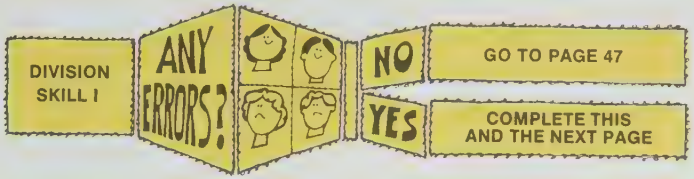
| A | B ($A \div 5$) |
|---|---|
| 57 | 1 |
| 114 | 2 |
| 171 | 3 |
| 684 | ? 12 |
| 1083 | ? 19 |
| 1425 | ? 25 |



44

Do these three division problems. **a** $2\overline{)68}$ (34) **b** $3\overline{)639}$ (213) **c** $4\overline{)288}$ (72)

Check your answers with the key on the next page.




**1.** $2\overline{)68}$

$$\begin{array}{r} 30 \\ 2\overline{)68} \\ \underline{60} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

How many 2s in 68?
20? $20 \times 2 = 40$
30? $80 \times 2 = 60$
40? $40 \times 2 = 80$ too many

$30 \times 2$ is 60. Write 60. Then subtract.

$$\begin{array}{r} ? \\ \hline 4 \\ 30 \\ 2\overline{)68} \\ \underline{60} \\ 8 \\ \underline{8} \\ 0 \end{array}$$ 34

How many 2s in 8? $4 \times 2 = 8$

O.K Finish it. Add the parts of your answer on top.

**2.** $3\overline{)639}$

$$\begin{array}{r} ? \\ \hline 3 \\ 10 \\ 200 \\ 3\overline{)639} \\ \underline{600} \\ 39 \\ \underline{30} \\ 9 \\ \underline{9} \\ 0 \end{array}$$ 213

How many 3s in 639?
$200 \times 3$
How many 3s in 39?
$10 \times 3$
How many 3s in 9?
$3 \times 3$

**3.** $4\overline{)288}$

$$\begin{array}{r} ? \\ \hline 2 \\ 70 \\ 4\overline{)288} \\ \underline{280} \\ 8 \\ \underline{8} \\ 0 \end{array}$$ 72

How many 4s in 288?
$70 \times 4$
How many 4s in 8?
$2 \times 4$



**lesson** Pages 45, 46

**goal** Diagnosis of ability and help in dividing by a 1-digit divisor

**page 45** Everyone knows how to handle these pages by now.

The pupil who makes a mistake is given help with the algorithm right on the page. The need for more practice in multiplying multiples of 10 and 100 may be indicated. For example:

$3 \times 4 =$ $3 \times 40 =$ $3 \times 400 =$
$7 \times 6 =$ $7 \times 60 =$ $7 \times 600 =$

Be alert for addition and subtraction errors.

$$\begin{array}{r} 30 \\ 2\overline{)68} \\ 60 \\ \hline 0 \end{array}$$ ⓪ subtraction error

$$\begin{array}{r} (38) \\ \hline 4 \\ 30 \\ 2\overline{)68} \\ \underline{60} \\ 8 \\ \underline{8} \\ 0 \end{array}$$ addition error

For additional practice select the odd-numbered problems from page 46 and use the even-numbered problems for the Progress Check.

**goal** **Progress Check**—dividing by a 1-digit divisor

**page 46** No need to worry about pupils directed to skip this page. The skills parallel those already checked in the diagnostic section of page 45.

| Problems | Correlate to diagnostic problem |
|---|---|
| 1–12 | **a** |
| 13–24 | **b** |
| 25–36 | **c** |

Use this chart to help you select problems for the Progress Check, for additional practice, for retesting—or for skipping.

# PROGRESS CHECK

Skill: Dividing a 2-digit by a 1-digit number

| | | | |
|---|---|---|---|
| 1. $3\overline{)93}$ = 31 | 2. $4\overline{)48}$ = 12 | 3. $6\overline{)66}$ = 11 | 4. $2\overline{)86}$ = 43 |
| 5. $3\overline{)36}$ = 12 | 6. $4\overline{)88}$ = 22 | 7. $5\overline{)55}$ = 11 | 8. $2\overline{)28}$ = 14 |
| 9. $3\overline{)39}$ = 13 | 10. $4\overline{)44}$ = 11 | 11. $3\overline{)96}$ = 32 | 12. $2\overline{)64}$ = 32 |

Skill: Dividing a 3-digit by a 1-digit number

| | | | |
|---|---|---|---|
| 13. $2\overline{)468}$ = 234 | 14. $4\overline{)844}$ = 211 | 15. $3\overline{)690}$ = 230 | 16. $3\overline{)366}$ = 122 |
| 17. $2\overline{)682}$ = 341 | 18. $2\overline{)448}$ = 224 | 19. $4\overline{)804}$ = 201 | 20. $2\overline{)264}$ = 132 |
| 21. $3\overline{)963}$ = 321 | 22. $2\overline{)822}$ = 411 | 23. $3\overline{)936}$ = 312 | 24. $5\overline{)550}$ = 110 |
| 25. $3\overline{)189}$ = 63 | 26. $5\overline{)255}$ = 51 | 27. $4\overline{)328}$ = 82 | 28. $4\overline{)244}$ = 61 |
| 29. $7\overline{)497}$ = 71 | 30. $8\overline{)560}$ = 70 | 31. $5\overline{)305}$ = 61 | 32. $3\overline{)276}$ = 92 |
| 33. $6\overline{)426}$ = 71 | 34. $3\overline{)249}$ = 83 | 35. $7\overline{)567}$ = 81 | 36. $9\overline{)819}$ = 91 |

Check your answers on page 53.
Go to the next page. Do the first two problems.

46

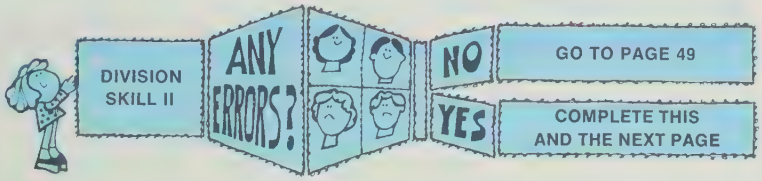
See activity 5, page 53c.

See activity 6, page 53c.

THESE TWO
VISION PROBLEMS.

**a** $34\overline{)788}$ 23 R6 **b** $49\overline{)8925}$ 182 R7

Check your answers with the key on the next page.




It helps to round your divisor when you have a two-digit divisor.

```
   ?   23 R6
   3
  20
34)788   About how many 30s in 788?   10 × 30 = 300?
 680     20 × 34                      20 × 30 = 600?   Good estimate
 108     How many 30s in 108?         30 × 30 = 900?   for 34 too.
 102     3 × 34
   6     This is a remainder. It goes with your answer.
```

```
     ?   182 R7
     2
    80
   100
49)8925   About how many 50s in 8925?  Good estimate for 49 too
4900      100 × 49
4025      About how many 50s in 4025?
3920      80 × 49
 105      About how many 50s in 105?
  98      2 × 49
   7      This is a remainder. It goes with your answer.
```

**2.**

47

**lesson** Pages 47, 48, 49

**goal** Diagnosis of ability and help in dividing by a 2-digit divisor

**page 47** You will want to check pupils who are directed to the bottom of the page for their ability to round the divisor to estimate the first partial quotient.

Select problems from page 48 if additional practice is needed.

**goal** **Progress Check**—dividing by a 2-digit divisor

**page 48** No one should do all these problems. Adjust practice assignments and your selection of problems for the Progress Check to meet the individual needs of the pupil.

| Problems | Correlate to diagnostic problem |
|---|---|
| 1–9, 19–28, 30–33, 35 | **a** |
| 10–18, 29, 34, 36 | **b** |

No need to practice a skill that is mastered. Skip unnecessary problems and get on to new ideas.

# PROGRESS CHECK

Skill: Dividing by a 2-digit divisor

| | | | | | |
|---|---|---|---|---|---|
| 1. 22 R14<br>21)476 | 2. 18 R24<br>39)726 | 3. 17 R6<br>55)941 | 4. 26 R5<br>28)733 | 5. 14 R6<br>42)594 | 6. 15 R42<br>53)837 |
| 7. 24 R16<br>18)448 | 8. 13 R52<br>71)975 | 9. 17 R24<br>45)789 | 10. 157 R9<br>31)4876 | 11. 296 R12<br>25)7412 | 12. 112 R40<br>66)7432 |
| 13. 137 R2<br>43)5893 | 14. 266 R12<br>27)7194 | 15. 154 R6<br>38)5858 | 16. 118 R65<br>72)8561 | 17. 227 R16<br>34)7734 | 18. 329 R28<br>29)9569 |
| 19. 74 R22<br>24)1798 | 20. 46 R21<br>48)2229 | 21. 62 R8<br>33)2054 | 22. 65 R7<br>79)5142 | 23. 49 R64<br>93)4621 | 24. 87 R34<br>49)4297 |
| 25. 22 R36<br>61)1378 | 26. 91 R34<br>55)5039 | 27. 78<br>16)1248 | 28. 14 R9<br>42)597 | 29. 252 R18<br>31)7830 | 30. 9 R41<br>72)689 |
| 31. 49 R28<br>53)2625 | 32. 16 R16<br>58)944 | 33. 62 R9<br>49)3047 | 34. 151 R8<br>13)1971 | 35. 20 R18<br>35)718 | 36. 178 R26<br>52)9282 |

Check your answers on page 53. Go on to the next page.

40

See activity 7, page 53c.

See activity 8, page 53d.

ink about putting some of the facts in each story
o a table. It might help you find the answers.

**1** The sign read: BUY 2 AND GET 1 MORE FREE. How many free ones would you get if you bought 4? 8? 12? How many free ones would you get if you bought 15?

| Buy | Free |
|---|---|
| 2 | 1 |
| 4 | 2 |
| 8 | 4 |
| 12 | 6 |
| 15 | 7 |

**2** Only 5 would fit into a box. How many boxes would you need to pack 15? 50? 60? 75? 88?

| Things | Boxes |
|---|---|
| 5 | 1 |
| 15 | 3 |
| 50 | 10 |
| 60 | 12 |
| 75 | 15 |
| 88 | 18 |

**3** One jar of coffee makes 32 cups of coffee. How many cups can be made from 5 jars? 10 jars? 12 jars? How many jars will be needed to make 640 cups of coffee?

| Jars | Cups |
|---|---|
| 1 | 32 |
| 5 | 160 |
| 10 | 320 |
| 12 | 384 |
| 20 | 640 |

**4** In some restaurants a service charge is added to the bill instead of the customer leaving a tip. In one restaurant the service charge is 15¢ for every dollar spent. How much would the service charge be if your bill is $2.00? $5.00? $10.00? $100.00?

| Bill | Service Charge |
|---|---|
| $1.00 | $0.15 |
| $2.00 | $0.30 |
| $5.00 | $0.75 |
| $10.00 | $1.50 |
| $100.00 | $15.00 |

**5** He gets a discount on everything he buys. If he buys something that costs $1.00, he pays $0.90. If he buys something that costs $2.00, he pays $1.80. How much does he pay for something that costs $3.00? $5.00? $10.00?

| Price | Discount price |
|---|---|
| $1.00 | $0.90 |
| $2.00 | $1.80 |
| $3.00 | $2.70 |
| $5.00 | $4.50 |
| $10.00 | $9.00 |

Research project: Does the average person need to be skillful in dividing by a divisor having more than 2 digits? Decide after you complete this chart.

| Where is division used? | By whom? | Kind of divisor? 1-digit | 2-digit | 3-or-more digit | Is a calculator used? |
|---|---|---|---|---|---|
| Cost of materials | Engineer | | | x | Yes |
| Cooking | Chef | x | x | | No |

**goal** Identifying number relationships, using all four operations

**page 49** The sophisticated thinking required to solve these problem situations may be too difficult for some pupils. Consider using the page as an extension activity for those who are capable of working independently while you work with those who need help with skill development.

**lesson** Pages 50, 51

**goal** Exploring the distributive property of multiplication over addition

**memo** Page 50 and problem 2 on page 51 explore the distributive property of multiplication over addition. Technical terminology is not introduced on either page. This program is dedicated to developing understanding and skill in operating with the properties. It endeavors to avoid vocabulary distractions. The decision to introduce such terminology is yours to make.

**page 50** The problems on pages 50 and 51 are related. You'll find they do not lend themselves well to independent study. Have fun with the subject matter. The prices should be meaningful to sharp shoppers. Beware—more than one operation is used in the same problem. Emphasize the importance of parentheses.

# Just for fun

Several years ago a mail-order catalogue used in the late 1800s was reprinted. It was great fun to look at the old catalogue and see how much things cost in those days.

Here are the prices for men's socks that appeared in the catalogue.
The cheapest cotton socks cost $0.08 a pair.
The most expensive cotton socks cost $0.45 a pair.
The most expensive silk socks cost $0.98 a pair.

The amount your great-grandfather would have paid for socks from this catalogue would depend on what things?
Type and quantity of socks purchased, postage
Did you remember the socks would be sent through the mail? That is part of the cost too. At that time postage was 2 cents a pair.

amount paid = number of pairs × cost per pair + postage
This is one math sentence that could be used if grandpa bought 3 pairs of cheap socks: $ = 3 × $0.08 + 3 × $0.02
Goodness! Where did all those numbers come from?

This is another math sentence that could be used for the same purchase: $ = 3 × ($0.08 + $0.02)
The parentheses mean that you complete that part of the problem first.
This sentence says that each pair really cost $0.10.
Can that be true? Will the answer be the same Yes
no matter which math sentence you use? Prove it. Yes
(3 x $0.08) + (3 x $0.02) = $0.30 3 x (0.08 + $0.02) = $0.30

1. Compute the cost for grandpa's socks if he bought—
   **a** 4 pairs of each kind $6.28 **b** 6 pairs of each kind $9.42
   **c** 12 pairs of expensive cotton socks and 6 pairs of silk socks $11.64

50

How much do socks cost now? Pretend you're going to buy socks for some man you know. You will go to the store so that you don't have to worry about postage. How much will 2 pair cost? 3 pair?

Answers will vary with type of socks purchased.

That same catalog had prices for ladies' hose, too.

The cheapest cotton hose cost $.05 a pair.
The most expensive cotton hose cost $.50 a pair.
The most expensive silk hose cost $2.47 a pair.

One pair weighed 3 ounces. The postage rate was $.01 per ounce.

Compute the cost for grandma's hose if she had bought—

**a** 2 pairs of each kind $6.22

**b** 6 pairs of each kind of cotton hose $3.66

**c** 12 pairs of the cheapest cotton hose and 2 pairs of silk hose $5.96

Could grandma have bought nylon hose at the turn of the century? No

Why? What do you think was the difference between the 5-cent and the 50-cent hose besides the price? Nylon wasn't invented until 1938. Quality, appearance, durability

Try to find out what the price range of hose is today. 59¢ to $3.00

Postage rates are much different now. Try to find out the cost of sending a letter that weighs 1 ounce by first-class mail now. What is the cost of airmail? What is the charge for a special delivery letter?

Postal charges change without much warning. Please check current costs.

Could grandma have sent a letter in the late 1800s by airmail? Why?

No; no airplanes. (Airmail service started in 1918.)

Why do you think mail-order catalogs were popular in the 1800s?

Answers will vary. Examples: Fewer towns, greater rural population, difficult means of transportation



**goal** Examining some economic changes since the late 1800s

**page 51** Pupils may be surprised by how little the cost of mail has changed compared with the cost of socks. Some of your students of history may find on these pages an idea appealing enough for some independent research.

**lesson** Page 52

**goal** **Checkout**—ability in all four number operations

**page 52** The page is a comprehensive check on the skills reviewed and practiced in chapters 1 and 2. Anyone who is still having trouble will need you to point out the problem. Then use any and all appropriate supplementary materials—programed learning materials, tape programs, drill kits—that are available in your school to help him overcome this weakness. Don't overlook an available natural resource—peer tutors.

Be liberal with praise for those who have shown improvement.

## CHECKOUT

**1.** Compute. Watch the operation signs. Skill: Subtracting 4-digit numbers

| a | b | c | d |
|---|---|---|---|
| 9785 − 8342 = 1443 | 7592 − 4335 = 3257 | 5641 − 1269 = 4372 | 6430 − 3657 = 2773 |

Skill: Adding 4-digit numbers

| e | f | g | h |
|---|---|---|---|
| 6324 + 2643 = 8967 | 4605 + 3258 = 7863 | 7348 + 1587 = 8935 | 5976 + 6349 = 12,325 |

**2.** Multiply. Skill: Multiplying by a 2-digit number

| a | b | c | d |
|---|---|---|---|
| 64 × 30 = 1920 | 57 × 46 = 2622 | 314 × 34 = 10,676 | 624 × 87 = 54,288 |

**3.** Divide. Skill: Dividing by 1- and 2-digit divisors

| a | b | c | d |
|---|---|---|---|
| 81 R0 | 71 R1 | 14 R4 | 237 R0 |
| 9)729 | 6)427 | 32)452 | 23)5451 |

Skill: Finding computation errors

**4.** Here's Jay's homework paper. Did he get all of them right? If there are errors, show how the problem should be completed.

a) 21 R24 (16) — 45)961 — 900 — 61 — 45 — 24 — 16

b) 702 × 50 = 35,100

c) 15 (12 1) — 1630 − 783 = 853 — 1847

52

See activity 9, page 53d.

See activity 10, page 53d.

ANSWERS PAGE 39

| | | | |
|---|---|---|---|
| **1.** 2120 | **7.** 713 | **13.** 782 | **19.** 2318 |
| **2.** 920 | **8.** 924 | **14.** 952 | **20.** 2613 |
| **3.** 4320 | **9.** 693 | **15.** 817 | **21.** 3456 |
| **4.** 1160 | **10.** 168 | **16.** 552 | **22.** 3600 |
| **5.** 693 | **11.** 759 | **17.** 1794 | **23.** 6942 |
| **6.** 504 | **12.** 484 | **18.** 2205 | **24.** 8352 |

ANSWERS PAGE 41

| | | | |
|---|---|---|---|
| **1.** 10,632 | **7.** 10,682 | **13.** 50,997 | **19.** 55,873 |
| **2.** 27,401 | **8.** 63,580 | **14.** 54,132 | **20.** 65,100 |
| **3.** 17,235 | **9.** 32,841 | **15.** 67,338 | **21.** 317,200 |
| **4.** 9065 | **10.** 68,244 | **16.** 28,043 | **22.** 128,700 |
| **5.** 18,225 | **11.** 8788 | **17.** 56,588 | **23.** 229,200 |
| **6.** 22,968 | **12.** 39,342 | **18.** 29,106 | **24.** 116,200 |

ANSWERS PAGE 46

| | | | |
|---|---|---|---|
| **1.** 31 | **10.** 11 | **19.** 201 | **28.** 61 |
| **2.** 12 | **11.** 32 | **20.** 132 | **29.** 71 |
| **3.** 11 | **12.** 32 | **21.** 321 | **30.** 70 |
| **4.** 43 | **13.** 234 | **22.** 411 | **31.** 61 |
| **5.** 12 | **14.** 211 | **23.** 312 | **32.** 92 |
| **6.** 22 | **15.** 230 | **24.** 110 | **33.** 71 |
| **7.** 11 | **16.** 122 | **25.** 63 | **34.** 83 |
| **8.** 14 | **17.** 341 | **26.** 51 | **35.** 81 |
| **9.** 13 | **18.** 224 | **27.** 82 | **36.** 91 |

ANSWERS PAGE 48

| | | | |
|---|---|---|---|
| **1.** 22 R14 | **10.** 157 R9 | **19.** 74 R22 | **28.** 14 R9 |
| **2.** 18 R24 | **11.** 296 R12 | **20.** 46 R21 | **29.** 252 R18 |
| **3.** 17 R6 | **12.** 112 R40 | **21.** 62 R8 | **30.** 9 R41 |
| **4.** 26 R5 | **13.** 137 R2 | **22.** 65 R7 | **31.** 49 R28 |
| **5.** 14 R6 | **14.** 266 R12 | **23.** 49 R64 | **32.** 16 R16 |
| **6.** 15 R42 | **15.** 154 R6 | **24.** 87 R34 | **33.** 62 R9 |
| **7.** 24 R16 | **16.** 118 R65 | **25.** 22 R36 | **34.** 151 R8 |
| **8.** 13 R52 | **17.** 227 R16 | **26.** 91 R34 | **35.** 20 R18 |
| **9.** 17 R24 | **18.** 329 R28 | **27.** 78 | **36.** 178 R26 |





Wouldn't it be nice to have all the right answers all the time?

# RESOURCES

## another form of evaluation

**for progress check** – page 39

| | | | |
|---|---|---|---|
| 1. $64 \times 40 = 2560$ | 2. $37 \times 50 = 1850$ | 3. $72 \times 30 = 2160$ | 4. $46 \times 20 = 920$ |
| 5. $23 \times 32 = 736$ | 6. $41 \times 22 = 902$ | 7. $33 \times 21 = 693$ | 8. $14 \times 12 = 168$ |
| 9. $31 \times 23 = 713$ | 10. $12 \times 41 = 492$ | 11. $32 \times 13 = 416$ | 12. $74 \times 11 = 814$ |
| 13. $25 \times 36 = 900$ | 14. $47 \times 19 = 893$ | 15. $23 \times 27 = 621$ | 16. $38 \times 24 = 912$ |
| 17. $48 \times 32 = 1536$ | 18. $92 \times 54 = 4968$ | 19. $74 \times 83 = 6142$ | 20. $38 \times 76 = 2888$ |
| 21. $77 \times 45 = 3465$ | 22. $63 \times 84 = 5292$ | 23. $98 \times 87 = 8526$ | 24. $56 \times 48 = 2688$ |

**for progress check** – page 41

| | | | |
|---|---|---|---|
| 1. $313 \times 42 = 13{,}146$ | 2. $418 \times 35 = 14{,}630$ | 3. $453 \times 54 = 24{,}462$ | 4. $356 \times 73 = 25{,}988$ |
| 5. $988 \times 52 = 51{,}376$ | 6. $231 \times 63 = 14{,}553$ | 7. $762 \times 94 = 71{,}628$ | 8. $847 \times 86 = 72{,}842$ |
| 9. $968 \times 98 = 94{,}864$ | 10. $623 \times 49 = 30{,}527$ | 11. $737 \times 62 = 45{,}694$ | 12. $583 \times 38 = 22{,}154$ |
| 13. $545 \times 68 = 37{,}060$ | 14. $798 \times 87 = 69{,}426$ | 15. $567 \times 46 = 26{,}082$ | 16. $856 \times 48 = 41{,}088$ |
| 17. $887 \times 36 = 31{,}932$ | 18. $574 \times 66 = 37{,}884$ | 19. $639 \times 95 = 60{,}705$ | 20. $864 \times 29 = 25{,}056$ |
| 21. $397 \times 400 = 158{,}800$ | 22. $924 \times 200 = 184{,}800$ | 23. $283 \times 700 = 198{,}100$ | 24. $671 \times 500 = 335{,}500$ |

**for progress check** – page 46

| | | | |
|---|---|---|---|
| 1. $4\overline{)84} = 21$ | 2. $5\overline{)55} = 11$ | 3. $3\overline{)69} = 23$ | 4. $2\overline{)48} = 24$ |
| 5. $3\overline{)33} = 11$ | 6. $2\overline{)88} = 44$ | 7. $4\overline{)44} = 11$ | 8. $3\overline{)63} = 21$ |
| 9. $2\overline{)46} = 23$ | 10. $3\overline{)96} = 32$ | 11. $3\overline{)66} = 22$ | 12. $2\overline{)26} = 13$ |
| 13. $3\overline{)369} = 123$ | 14. $2\overline{)862} = 431$ | 15. $4\overline{)840} = 210$ | 16. $4\overline{)484} = 121$ |
| 17. $3\overline{)693} = 231$ | 18. $4\overline{)488} = 122$ | 19. $2\overline{)648} = 324$ | 20. $3\overline{)906} = 302$ |
| 21. $4\overline{)844} = 211$ | 22. $3\overline{)393} = 131$ | 23. $4\overline{)440} = 110$ | 24. $2\overline{)426} = 213$ |
| 25. $4\overline{)168} = 42$ | 26. $6\overline{)246} = 41$ | 27. $5\overline{)355} = 71$ | 28. $2\overline{)186} = 93$ |
| 29. $8\overline{)488} = 61$ | 30. $9\overline{)729} = 81$ | 31. $6\overline{)306} = 51$ | 32. $4\overline{)288} = 72$ |
| 33. $7\overline{)427} = 61$ | 34. $4\overline{)240} = 60$ | 35. $8\overline{)568} = 71$ | 36. $7\overline{)637} = 91$ |

**for progress check** – page 48

| | | |
|---|---|---|
| 1. $29\overline{)536} = 18$ R14 | 2. $35\overline{)619} = 17$ R24 | 3. $58\overline{)934} = 16$ R6 |
| 4. $38\overline{)883} = 23$ R9 | 5. $41\overline{)794} = 19$ R15 | 6. $23\overline{)655} = 28$ R11 |
| 7. $19\overline{)709} = 37$ R6 | 8. $42\overline{)981} = 23$ R15 | 9. $63\overline{)925} = 14$ R43 |
| 10. $52\overline{)6052} = 116$ R20 | 11. $34\overline{)8677} = 255$ R7 | 12. $68\overline{)8486} = 124$ R54 |
| 13. $24\overline{)3683} = 153$ R11 | 14. $71\overline{)9821} = 138$ R23 | 15. $14\overline{)5141} = 367$ R3 |
| 16. $82\overline{)9187} = 112$ R3 | 17. $43\overline{)7111} = 165$ R16 | 18. $46\overline{)8717} = 189$ R23 |
| 19. $42\overline{)1798} = 42$ R34 | 20. $84\overline{)2292} = 27$ R24 | 21. $22\overline{)1895} = 86$ R3 |
| 22. $97\overline{)4251} = 43$ R80 | 23. $39\overline{)2146} = 55$ R1 | 24. $53\overline{)4297} = 81$ R4 |
| 25. $76\overline{)3781} = 49$ R57 | 26. $33\overline{)1039} = 31$ R16 | 27. $61\overline{)4218} = 69$ R9 |
| 28. $24\overline{)795} = 33$ R3 | 29. $13\overline{)3078} = 236$ R10 | 30. $59\overline{)986} = 16$ R42 |
| 31. $35\overline{)2526} = 72$ R6 | 32. $18\overline{)944} = 52$ R8 | 33. $94\overline{)7403} = 78$ R71 |
| 34. $31\overline{)3971} = 128$ R3 | 35. $43\overline{)878} = 20$ R18 | 36. $25\overline{)2829} = 113$ R4 |

**for checkout** – page 52

1. Compute. Watch the operation signs.

| | | | |
|---|---|---|---|
| a) $8659 - 4517 = 4142$ | b) $7483 - 2369 = 5114$ | c) $6942 - 3576 = 3366$ | d) $9130 - 4673 = 4457$ |
| e) $5264 + 1723 = 6987$ | f) $3409 + 2365 = 5774$ | g) $6528 + 2795 = 9323$ | h) $7451 + 6689 = 14{,}140$ |

2. Multiply.

| | | | |
|---|---|---|---|
| a) $56 \times 40 = 2240$ | b) $24 \times 76 = 1824$ | c) $231 \times 46 = 10{,}626$ | d) $597 \times 84 = 50{,}148$ |

3. Divide.

a) $7\overline{)497} = 71$ b) $8\overline{)649} = 81$ R1 c) $52\overline{)967} = 18$ R31 d) $34\overline{)8908} = 262$

4. Here's Mary's homework paper. Did she get all of the problems right? If there are errors, show how the problem should be completed.

| | | | |
|---|---|---|---|
| | | | 17 R9 |
| a) | 506 | b) 4706 | c) 10 |
| | × 40 | − 958 | 27)468 |
| | 2240 | 3758 | 270 |
| | 20,240 | 3748 | 198 |
| | | | 189 |
| | | | 9 |

## activities

**1. things** spirit master

Self-checking activity. Prepare a spirit master of grids as shown. Have the pupils select their own 2-digit factors.

| | | | |
|---|---|---|---|
| | 38 | 56 | |
| 25 | (950) | (1400) | _____ |
| 43 | (1634) | (2408) | + _____ |
| _____ + _____ | | | |

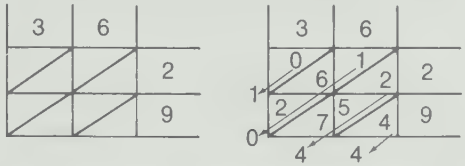
The numbers in parentheses are products obtained as follows:

| | |
|---|---|
| 38 × 25 | 56 × 25 |
| 38 × 43 | 56 × 43 |

To check: Add the products in each row and each column. Record the sums on the appropriate line. Then add the sums vertically and horizontally. This sum should be the same in both directions.

**2.** The lattice method of multiplication was used widely about 500 years ago. The digits of the partial products are arranged diagonally inside the lattice. Each digit is placed systematically, eliminating place-value errors. This method makes a great check for computations performed by the customary algorithm.

To multiply 29 × 36 make a lattice as shown.




Only the facts are needed to fill in the boxes: 2 × 6, 2 × 3, 9 × 6, and 9 × 3. Tens are recorded above the diagonal, ones below the diagonal. To find the product, add from right to left along the diagonal lines. 29 × 36 = 1044. Renaming in addition may be necessary.

Multiplication with 3-digit factors is possible by simply adding another column and another row to the lattice and using the same procedure.

**3.** You are a detective. Can you find the missing digits?

| | | |
|---|---|---|
| 356 | 145 | 417 |
| × 29 | × 68 | × □8 |
| 3□□□ | □1□□ | □□□□ |
| □□□□ | □□□□ | □□□1□ |
| 1□□□□ | □□6□ | □□8□□ |

Solutions:

| | | |
|---|---|---|
| 356 | 145 | 417 |
| × 29 | × 68 | × 38 |
| 3204 | 1160 | 3336 |
| 7120 | 8700 | 12510 |
| 10324 | 9860 | 15846 |

**4.** The complementary method of multiplication encourages mental computation. To find the product of two 2-digit factors proceed as follows:

| Factors | Complement |
|---|---|
| 96 | 11 |
| × 89 | 4 |

1. Find the difference between each factor and 100.
2. Record these differences (complements) inversely as shown.
3. Multiply the complements.
4. Subtract the complement of one factor from the other factor.
   $96 - 11 = 85$ $89 - 4 = 85$
5. Now to name the product. Record the product found in step 3 to show ones and tens. Record the difference found in step 4 to show hundreds and thousands.

| thousands | hundreds | tens | ones |
|---|---|---|---|
| 8 | 5 | 4 | 4 |

For 3-digit factors, find the complement by subtracting from 1000.

Challenge the youngsters to find the rule for naming the product for this example.

| | |
|---|---|
| 997 | 16 |
| × 984 | 3 |
| (981,048) | 48 |

(If the product of the complements is not a 3-digit number, show zero hundreds.)

**5. things** 10 same-size boxes (milk cartons); slips of paper

With pupils who are having difficulty naming the greatest multiple that can be subtracted, focus on estimating the quotient.

Write a division problem on each slip of paper. Make sure that the quotients are all 2-digit. The pupil selects a slip, estimates the quotient, and places the slip in the appropriate box. For example: $6\overline{)129}$. The pupil thinks: There are more than 20 sixes, but less than 30. The slip is placed in the box between 20 and 30.

For practice in estimating 3-digit quotients, label the opposite faces of the boxes as shown and prepare a set of slips containing appropriate problems.

**6.** Until a few hundred years ago division was considered too difficult for everyone to learn. A look at one of the methods used will tell you why. Your job—look in books on the history of mathematics and find out all you can about the galley method for division. Why was it given this name? Try to use the method.

References are Carl B. Boyer, *A History of Mathematics* (New York: Wiley, 1968), p. 241; Robert Swain and E. D. Nichols, *Understanding Arithmetic* (New York: Holt, Rinehart & Winston, 1965), p. 95.

**7. things** game board; small cards; markers

Each group of 2 to 4 players will need a game board and a set of 33 cards. Make three cards for each number from 2 through 12.

| | | | | | |
|---|---|---|---|---|---|
| Move back 2 spaces. | 13 | 77 | 36 | 29 | Move ahead 1 space. Take extra turn. |
| 26 | 99 | 3 | Move back 5 spaces. | 48 | 50 |
| 57 | 115 | | | 70 | 86 |
| 19 | 7 | HOME | | 12 | 72 |
| 64 | 16 | Advance 1 space. Lose next turn. | 81 | 79 | 100 |
| Move back 3 spaces. | 8 | 85 | 20 | 33 | START 17 |

To begin, each player places his marker on START. The cards are shuffled and placed in a stack facedown. In turn, each player turns over the top card and divides the number in the square containing his marker by the number on the card. The remainder tells the player how many spaces he may move his marker, always following the arrows. If the remainder is zero, the marker is not moved.

After each play, the divisor card is placed facedown under the stack. Play continues until one player succeeds in reaching HOME.

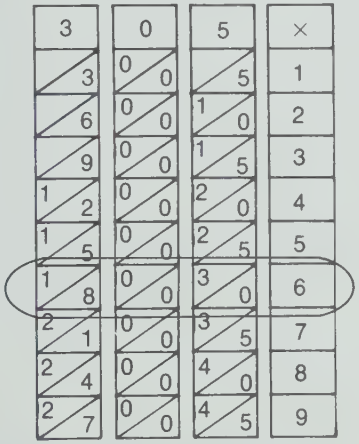
Players should predetermine a rule for an incorrect computation.

**8. things** for each pupil: tongue depressors

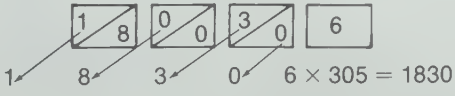
Research project: Find out all you can about John Napier and his bones.

As a result of the research, pupils may want to make their own set of Napier's bones, using tongue depressors.

To form the bones, write a numeral from 0 through 9 at the top of 10 depressors. Write the multiples of the number written at the top in the boxes below as shown—tens above the diagonal, ones below the diagonal. The last depressor is the multiplier bone.




To multiply 305 by 6, line up the multiplier with the bones for 3, 0, and 5 in the appropriate place-value positions. Locate the 6-row on the multiplier bone. Move across the row from right to left, adding along the diagonal lines to find the product.




**9.** Someone made a mistake in each of these problems. You want to help this person. Can you find the errors and tell him what he should have done?

| | | |
|---|---|---|
| 605 | 7601 | 16 |
| × 30 | −5431 | 6 |
| 18,450 | 2230 | 10 |
| (18,150) | (2170) | 35)563 |
| | | 350 |
| | | 213 |
| | | 210 |
| | | (16 R3) |

**10.** Many years ago the scratch method of addition was very popular. It's a fun method to use. Find out why "scratch" is an appropriate name. Try to use the method on columns of four or five really big numbers.

One reference is Robert Swain and E. O. Nichols, *Understanding Arithmetic* (New York: Holt, Rinehart & Winston, 1965), p. 71.

# additional learning aids

**operation**—chapter objectives 1, 2, 3

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: W-32, 33, 34, 35, 36
*Computapes*, SRA (1972)
Module 4, Lessons: MD 23–31, 34–36
*Computational Skills Development Kit*, SRA (1965)
Multiplication cards: 1–25
Division cards: 1–22
*Cross-Number Puzzles (Story Problems)*, SRA (1966)
Whole Number cards: 1–5, 11–15
*Cross-Number Puzzles (Whole Numbers)*, SRA (1966)
Multiplication cards: 1–17
Division cards: 1–14
Mixed-practice cards: 1–10
*Diagnosis: an instructional aid—Mathematics Level B*, SRA (1972)
Probes: M-3, 4
*Math Applications Kit*, SRA (1971)
Everyday Things cards: 1, 15, 20, 38, 39
*Mathematics Involvement Program*, SRA (1971)
Card: 367
*Skill Modes in Mathematics*, SRA (1974)
Level I, Molecules: A, B, C, D
Level II, Molecules: A, C, D
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 12, 13, 14, 17, 18, 19, 23, 56, 61

**other learning aids** (described on page 72j)—I Win (sets 2 and 3), Mathfacts Games*, Napier's Rods, Numble**, Orbiting the Earth (multiplication and division), Rally with Remainders, Ting, Veri-Tech Senior (multiplication book), Winning Touch

*Trademark of Milton Bradley Co.
**Trademark of Sigma Scientific, Inc.

# 3 GEOMETRY CONGRUENCE

**before this chapter the learner has –**

1. Sorted a set of geometric shapes by shape (triangles, circles, rectangles)
2. Sorted a set of similar geometric shapes by size
3. Identified shapes that are symmetrical by folding a model
4. Made a tracing of a figure

**in chapter 3 the learner is –**

1. Testing for congruent figures in a set of similar figures by tracing and then matching
2. Testing for congruent figures in a set of figures that have been flipped or turned by tracing and then matching
3. Making a congruent copy of a figure on graph paper
4. Exploring the concepts of area and perimeter

**in later chapters the learner will –**

1. Find the area and the perimeter for a figure
2. Identify congruent figures that have been moved by a flip, turn, or slide

# Notes & Things

This chapter should be a joy for one and all! The pure mathematical concept of congruence is secondary to the exploration of the idea through tracing and matching, turning and flipping figures, and the investigation of some of the applications of this concept. (The learners will have plenty of time for more formal study of congruence in later levels.)

A look at the real world points out the need for uniformity – replacement parts, standard-size doors, standard-size containers. The demands of our society for manufactured goods have created the notion of mass production. Mass production yields objects of uniform size and shape. Same size and shape is the essence of the concept of congruence. Yet when we try testing any two real objects for congruence, we run into difficulty. We can try fitting, but not always with 100 percent success. Ideas related to congruence can be explored far more easily on a flat surface.

The chapter's approach is one of discovering by doing. Through a set of sequential experiences, the students will look at shape, at size, and at patterns. They will prove two figures are congruent by making a movable model such as a tracing, and fitting the model exactly over another figure. Area and perimeter are also related to congruence. The student will be introduced to finding area by computation in preparation for more study of these ideas that comes in a later chapter.

This approach is a far cry from formal proofs and geometric constructions, and yet it will appropriately prepare the learner for that study.

## things

2 identical milk cartons
2 identical geometric solids with flat surfaces
graph paper
straightedges
paper straws or plastic straws and pipe cleaners

For the extra activities you will want to have these things available:

mirrors
geoboard
rubber bands of different colors
geopaper
colored pencils
potatoes
paint
needlework yarn
scrap tile
5 x 8 index cards
tracing paper

**goal** Think about and explore ideas through a picture clue

**page 54** Let the students decide for themselves what objects are shown in the photograph. Maybe no one will know and that's O.K. too, because the important questions relate to size and shape.

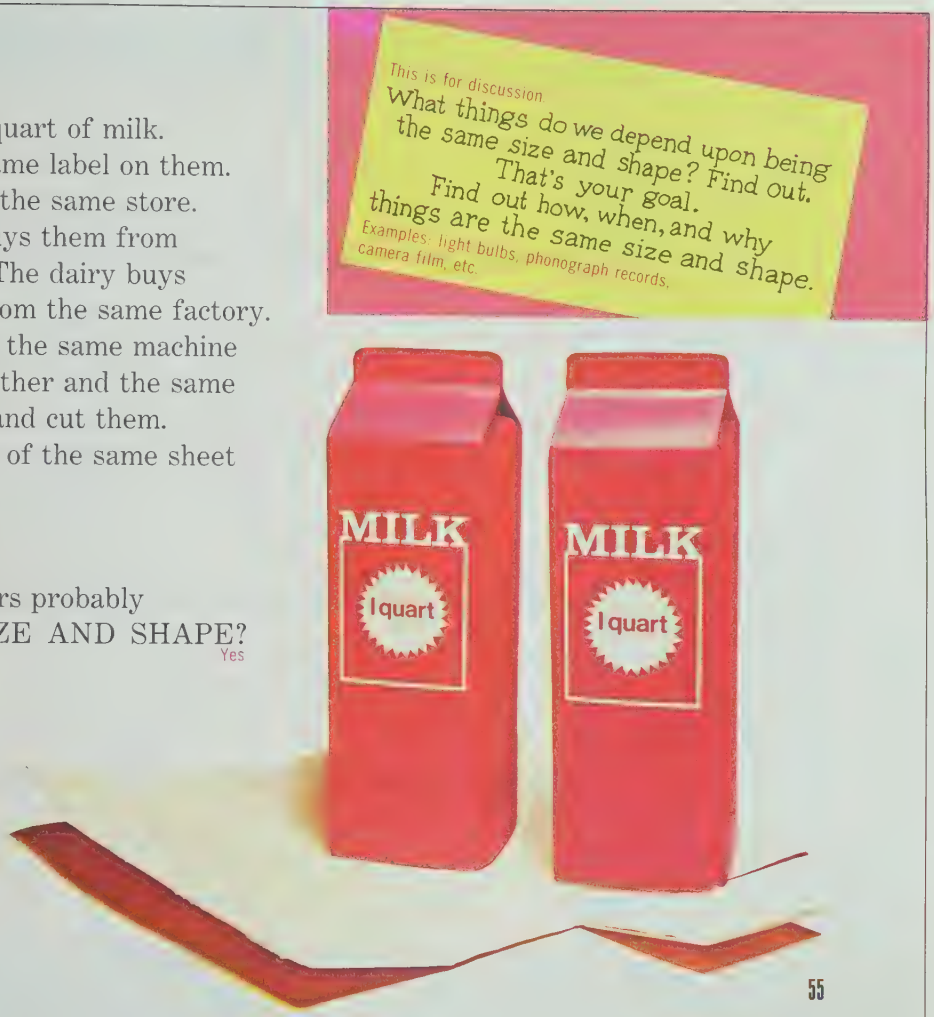
Younger children have trouble seeing shapes drawn or photographed in perspective. Find out if any members of your group still are having problems by asking some direct questions. Is each object in the photograph probably the same size and shape? If you measured the objects in the photograph would each have the same size? If you measured several of the real objects, would they probably be the same size? Do you think these objects are manufactured or made by hand?

Are all things that are the same size necessarily the same shape? Are all things the same shape necessarily the same size? What are some things that are the same size and the same shape?

Go on into the chapter itself. Everyone should be ready!

Two containers.
Each holds one quart of milk.
Both have the same label on them.
Both come from the same store.
And the store buys them from
the same dairy. The dairy buys
the containers from the same factory.
The factory uses the same machine
to put them together and the same
machine to fold and cut them.
They are cut out of the same sheet
of cardboard.

Are the containers probably
THE SAME SIZE AND SHAPE?
Yes


This is for discussion
What things do we depend upon being the same size and shape? Find out.
That's your goal.
Find out how, when, and why things are the same size and shape.
Examples: light bulbs, phonograph records, camera film, etc.
MILK
1 quart
MILK
1 quart

55

**lesson** Page 55

**goal** Determining whether 2 objects have the same size and same shape

**things** 2 identical milk cartons
2 identical geometric solids with flat surfaces

**page 55** Testing real objects for same size and same shape is a difficult task. Let your pupils discover this for themselves by examining 2 actual (identical) cartons. Can you prove that these containers really have the same size and same shape? How? Does it help to lay one on top of the other? Can you match each part? Now try 2 solid geometric figures. Are these easier to match?

Mass-produced objects are not always produced to exact measure—there is a "measure of error" allowance. Under the production conditions discussed in the text we can **say** that the products have the **same size** and the **same shape**, but we cannot actually prove our statement.

The last question should open the door to much discussion. What if the replacement part of an automobile is not the same size and shape as the original?

**lesson** Pages 56, 57, 58, 59

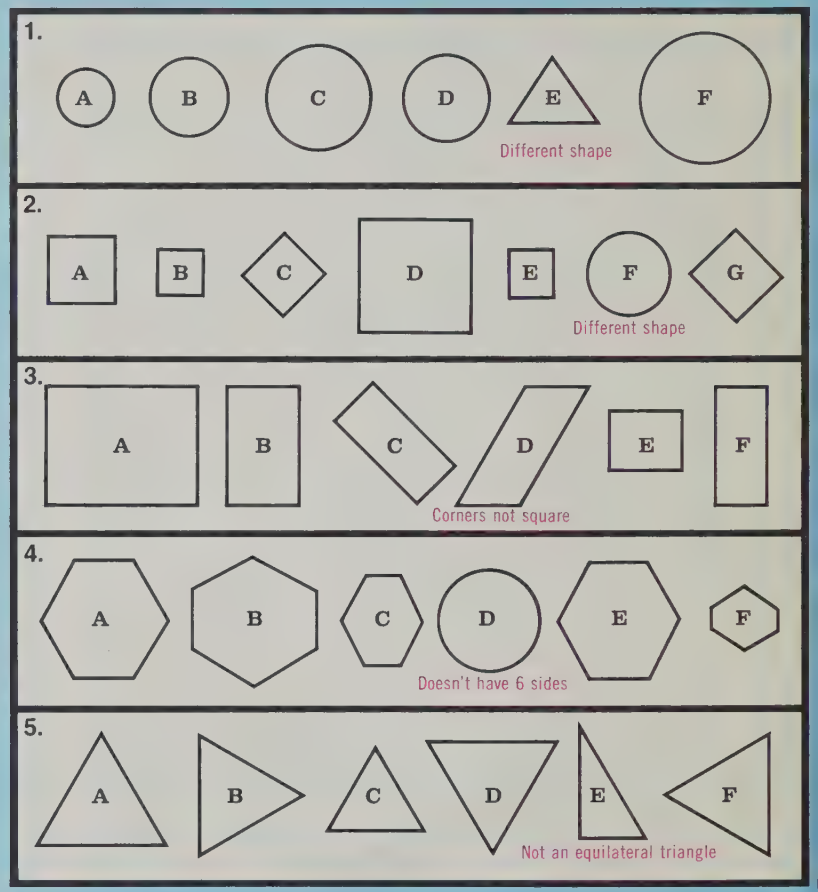
**goal** Identifying figures having the same shape

**memo** Pages 56, 57, and 58 appear simple, but they are very important. They provide readiness for the concept of congruence—figures that have the same size **and** the same shape.

**page 56** The focus is on **same shape**. Figures that have the same shape, but not necessarily the same size, are similar. There is no need to develop this vocabulary now—merely an awareness. Which figure does not belong in each set? Why not? How are the other figures in the set alike? How are they different?

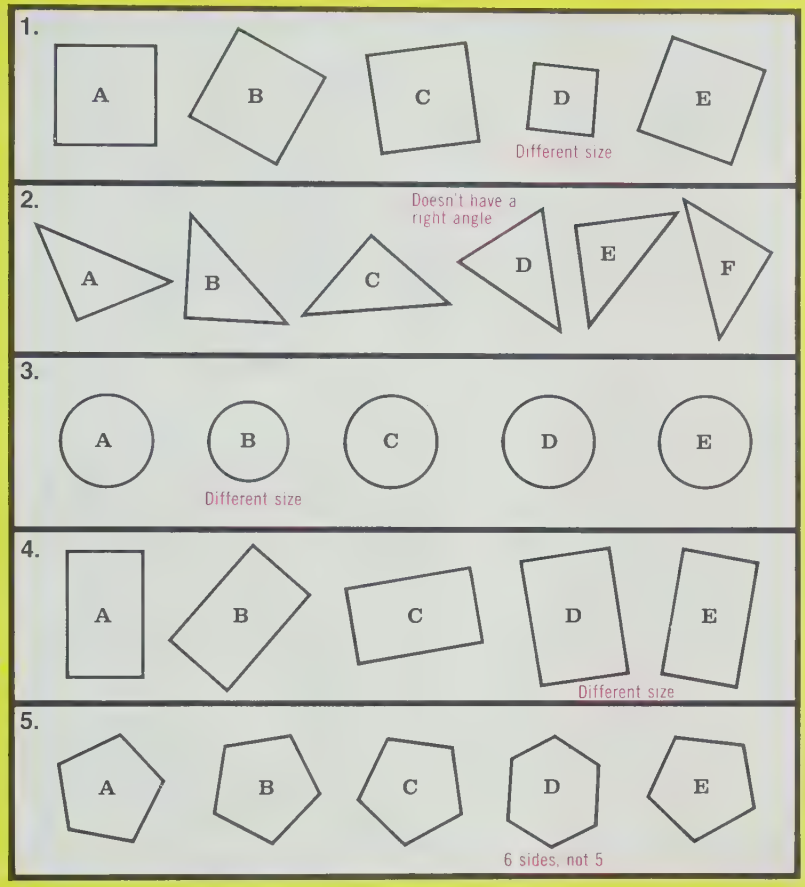
Careful with problem 5—triangle E has a square corner. Prove this figure is different from the others by having the pupils make a tracing and then lay the tracing over the corners of the other triangles.

Look at each row.
Name the figure that does not belong.
Why doesn't it belong?




56

Look at each row.
Name the figure that
does not belong.
Why doesn't it belong?




57

**goal** Identifying figures having the same size

**page 57** The focus is on **same size**—with two examples based on **same shape** included. (No one will fall into a rut this way.) The pupils should trace one figure and lay the tracing over each of the remaining figures to prove his answers. Simply counting the number of sides will do the trick for problem 5.

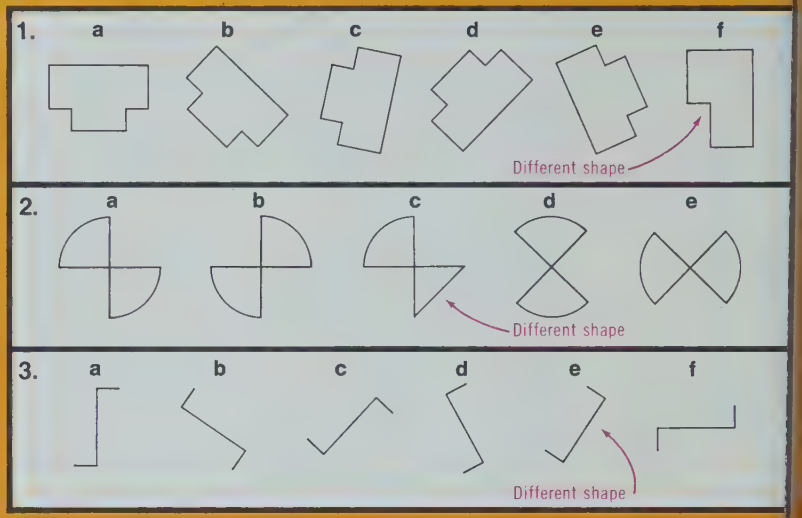
**goal** Identifying figures having the same size or the same shape

**page 58** Encourage your pupils to use tracing and matching to find their own answers.

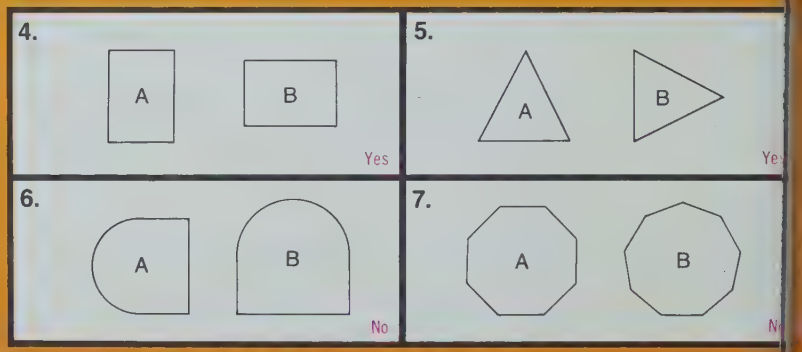
Problem 3 includes a figure that has been flipped. Challenge everyone to trace figure **3a** and try to make the tracing fit figure **3f**. Then ask them what they had to do with their paper. (Flip it.)

Problems 4 and 5 involve a different motion to make the tracing fit. Caution the youngsters to observe **how** they must move their papers to make the figure fit. These problems involve a turn.

Look at each row. Name the figure that does not belong. Why doesn't it belong?




Look at each pair of figures. Is figure A the same size and shape as figure B?




58

**1.**
Three of these rabbits are the same shape and the same size. Find them. Would it help to do some tracing? Yes

**2.**
Here is a pattern of a car. Trace it.
Find the other two pictures of a car that have been drawn from the same pattern.

**3.**
If a picture is drawn from a pattern, does it have the same size and shape as the pattern?
It should.

59

**goal** Identifying figures that have the same size and the same shape

**page 59** Which rabbit do I begin with? Let everyone make that decision for himself. Praise the one who realizes that a critical look at all the rabbits first will narrow down the testing problem.

Problem 2 may be somewhat simpler than problem 1—the pupil knows which model must be matched from the pattern.

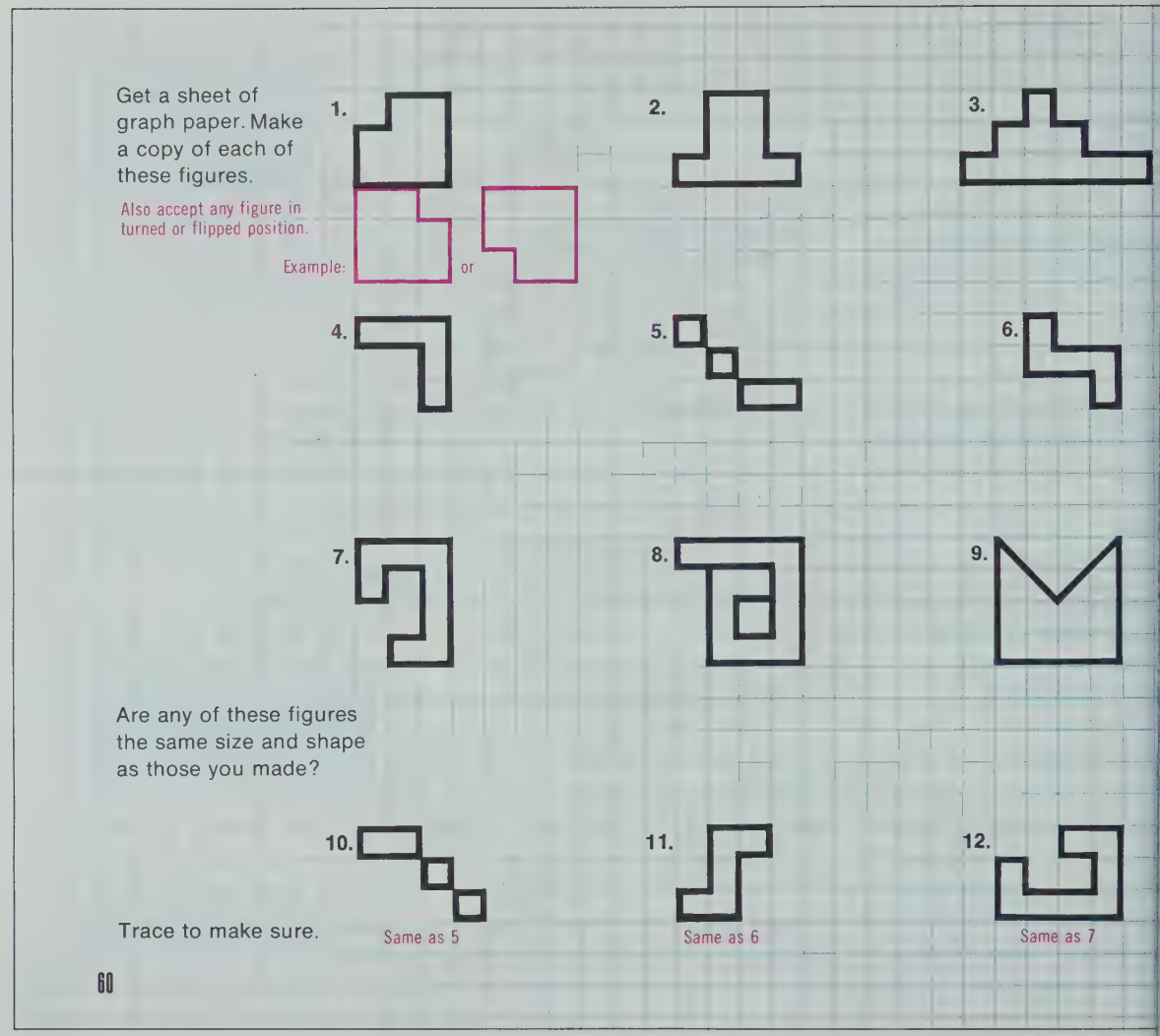
**lesson** Page 60

**goal** Making a congruent copy of a figure on graph paper

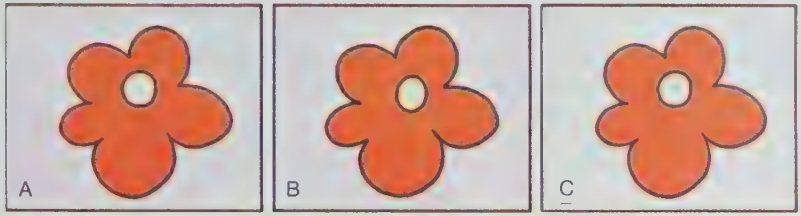
**things** graph paper
straightedges

**page 60** Each figure can be reproduced by simply counting square units and drawing matching segments. A straightedge will help keep the segments straight. The copy can be tested by laying it over the figure on the page.

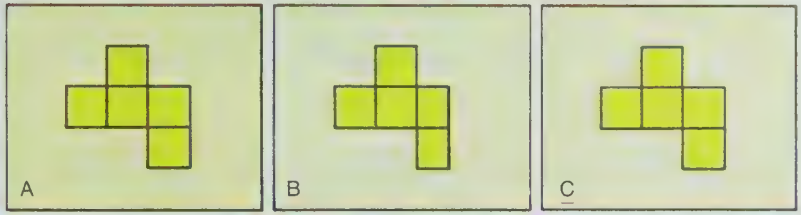
Challenge your pupils to observe how they move their tracings for figures 10, 11, and 12 to find the figures that match. A turn is made each time.



**1.**
Trace A. Which figure is the same as A?

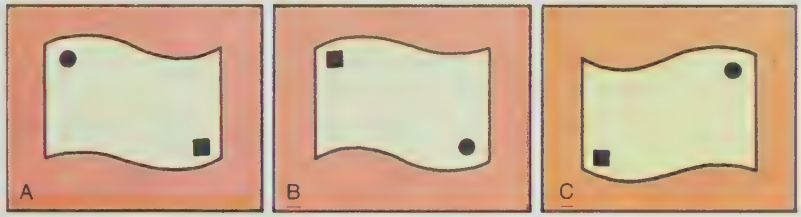



**2.**
Trace A. Which figure is the same as A?




**3.**
Trace A. Which figure is the same as A?

Both B and C

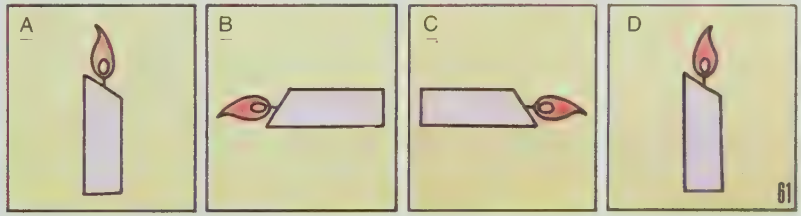



(turned) (flipped)

When a tracing of a figure matches another figure, you can say the two figures are congruent. That means they are the same size and shape even though they may be in a different position.

**4.**
Which of these figures are congruent?

(turned) (turned and flipped)




61

**lesson** Pages 61, 62

**goal** Introduction to the term CONGRUENT for figures that have the same size and same shape

**memo** Technical vocabulary for motions in geometry is not introduced at this level. Figures that have been **turned** or **flipped** are naturally involved when identifying figures that are congruent. These words are far more descriptive for the pupil than the words rotations and reflections.

**page 61** Some students have fine visual perception and need not trace each figure – however, do discourage guessing.

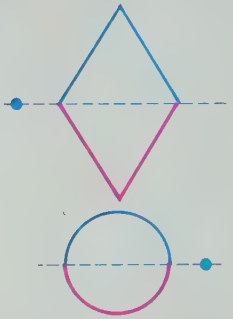
**Congruent** is now part of the pupil's vocabulary. The youngster who makes a sloppy tracing is literally correct when he states, "This figure is not congruent," even though the answer key may indicate otherwise. No figure in the book may be congruent to his particular tracing.

Do not emphasize this point – just accept his answer as correct. Congruent figures must match exactly in size and shape.

**goal** Introduction to the motion of flipping a figure

**page 62** On preceding pages the youngsters have had to FLIP some of their tracings in order to identify all the figures of the same size and shape. Here this action is combined with tracing to create a new figure. Plain notebook paper will serve as tracing paper. Praise the student who recognizes the dotted line in the three examples as a line of symmetry—no need to discuss this unless you want to. Are the halves of a symmetrical figure congruent?

Each of these drawings has a broken line on one side and a dot on that line. Use tracing paper to trace the drawing, the line, and the dot.



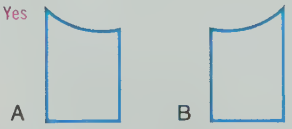
Now flip your tracing paper so that the line you drew matches the line in the drawing. Make the dot you drew also match the dot on the picture. The original picture looked like a triangle. What does the new image look like? A diamond

Try it again. Trace. Flip. Match the line. Match the dot. The original picture looked like a half-circle. What does the new image look like? A circle

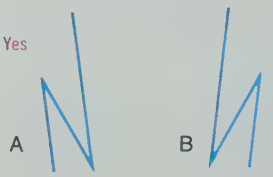
Look carefully at this one. Trace every mark shown in the figure. Then flip your tracing. Match the line and the dot.

Now try these. Can B be a flip of A? Trace and make sure.

**1.** Yes




**2.** Yes




Do you see a flip of you when you look in a mirror? Yes

62

**things** mirrors

Examine the figures on the page with a small mirror. Will a mirror show a flip?

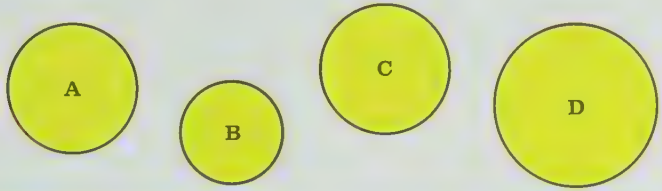
What about this example?

Will a mirror do the trick here? Why not? Try two mirrors. Do they help to show a flip? Can figures be flipped in any direction?

**1**

Are circles always the same shape? Are they always the same size? Which of these circles are congruent? A and C

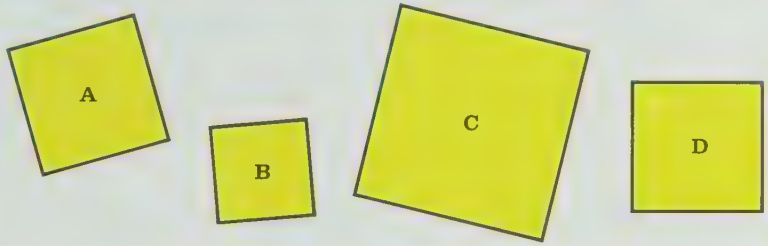
Yes
No




**2**

Are squares always the same shape? Are they always the same size? Which of these are congruent? A and D

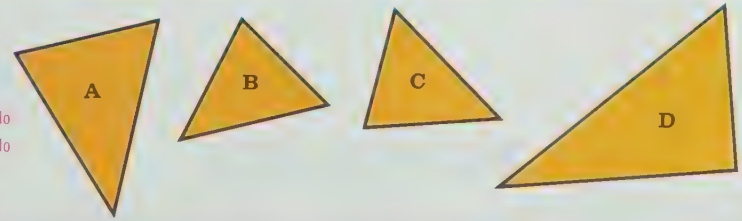
Yes
No




**3**

Are triangles always the same shape? Are they always the same size? Which of these triangles are congruent? B and C

No
No




**4** Does it make any difference in the looks of a circle or a square if either of the shapes is flipped? No

63

**lesson** Pages 63, 64, 65

**goal** Identifying some characteristics of circles, squares, and triangles

**page 63** Discourage guessing. If in doubt, pupils should test by tracing and matching the figures. Have them try flipping a tracing of a circle and a square to be **sure** of their answer. Ask what is alike about all triangles. (3 sides, 3 corners) What makes some triangles different? (Length of sides, size of corners) Will tracing help to find out how they are alike and how they are different?

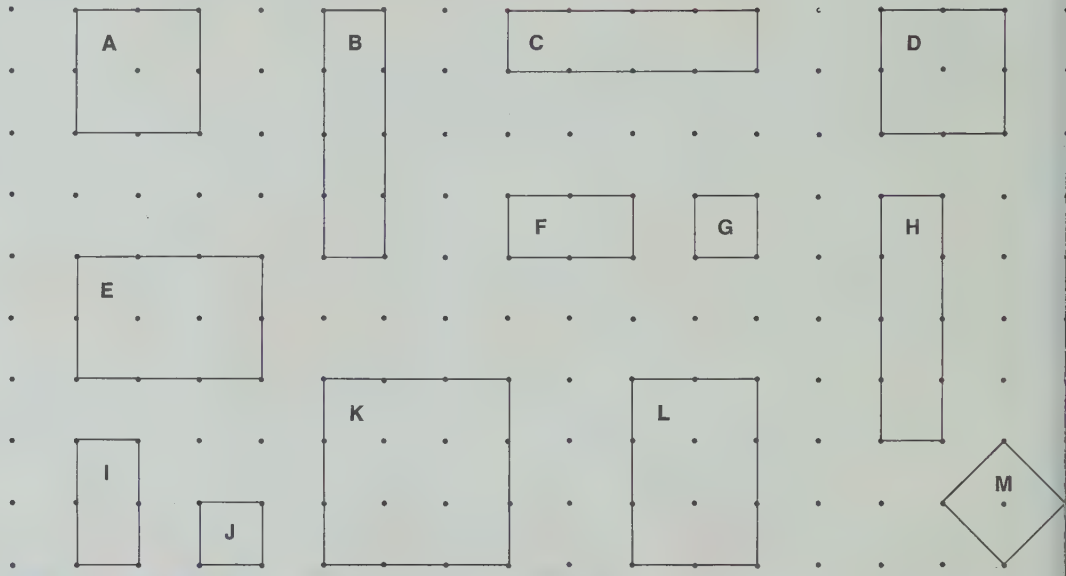
Ask why flipping doesn't make a difference for circles and squares. What about triangles?

**goal** Identifying congruent rectangles; **Progress Check** – identifying congruent figures

**page 64** More capable students could approach this page with area sentences rather than tracing or counting – if they already know about area. Figures with equivalent sentences are congruent. Example: $3 \times 2 = 6$ and $2 \times 3 = 6$ are equivalent sentences. For two figures to have equal areas is not enough – wait for page 68 to make this discovery.

Counting units and matching the length of each side is another way to identify congruent rectangles. What is alike about all rectangles? (4 square corners, 4 sides) What parts of *every* rectangle are congruent? (Corners) What makes some rectangles different? (Length of sides)

Are rectangles always the same shape? No
Are they always the same size? No
Which of these rectangles are congruent? (A,D) (E,L) (F,I) (G,J) (B,C,H)



## PROGRESS CHECK

Which pairs of figures are congruent? Skill: Finding congruent figures by tracing and matching

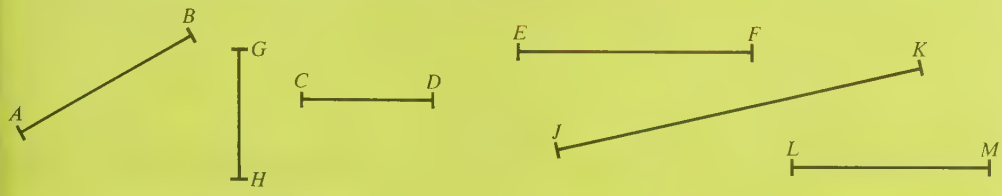
Pairs 1, 3, 5

64

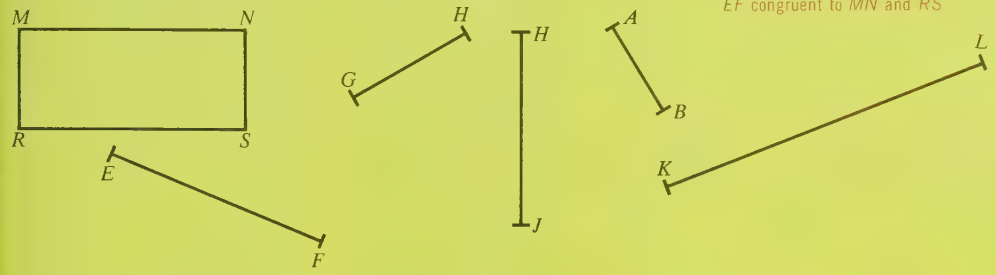
See activity 1, page 72a.

See activity 2, page 72a.

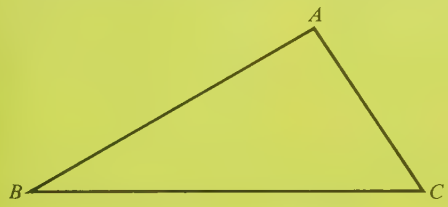
**1.** Which line segments are congruent? *AB* and *LM*, *CD* and $\overline{GH}$

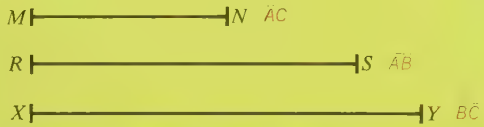



**2.** Which two sides of the rectangle are congruent? $\overline{MN}$ and $\overline{RS}$, *MR* and *NS*
Is there another line segment that is congruent to these sides? *AB* congruent to *MR* and *NS*.
*EF* congruent to *MN* and *RS*




**3.** Are any sides of the triangle congruent? What is the greatest number of congruent sides a triangle could have? No 3




Are any of these line segments congruent to any side of the triangle?

65

**goal** Identifying congruent line segments

**things** paper straws or plastic straws and pipe cleaners

**page 65** Use tracing rather than rulers to test for congruent segments.

Problem 3 should lead to quite a discussion. Challenge pupils to prove their answers. Here are two possible techniques:

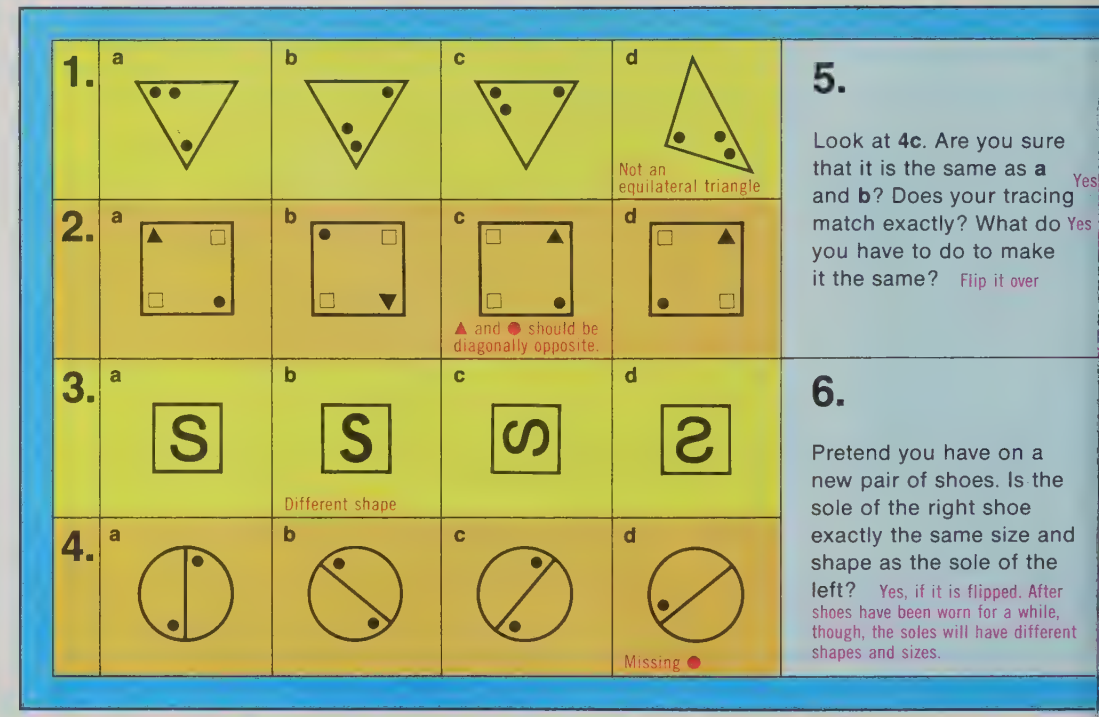
- Flatten paper straws, cut them to desired lengths, and lay out the figure on a flat surface.
- Cut plastic straws to desired lengths and connect each corner with a bent piece of pipe cleaner.

**lesson** Pages 66, 67

**goal** Identifying congruent figures involving a pattern

**page 66** Something new has been added—patterns. Shape **and** the pattern within the shape are both important for problem 1. The shapes are congruent for each of the remaining problems. For these, congruence depends on pattern only. Discuss the importance of tracing the pattern exactly when making a test model.

Look at each row. Name the figure that does not belong. Why doesn't it belong? Be careful—these are tricky. Trace to make sure.




**5.**

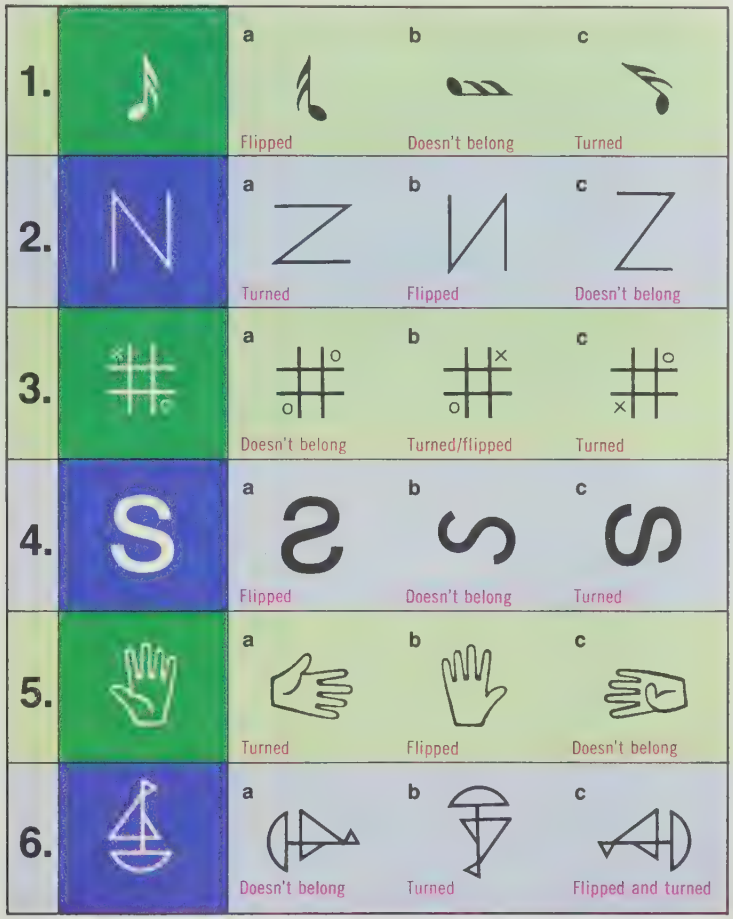
Look at **4c**. Are you sure that it is the same as **a** and **b**? Does your tracing match exactly? What do you have to do to make it the same? Yes Yes Flip it over

**6.**

Pretend you have on a new pair of shoes. Is the sole of the right shoe exactly the same size and shape as the sole of the left? Yes, if it is flipped. After shoes have been worn for a while, though, the soles will have different shapes and sizes.

66

Look at the pattern. Then look at the tracing. One tracing was turned. One tracing was flipped. And one got messed up and just doesn't belong. What happened to each tracing?




67

**goal** Identifying tracings that have been flipped, turned, or are not congruent

**page 67** Continue to encourage making a tracing to prove an answer.

Problem 5 provides quite a challenge. By tracing and testing for congruence, the pupil will reply:

a) turned b) doesn't belong
c) doesn't belong

But the pupil who works with his 3-dimensional hand will reply:

a) turned b) flipped
c) doesn't belong

This problem points out that not all that is seen in the three-dimensional real world can be set down on a flat piece of paper. There are two sets of answers, either of which is correct. This problem should lead to quite a debate.

**things** potatoes; paint; paper; needlework yarn; cardboard; scrap tile; mirror

Flips, turns, and slides can be shown in interesting art patterns. These patterns can be made in a number of ways:

- Potato prints. Cut a pattern into a thick slice of potato or foam rubber, dip the pattern into paint, and print on paper to create a design.
- Needlework patterns. Designs can be sewn on heavy cardboard.
- Tile patterns. Use colored cardboard or scrap tile obtained from a local store.

A mirror will help verify illustrations of flips.

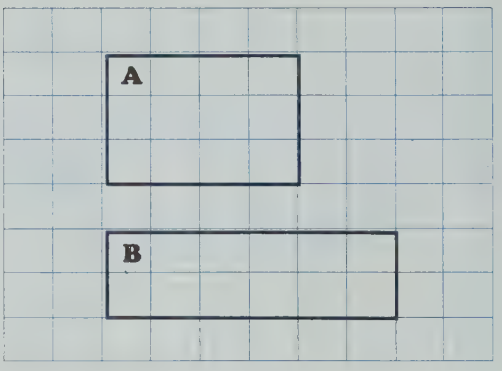
**lesson** Pages 68, 69

**goal** Examining area as it relates to congruent figures

**page 68** The focus is on AREA. There is no need to introduce the special name perimeter for the distance around a figure now. This is done in a later chapter. Concentrating on **area**, **height**, and **base** is enough for now.

Some sophisticated ideas are presented. Consider what is alike about rectangles A and B. (Area) What is different? (Distance around, shape) What is alike about rectangles C and D? (Area, distance around or shape, size) What is different? (Nothing) If you know two figures are congruent, does this tell you something about their area? (The same) About the distance around each of these figures? (The same)

Area is the number of square units covered by a plane figure.

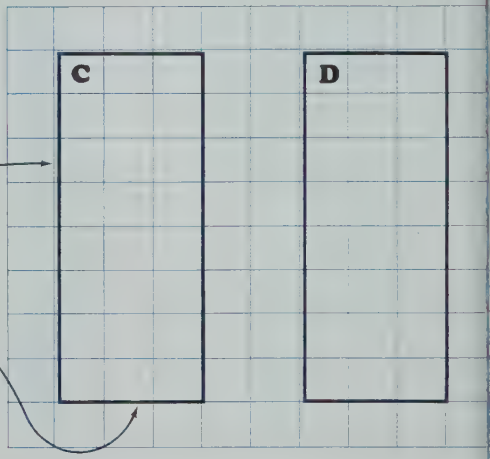



Time out to look at the area of a rectangle.

1. Does the distance around a rectangle tell you anything about the number of squares covered by the rectangle? No
   Check rectangles A and B to make sure.
2. How many squares are covered by rectangle A
   The area of A is ? square units. 12
3. What is the area of rectangle B? 12 square units
4. Are rectangles A and B congruent? No
   How do you know? Different shapes

5. Are rectangles C and D congruent? Yes
   How do you know? Same size and shape
6. Does rectangle C have the same area as rectangle D? Yes
7. How many squares here? 8
   This is the *height* of rectangle C.
   How many squares here? 3
   This is the *base* of rectangle C.

Is there a way to find the number of squares covered other than by adding? Multiply height times the base.




68

**things** graph paper

Challenge the youngsters to make figures that have an area of 36 square units but different perimeters. The shape need not be rectangular. Are any of the figures congruent?

THINK Then answer.

. If two rectangles cover the same number of squares, are they congruent?
No, shapes may be different.

. Can a rectangle and a square cover the same number of squares? Yes

. If a rectangle and a square cover the same number of squares, does that mean that they are congruent? No, shapes may be different.

. Can a rectangle and a square be congruent? Yes

5. Can any sides of a square be congruent? Yes
If so, which ones? All of them

6. Can any sides of a rectangle be congruent? Yes
If so, which ones? If a square, all sides; if not a square, opposite sides

7. Can any sides of a triangle be congruent? Yes
If so, which ones? 2 if isosceles, 3 if equilateral

8. Would any two congruent figures cover the same number of squares? Yes

69

**goal** Examining ideas related to area and congruency

**things** graph paper

**page 69** Time to **think** about the activities on the preceding pages and put it all together. Not sure about an answer? Then use graph paper and experiment.

Problem 1 – Make as many rectangles as possible that cover 18 square units. (1 by 18, 2 by 9, 3 by 6)

Problem 3 – Make as many rectangles as possible that cover 16 square units. (1 by 16, 2 by 8, 4 by 4)

These figures will help answer problems 4, 5, and 6 also.

**lesson** Pages 70, 71

**goal** Examining congruency in the real world

**page 70** There's lots to talk about on this page—perhaps even to argue about. You may want to have some of the items named in the problems on hand to help settle the arguments. A pair of rubber gloves with a different color inside will be especially helpful for problem 3.

Congruent shapes are equal in all ways. They are exact copies of one another. They are the same size *and* the same shape. They must match exactly.

Can solids be congruent?

**1**

Think about cans of fruit juice that are the same brand. Are the cans the same size?* Are they the same shape? Yes Do they have to be? No

*Usually the producer has more than 1 size.

**2**

Think about the bases of light bulbs for a lamp. Are they the same size? Yes Are they the same shape? Yes What would happen if they weren't the same size and shape?

Each bulb or socket would have to be individually made to fit.

**3**

Think about a pair of gloves. Are the two gloves the same size? Yes Are they the same shape?

Yes—one is a flip of the other.

**4**

Think about a coin—a dime, for example. Are all new dimes the same size? Are they the same shape? What would happen if they weren't the same size and shape?

Yes Yes

**5**

Think about floor tiles. If you were going to put them down on a floor, would you depend on their being the same size and shape? What would happen if they weren't?

Yes

You wouldn't know how many to buy, and it would be harder to completely cover the floor.

**6**

Think about film for a movie camera. Does it all need to be the same size and shape? Why?

Yes; so that it will fit standard-size cameras

**7**

Are machine-made items more likely to be congruent than handmade items? Why?

Yes

You couldn't use them in the coin slots of savings banks, vending machines, or parking meters.

Patterns and dies are the same size and shape, although tolerance of as little as .01 cm is expected.

70

Answers will vary. Examples given.

What would happen if—

**1**

Pages of a book were not congruent?
Higher manufacturing cost, harder to use

**2**

All coins were congruent?
Harder for humans and machines to differentiate between coins

**3**

The top of a shoe box were congruent to the bottom of the box? It wouldn't close

**4**

Each plate in a set of dishes were a different size and shape? Difficult to store

**5**

The size and shape of every candy bar were different?
Harder to package and stock on shelves and in vending machines

**6**

All doors were different in size and shape? More expensive, since all would have to be custom-made

**7**

Every "quart" were different in size and shape? Difficult to store

**8**

All theater seats were different in size and shape?
Hard to make rows

**9**

The shape of the front tires of a car were different from the shape of the back tires? The car wouldn't ride evenly or smoothly. (Allow difference in size and shape of snow tires on back wheels.)

**10**

All people were the same in size and shape?
The world would be awfully dull

Summary: The demands of our society for the less expensive, mass-produced goods have created many objects of the same size and shape.

71

**goal** Emphasizing the importance of congruence in mass production

**page 71** Turn imaginations loose! You may even have your pupils do a bit of playacting in order to realize the full impact of some of the situations that are described.

**lesson** Page 72

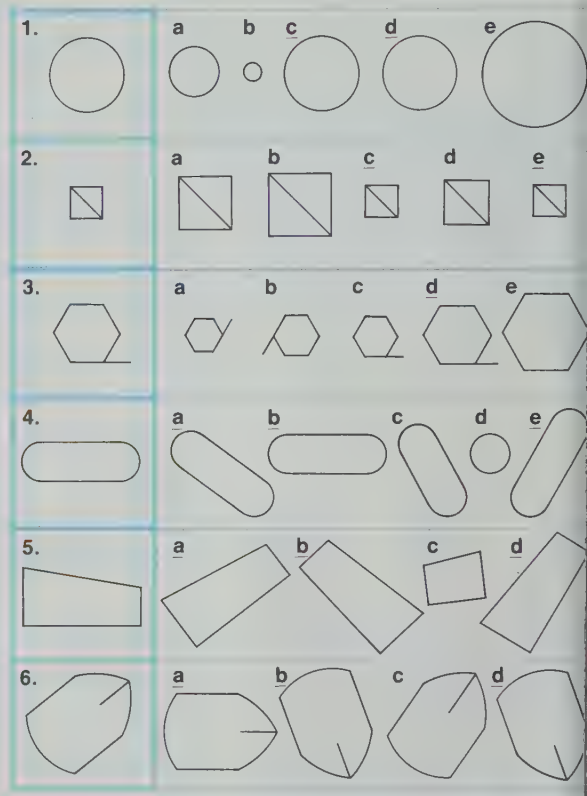
**goal** **Checkout** – identifying congruent figures

**page 72** Some of the problems are very tricky. Encourage everyone to make a movable model by tracing and test to be sure of their answers. Make allowances for pupils who experience difficulty in making a fairly accurate tracing. This may be a visual or motor-coordination problem, rather than one of understanding.

## CHECKOUT

Skill: Finding congruent figures in a set of figures that has been flipped or turned

In each row, which figures are congruent with the one in the box?




72

See activity 3, page 72b.

See activity 4, page 72b.

# RESOURCES

## another form of evaluation

**for progress check**—page 64

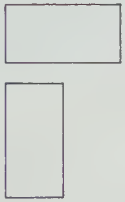
Which pairs of figures are congruent? 1, 2, 4

1.

2.

3.

4.



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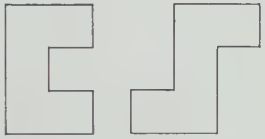
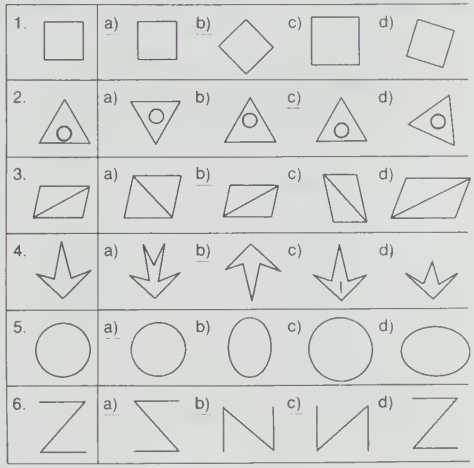


**for checkout**—page 72

In each row, which figures are congruent with the one in the box?




## activities

**1. things** geoboard; rubber bands of different colors; geopaper; colored pencils

Individual activity. (Provide the pupil with the following directions.)

1. Make a 4-by-4 square.
2. How many congruent squares can you find? Use a different color to show each one. Record your findings on the geopaper.
3. Start again. This time find squares that are **not** congruent. (1-by-1, 2-by-2, 3-by-3, 4-by-4)
4. Now make a 4-by-6 rectangle.
5. Look for congruent rectangles. Using different colors will help again.
6. Here's a real challenge. Your 4-by-6 rectangle contains 18 rectangles that are not congruent. Can you find them?

Did you remember that a square is a rectangle?

**2. things** geoboard and rubber bands or geopaper

For this activity use a 5-by-5 array of dots. One more rule—the figures you make **cannot** be congruent.

1. Make as many triangles as you can with exactly one dot inside the triangle.
2. Make as many rectangles as you can with an even number of dots inside each rectangle.
3. Do the same thing, but this time have an odd number of dots inside each rectangle.
4. Now make rectangles with an even number of dots on the perimeter.
5. Finally, make rectangles with an odd number of dots on the perimeter.

Pupils can work in pairs and verify each other's findings.

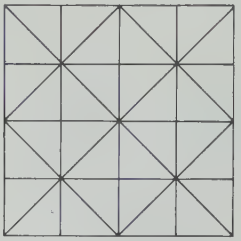
**3. things** $5 \times 8$ index cards

Have each pupil make a card containing figures of the same shape but not necessarily of the same size. Use any of the materials suggested in the activity on page 67.

When the cards are completed, have the youngsters exchange them and challenge a friend to find the congruent figures.

**4. things** copy of figure shown; tracing paper

Challenge: How many triangles can you find in this square? (96)



1. Do the triangles all have the same shape? (Yes)
2. Are the triangles all the same size? (No)
3. Trace each size triangle. How many triangles can you find that are congruent to each of these triangles?

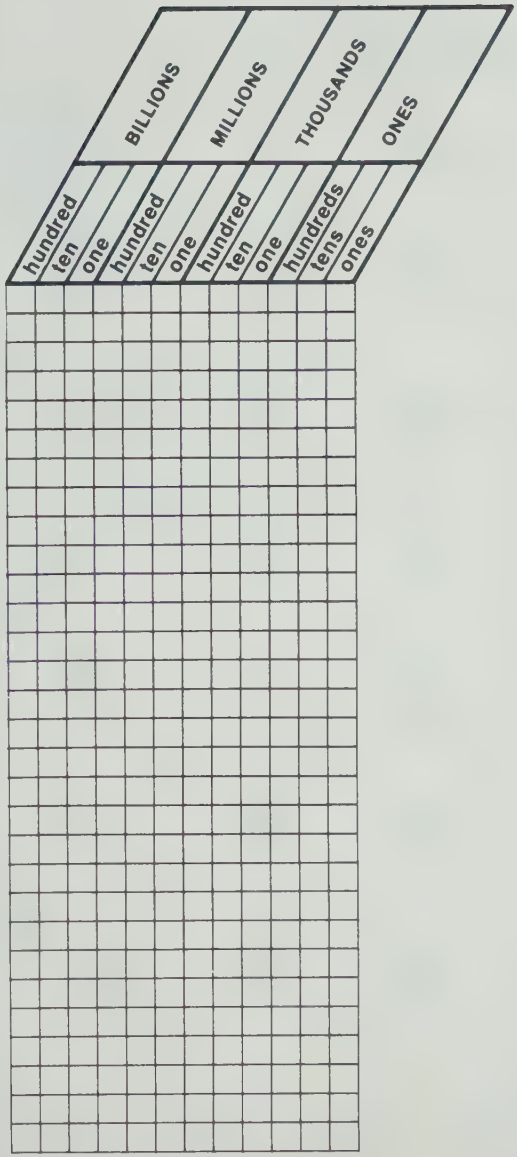
# additional learning aids

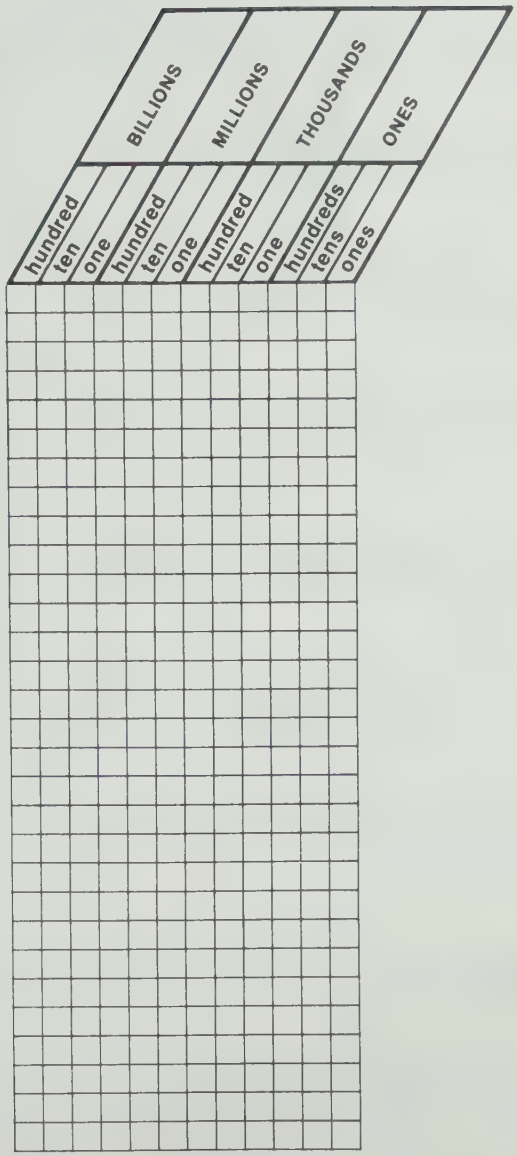
**geometry**—chapter objectives 1, 2, 3, 4

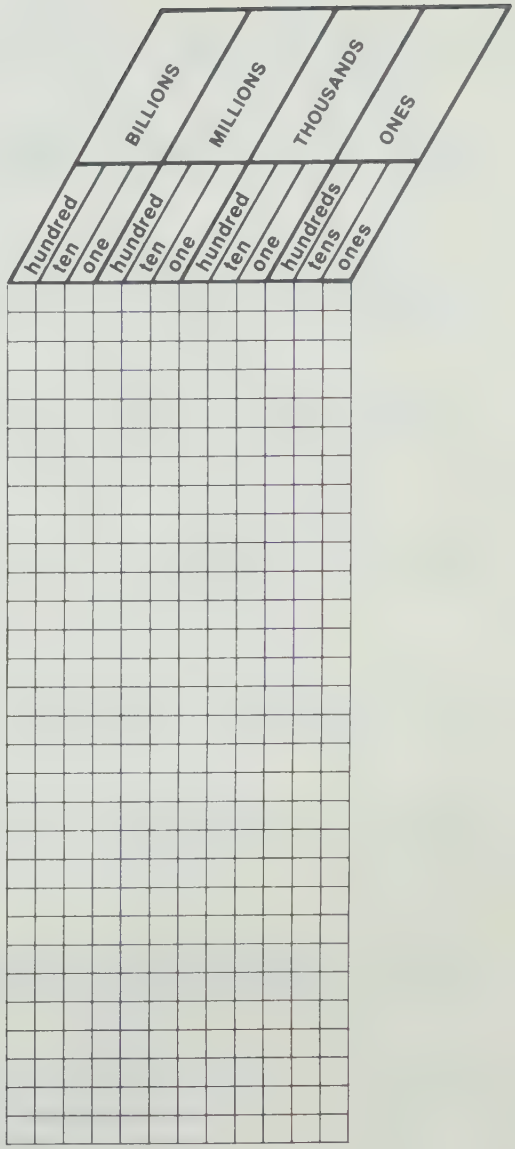
**SRA products**

*Mathematics Learning System*,
*Activity Masters, level B*, SRA (1974)
Spirit masters: G-6, 7, 8; M-9, 10
*Math Applications Kit*, SRA (1971)
Appeteasers cards: 8
Science card: 6
*Mathematics Involvement Program*, SRA (1971)
Cards: 196, 206, 306
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 6, 15, 16, 21, 35, 36, 42, 43, 52, 53, 64

**other learning aids** (described on page 72j)—Geoboard Activity Cards (intermediate set), Geoboards and Motion Geometry, Geoboard Kit, Learn to Fold—Fold to Learn, Mira, Mira Math for Elementary School, Mirror Magic, Paper and Pencil Geometry, Polygons, Shape Tracers, Tangramath, Tangrams

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## Place Value

Use this game board. Find a token to use as a marker. Pick a partner.

| | | | | | | |
|---|---|---|---|---|---|---|
| 130 | 713 | 542 | 985 | 425 | 249 | 476 |
| 316 | 308 | 867 | 960 | 567 | 609 | 176 |
| 713 | 542 | 958 | 542 | 590 | 476 | 248 |
| 790 | 613 | 591 | 349 | 830 | 671 | 258 |
| 824 | 179 | 645 | 825 | 325 | 806 | 718 |
| 956 | 417 | 832 | 470 | 879 | 102 | 910 |
| 130 | 238 | 382 | 630 | 407 | 315 | 340 |

Each of you can put your token anywhere on the board. Put the digit cards facedown in a pile. Pick a card. You can move one space in any direction, but you must move to a number that has your digit. You get as many points as the value of the digit in your number. (70 points for 7 in 476, 700 points for 713, and so on). Take turns. Use all the cards. Highest score wins.

Cut these digit cards apart.

| | | |
|---|---|---|
| 0 | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |
| 9 | 0 | 1 |
| 2 | 3 | 4 |
| 5 | 6 | 7 |
| 8 | 9 | WILD |



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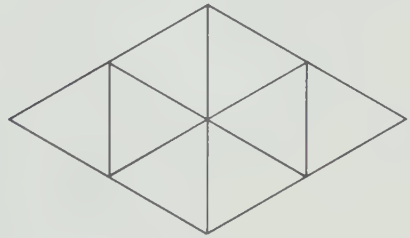
**Start a Super Sleuth box. Collect interesting problems and puzzles.
Put one on each card. Here are two problems to start with.**

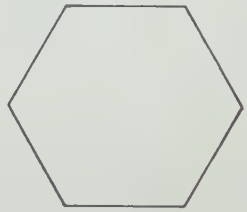
Nuts and bolts in a hardware store are arranged by their diameter size, smallest to largest.
Here is a cabinet where someone has mixed up the drawers. Put them in order from smallest to largest. Start with the smallest at the top.

| |
|---|
| $\frac{3}{8}$ inch |
| $\frac{3}{4}$ inch |
| $\frac{5}{16}$ inch |
| $\frac{1}{4}$ inch |
| $\frac{15}{16}$ inch |
| $\frac{13}{16}$ inch |
| $\frac{1}{8}$ inch |
| $\frac{5}{8}$ inch |
| $\frac{1}{16}$ inch |
| $\frac{7}{8}$ inch |
| $\frac{9}{16}$ inch |
| $\frac{3}{16}$ inch |
| $\frac{7}{16}$ inch |
| $\frac{11}{16}$ inch |
| $\frac{1}{2}$ inch |

Arrange 16 toothpicks like this—
Take 4 away. Don't move any others.
Leave 4 congruent triangular shapes.



Arrange 6 toothpicks like this—
Move only 2. Add 1 more.
Make two congruent diamond shapes.





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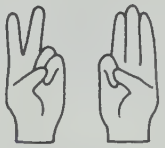
**Here are still other types of Super Sleuth ideas.**

You can count on your fingers, but can you multiply on your fingers? If you know the multiplication facts through 5, you can do any fact on your fingers.

Try $7 \times 8$.

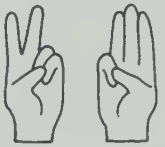
Make 2 fists. Each fist stands for 5. Raise two fingers on your left hand. That stands for 7. (Fist is 5 plus 2 fingers up—that's 7.) Show 8 with your right hand. (Fist is 5, 3 fingers up).

$7 \times 8$



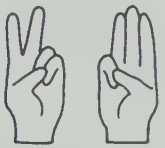
Add the number of raised fingers. That will be the number of tens in the final product. (2 fingers + 3 fingers is 5 fingers—that's 5 tens).

$2 + 3 = 5$(tens)



Now multiply the fingers not raised. (Don't forget your thumb.) 3 fingers down on your left hand and 2 fingers down on your right. $3 \times 2 = 6$. This is the number of ones in the final product.

$3 \times 2 = 6$(ones)



You have 5 tens and 6 ones. That's 56. $7 \times 8 = 56$

Your turn. Find out if your fingers can show these products.

1. $7 \times 9$
2. $6 \times 8$
3. $9 \times 6$
4. $7 \times 7$
5. $8 \times 8$
6. $9 \times 8$
7. $9 \times 9$
8. $6 \times 6$

Here is a list found in an old recipe book in the 1860s. It is called "Food for a Barn Raising." When a new barn was built, friends and neighbors gathered to help. At the end of the day, they shared this meal:

115 lemon pies
500 fat cakes (doughnuts)
15 large cakes
3 gallons of applesauce
3 gallons of rice pudding
3 gallons of cornstarch pudding
16 chickens
3 hams
50 pounds of roast beef
300 rolls
16 loaves of bread
6 pounds of dried prunes, stewed
1 large crock of stewed raisins
5 gallon stone jars of white potatoes; the same amount of sweet potatoes

Enough food for 175 men.

Suppose you were having a house-raising party with only 25 people. Reduce this meal to the appropriate amounts.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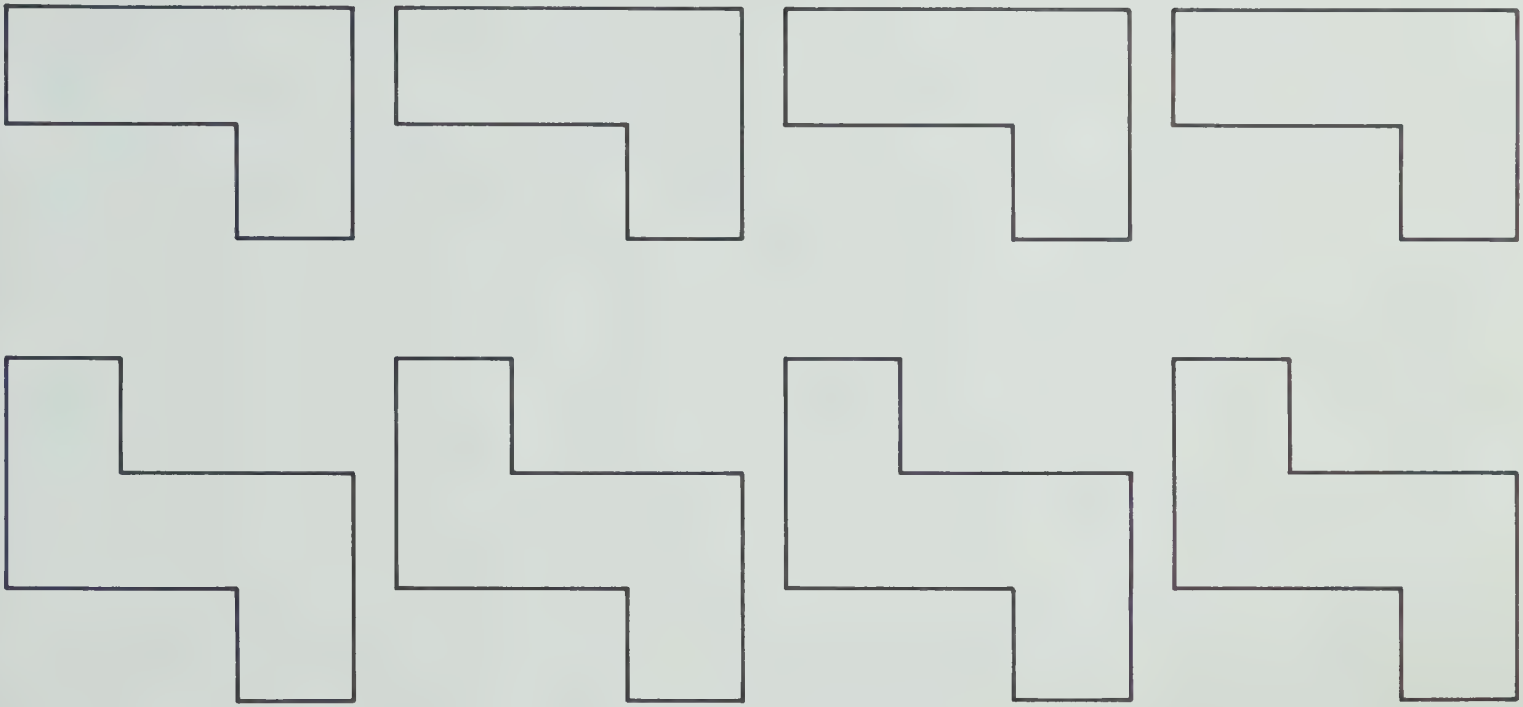
Paste this sheet on card board.
Cut out the puzzle pieces.
Put them together to form a square shape.



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# Other Learning Aids

## whole-number notation

**Abacus board** (Creative Publications) Counting board useful for teaching place value
**Chip Trading** (Scott Scientific) Game to develop an understanding of place value
**Place Value I and II** (Creative Publications) Self-correcting cards to provide practice in reading numbers through hundred millions

## whole-number operations

**Dial-A-Matic Adding Machine** (Sigma Scientific) A simple calculator for practice in addition and subtraction
**Dividing Machine** (Developmental Learning Materials) Self-checking machine to be used for practice with the division facts
**Good Time Mathematics** (Holt, Rinehart & Winston) A multimedia program to give activity-based learning experiences
**I Win** (Scott, Foresman) [sets 1, 2, and 3] Card game for practice in basic operations
**Japanese Abacus** (Creative Publications) An abacus for place value and basic operations
**Mathfacts Games** (Milton Bradley) Self-checking games for multiplication and division facts
**Napier's Rods** (Sigma Scientific) Rods for practice in multiplication
**Numble** (Sigma Scientific) Crossword-type number game for basic operations
**Orbiting the Earth** (Scott, Foresman) Game to provide practice in all four operations
**Prime-O** (Creative Publications) Card game to provide practice in prime factorization
**Rally with Remainders** (Math Shop) A self-correcting game providing division practice
**Sequence** (Math Shop) Puzzle-type game that uses addition combinations to find patterns
**Ting** (SEE) A jigsaw puzzle for multiplication
**Triscore** (Creative Publications) Games for practice in the basic operations
**Veri-Tech Senior** (ETA) [addition, subtraction, and multiplication books] A self-checking device that provides practice with operations
**Winning Touch** (Ideal) Game for reinforcement of multiplication facts

## fractional-number notation

**Decimal Fraction Dominoes** (Mind/Matter Corp.) Game for practice in recognizing relationships of fractions and decimals
**Decimal/Fraction Matching Cards** (SEE) Cards to aid in learning about decimals and fractions
**Experiments in Fractions** (Math Shop) Activities for notation and operations
**The Fat Fraction Game** (ETA) Card game for drill in simplifying fractions
**Fraction Bars Student Activity Book** (Creative Publications) Games and activities to teach fractions and their operations
**Fraction Dominoes** (SEE) Game involving matching a fractional numeral with its model
**Fraction Line Set** (Sigma Scientific) Activity to help visualize operations by computing with fraction strips
**Fraction Tally** (Math Shop) Game for practice in addition and subtraction
**Fractional Number Cards** (Math Shop) Cards used with a geoboard for finding equivalent fractions
**Tripletts** (Math Shop) A rummy-type card game for identifying equivalent fractions

## fractional-number operations

**Action Fraction Games** (Constructive Playthings) Game to develop concepts and skills
**Fraction Multifax** (Math Shop) Game to reinforce multiplication of fractional numbers
**Mathimagination** (Math Shop) [book D—Fractions] Puzzles to reinforce operations

## geometry

**Geoboard Activity Cards** (Creative Publications) [intermediate set] Activities for the geoboard
**Geoboard Kit** (Cuisenaire) Plastic geoboards and activity cards that show geometric concepts
**Geoboards and Motion Geometry** (Scott, Foresman) Resource book dealing with congruence, coordinates, transformations, and area
**Great Shapes** (Cuisenaire) Game to develop principles of patterns, symmetry, and so on
**Learn to Fold—Fold to Learn** (Lyons & Carnahan) Workbook of paper-folding activities to demonstrate geometric figures and symmetry
**Mira** (Creative Publications) An aid for investigating properties of plane geometry
**Mira Math for Elementary School** (Creative Publications) Activities for the Mira
**Mirror Magic** (Lyons & Carnahan) Workbook activities for exploring the concept of symmetry
**Paper and Pencil Geometry** (Lyons & Carnahan) Activities to develop geometric concepts
**Polygons** (Math Shop) Cards for understanding of basic properties of geometric figures
**Rotation, Translation and Reflection Kit** (Invicta) Activity cards dealing with motion geometry
**Shape Tracers** (Math Shop) Set of basic geometric shapes
**Tangrams** (Creative Publications) Puzzle to aid in the discovery of geometric properties
**Tangramath** (Creative Publications) Book of tangram shapes to assist in learning concepts of shapes, congruence, similarity, and area

## measurement

**The Fatal Foot** (Math Shop) Game for drill in addition of linear measures
**Geometric Ruler** (Math Shop) Folding rule to demonstrate perimeter-area relationships
**The Ghastly Gallon** (Math Shop) Game for practice in operations with liquid measures
**Introducing the Metric System with Activities** (Math Shop) Activities to develop basic understanding of the metric system
**Learning about Measurement** (Lyons & Carnahan) A workbook of activities using the metric and customary systems
**Metric Place Value Chart** (Ideal) Chart for meaning of metric system of measures
**The Perilous Pound** (Math Shop) Game for practice in regrouping weight measures

## statistics and probability

**Block Graph** (ESA) Demonstration set for introducing bar graphs, averages, and so on
**Histogram Board** (ESA) Board for making bar diagrams—to be used with Stern Unit Cubes
**Making and Using Graphs and Nomographs** (Lyons & Carnahan) A workbook for skills in making and reading graphs
**Probability Maze** (ESA) A board to illustrate probability and statistics
**Probability Set** (Invicta) Apparatus to demonstrate simple probability
**Towards Probability** (Cuisenaire) Book of easy experiments dealing with probability

## problem solving and applications

**Foo** (Cuisenaire) Card game for order of operation in open math sentences
**Heads Up** (Creative Publications) Game providing practice with equations
**True or False** (ESA) Game for deciding true or false statements

# 4 DECIMALS

**before this chapter the learner has –**

1. Read and written numbers in money notation
2. Compared amounts of money written in money notation
3. Rounded numbers to the nearest 10, 100, or 1000
4. Worked with fractions having a denominator of 10, 100, or 1000
5. Read and told the value of each digit of a number having 9 or less digits
6. Found the product for 3 or more factors
7. Renamed a common fraction as a decimal fraction and a decimal fraction as a common fraction

**in chapter 4 the learner is –**

1. Mastering reading, writing, and telling the value of each digit for decimal fractions expressed to thousandths
2. Comparing and ordering decimal fractions expressed to thousandths
3. Mastering renaming a common fraction as a decimal fraction and a decimal fraction as a common fraction
4. Exploring writing standard numerals in exponent and factor forms
5. Exploring writing numerals in base five
6. Rounding decimals

**in later chapters the learner will –**

Compute with decimal fractions

# Notes & Things

This chapter's title could have been "The Study of our Numeration System," but that title probably would not have meant very much to the learners even though it would have more properly described the content for you. This chapter is the synthesis of number ideas relating to whole numbers, both common and decimal fractional numbers, and the corresponding notation.

The pupils who have studied earlier levels in this program will not be surprised to see a review of rounding. They will know this skill is important because it is the prerequisite to estimation. The review is included in this chapter, however, to focus on the place value of numbers.

The pupil pages feature money as well as batting averages as a way to think about decimals. These topics should provide some good motivation. You may be delighted to find out how much your pupils know about planets, space travel, and other scientific topics that are featured to point out applications of large numbers.

Please know that the introduction of exponential notation is not intended for mastery but rather as an introduction of things to come in later levels. The pupils need to understand that we have a lot of different ways to record numbers and so far they have used just a few of these ideas. There are more to come.

The final light touch with base five can be used as it is or can be deemphasized or expanded according to your pupils' interests and your wishes.

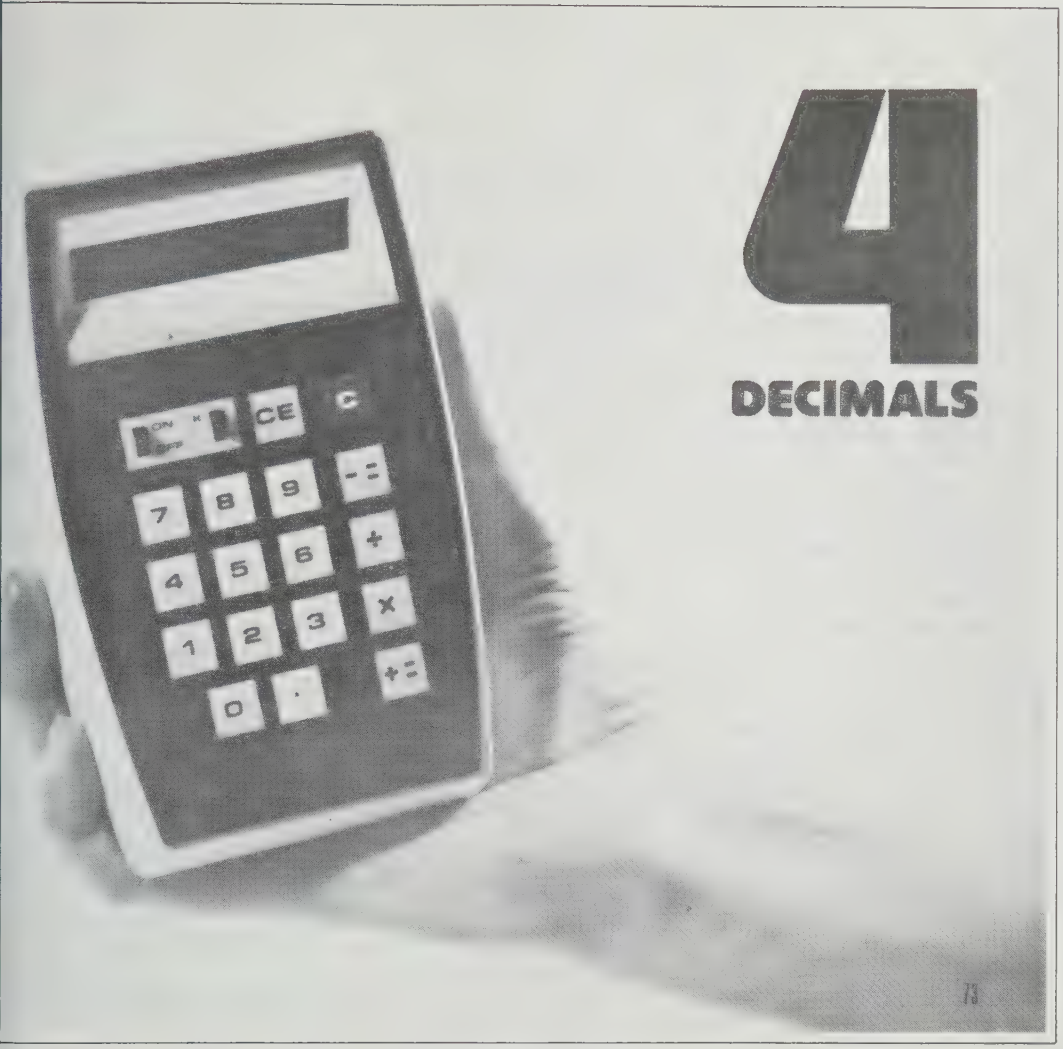
The learners may finally be at an age and achievement level when they can see and understand how simple and neat our numeration system really is. Then they will truly be ready to move on in their study of mathematics beyond the skills of arithmetic.

## things

wood cubes or sugar cubes
graph paper

For the extra activities you will want to have these things available:

- newspapers and magazines
- markers of 2 colors
- wood cubes of 2 colors




# 4 DECIMALS

**goal** Think about and explore ideas through a picture clue

**page 73** Today's children are growing up in a world of computers. Banking processes are increasingly handled by computer, utility and many other bills are processed by computer, and hand calculators such as that shown are becoming familiar in many small businesses, schools, and even some homes. The children may know more about their functions than you would think.

In learning about decimals, pupils will be able to understand some of the principles on which a computer operates. It moves from the realm of magic and gadgetry into that of logic and mathematics. Questions about the photograph on this page, and computers in general, can introduce the subject.

*What is in the photograph? What are its uses? What is its source of power? Notice the arrangement of the digits. Why are they arranged in this way? What other symbols are on the keyboard? What do they tell you about the uses of this little machine? What is the symbol next to the zero?*

What are the other buttons for? (On this calculator, C means Clear—clear the computer of all figures to start again; CE means change—in case of error you can change the last figure you entered without wiping out everything else; K means constant—one figure can be kept constant throughout a series of operations without entering it every time.) These details may be more than most pupils can understand at this time.

If a hand calculator is available, allow

**lesson** Pages 74, 75, 76

**goal** Examining decimal fractions on a number line

**memo** Pages 74, 75, and 76 will help you determine how much your pupils remember about decimals from the level 5 materials of this program. The pace at which you proceed depends on their level of understanding of these basic concepts.

Note the absence of common fractions. This is deliberate. The emphasis is on decimal fractions as a part of our system of numeration rather than as another way to write a common fraction.

**page 74** Imagine that a more powerful magnifying glass has been used for each number line. Look for a number pattern each time the unit is divided into 10 segments. How does this affect the size of the segments? There are 10 times more pieces—but the pieces are much smaller. This idea relates directly to the place-value chart on the next page.

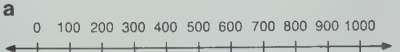
One of the ten parts shows one hundred.

**If you moan when you see a fraction like $\frac{1}{10}$, you'll like these pages.**

YOUR GOAL

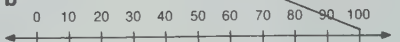
**is to learn about another way to deal with parts less than 1. And you'll learn another way to write big numbers, too.**

a



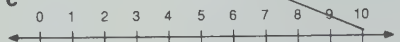

Take part of the number line between 0 and 100. Enlarge it.
One of the ten parts now shows
One ten

b



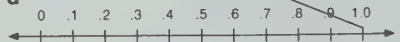

Take a part between 0 and 10. Enlarge it.
One of the ten parts now shows
One one

c



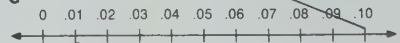

Take a part between 0 and 1. Enlarge it.
One of the ten parts now shows
One tenth

d



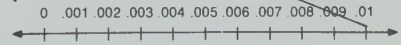

Take a part between 0 and .1. Enlarge it.
One of the ten parts now shows
One hundredth

e




Take a part between 0 and .01. Enlarge it.
One of the ten parts now shows
One thousandth

f

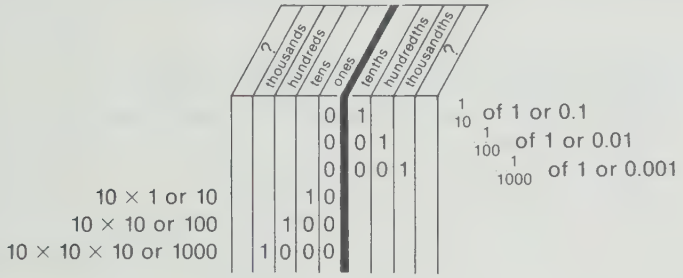



Could you enlarge the part between 0 and .001? Yes

74

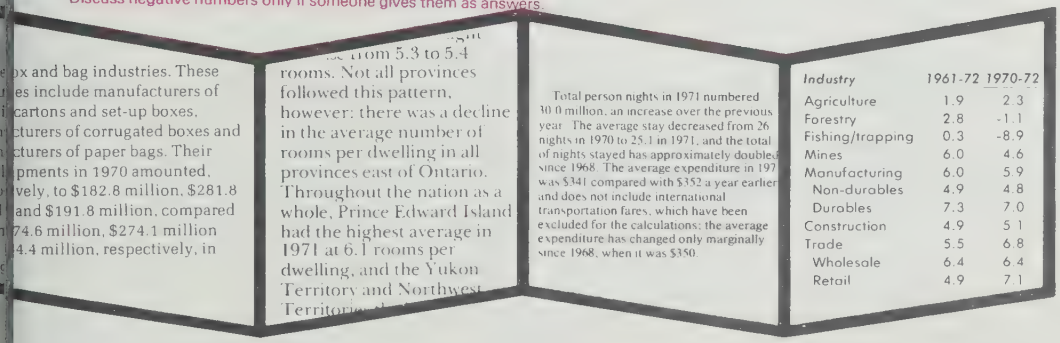
Our number system has only ten digits.
The value of a digit depends on its place in relation to other digits and its relation to a decimal point.

← larger numbers ← • → smaller fractional parts →




List all the decimal fractions you can find below.
On your list, put a ✓ by the smallest number. Put an X by the largest number. ✓ 0.3, -8.9 * x $281.8 million

*Discuss negative numbers only if someone gives them as answers.



...x and bag industries. These ...es include manufacturers of ...cartons and set-up boxes, ...cturers of corrugated boxes and ...cturers of paper bags. Their ...pments in 1970 amounted, ...tively, to $182.8 million, $281.8 ... and $191.8 million, compared ...74.6 million, $274.1 million ...4.4 million, respectively, in

... from 5.3 to 5.4 rooms. Not all provinces followed this pattern, however: there was a decline in the average number of rooms per dwelling in all provinces east of Ontario. Throughout the nation as a whole, Prince Edward Island had the highest average in 1971 at 6.1 rooms per dwelling, and the Yukon Territory and Northwest Territories ...

Total person nights in 1971 numbered 30.0 million, an increase over the previous year. The average stay decreased from 26 nights in 1970 to 25.1 in 1971, and the total of nights stayed has approximately doubled since 1968. The average expenditure in 197... was $341 compared with $352 a year earlier and does not include international transportation fares, which have been excluded for the calculations; the average expenditure has changed only marginally since 1968, when it was $350.

| *Industry* | 1961-72 | 1970-72 |
|---|---|---|
| Agriculture | 1.9 | 2.3 |
| Forestry | 2.8 | -1.1 |
| Fishing/trapping | 0.3 | -8.9 |
| Mines | 6.0 | 4.6 |
| Manufacturing | 6.0 | 5.9 |
| Non-durables | 4.9 | 4.8 |
| Durables | 7.3 | 7.0 |
| Construction | 4.9 | 5.1 |
| Trade | 5.5 | 6.8 |
| Wholesale | 6.4 | 6.4 |
| Retail | 4.9 | 7.1 |

75

**goal** Examining writing decimal fractions and whole numbers

**page 75** The focus is on the decimal point **and** on place value. These can be combined with the 10 digits to write any whole number or fraction. The **decimal point** signals a fractional part. Emphasize that each position moving to the right of the decimal is 10 times smaller than the preceding position and that each position moving to the left of the decimal is 10 times larger than the preceding position.

Why isn't there a ones position in fractions? The lack of a numeral in the ones position to the right of the decimal point causes some pupils trouble in reading decimals. Concentrate on developing the concept of place value rather than putting heavy emphasis on the ability to read the numerals aloud. That comes later.

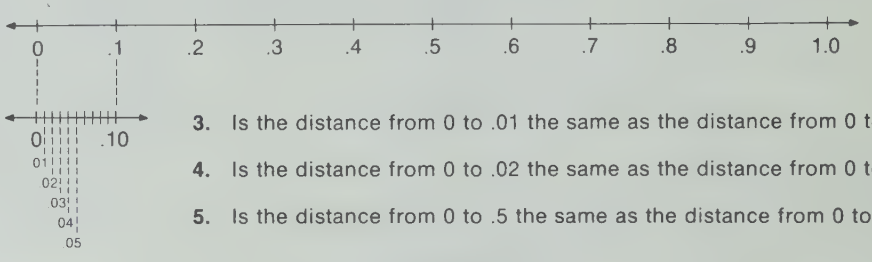
**goal** Comparing decimal fractions

**page 76** Youngsters with good imaginations will have an advantage. Reproduce part of the number line shown to help build understanding and to show how difficult this task is – especially when you try to show and label thousandths. Ask in which direction the whole numbers can be shown on the number line. Encourage everyone to refer to the number line in answering the questions.

## THINK

Imagine one number line marked with tenths and another
marked with hundredths and another
marked with thousandths put on top of one another

1. Would the distance from 0 to .5 be the same as the distance from 0 to .50? Yes
2. Would the distance from 0 to .50 be the same as the distance from 0 to .500? Yes




3. Is the distance from 0 to .01 the same as the distance from 0 to .1? No
4. Is the distance from 0 to .02 the same as the distance from 0 to .2? No
5. Is the distance from 0 to .5 the same as the distance from 0 to .05? No

## LOOK OUT!

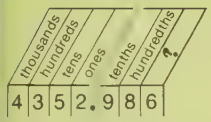
6. Is the distance from 0 to .1 the same as the distance from 0 to 1.0? No
7. Is the distance from 0 to .01 the same as the distance from 0 to 1.0? No

*8. Copy each pair of numbers. Replace ? with $>$, $<$, or $=$ to make each sentence true.

**a** $.1 \overset{<}{?} .4$ **b** $1.0 \overset{>}{?} .9$ **c** $.10 \overset{<}{?} 1.0$ **d** $1.0 \overset{<}{?} 1.1$

**e** $.03 \overset{<}{?} 3.0$ **f** $.20 \overset{>}{?} .02$ **g** $.1 \overset{=}{?} .10$ **h** $1.5 \overset{=}{?} 1.50$

76




What is the place value of the 5?
The number it represents is written 50. Tens

What is the place value of the 9?
The number it represents can be written .9 or $\frac{9}{10}$. Tenths

What is the place value of the 4?
The number it represents is written 4000. Thousands

What is the place value of the 6?
6 thousandths can be written .006 or $\frac{6}{1000}$. Thousandths

Express each of these fractions as a decimal.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 1. | $\frac{2}{10}$ .2 | $\frac{2}{100}$ .02 | $\frac{2}{1000}$ .002 | $\frac{20}{100}$ .20 | $\frac{20}{1000}$ .020 |
| 2. | $\frac{5}{10}$ .5 | $\frac{5}{100}$ .05 | $\frac{5}{1000}$ .005 | $\frac{55}{100}$ .55 | $\frac{55}{1000}$ .055 |

Express each of these decimals as a fraction.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 3. | .651 $\frac{651}{1000}$ | .908 $\frac{908}{1000}$ | .550 $\frac{550}{1000}$ | .938 $\frac{938}{1000}$ | .025 $\frac{25}{1000}$ |
| 4. | .428 $\frac{428}{1000}$ | .230 $\frac{230}{1000}$ | .55 $\frac{55}{100}$ | .026 $\frac{26}{1000}$ | .005 $\frac{5}{1000}$ |
| 5. | .337 $\frac{337}{1000}$ | .066 $\frac{66}{1000}$ | .055 $\frac{55}{1000}$ | .410 $\frac{410}{1000}$ | .600 $\frac{600}{1000}$ |

Tell how many tenths, hundredths, or thousandths.

| | a | b |
|---|---|---|
| 6. | .4 = ? tenths 4 | .873 = ? thousandths 873 |
| 7. | .429 = ? thousandths 429 | .56 = ? hundredths 56 |
| 8. | .9 = ? tenths 9 | .385 = ? thousandths 385 |
| 9. | .50 = ? hundredths 50 | .27 = ? hundredths 27 |

Tell what place value the last digit in each number represents.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 10. | .399 Thousandths | .21 Hundredths | .3 Tenths | .95 Hundredths | .8 Tenths |
| 11. | .160 Thousandths | .20 Hundredths | .766 Thousandths | .001 Thousandths | 10.1 Tenths |

77

See activity 1, page 100b.

**lesson** Pages 77, 78, 79

**goal** Reading and writing decimal fractions

**page 77** The focus continues to be on place value and the decimal point. The digits of a decimal fraction tell the numerator; the number of positions to the right of the decimal tell the denominator.

There are lots of problems, so be selective. A row from each group will survey the pupil's ability. The remaining problems can be used for additional practice of the type that will meet each pupil's needs.

Use this chart to guide assignments over the next several pages.

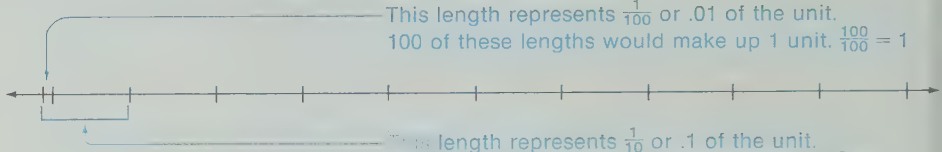
| Trouble with rows | Go to page |
|---|---|
| 1–5 | 78 |
| 6–11 | 79 |
| No trouble at all | 80 |

**goal** Practice in reading and writing decimal fractions

**memo** Use this page with pupils who had trouble with rows 1 through 5 on page 77.

**page 78** Talk about the illustration. Stress the difference in size between .1 and .01 of a unit. Note that the focus is only on tenths and hundredths. Emphasize the number of positions used to write tenths and the number of positions needed to write hundredths.

Suppose that the length of the whole segment drawn on this page represents 1 unit.


This length represents $\frac{1}{100}$ or .01 of the unit.
100 of these lengths would make up 1 unit. $\frac{100}{100} = 1$
This length represents $\frac{1}{10}$ or .1 of the unit.
10 of these lengths would make up 1 unit. $\frac{10}{10} = 1$

1 can be written as 1 or 1.0 or 1.00 or $\frac{1}{1}$ or $\frac{10}{10}$ or $\frac{100}{100}$.

10 means 1 ten ---------- .1 means 1 tenth
100 means 1 hundred -------- .01 means 1 hundredth

Write each of these fractions as a decimal.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 1. | $\frac{4}{10}$ .4 | $\frac{30}{100}$ .30 | $\frac{11}{100}$ .11 | $\frac{20}{100}$ .20 | $\frac{1}{10}$ .1 |
| 2. | $\frac{53}{100}$ .53 | $\frac{8}{100}$ .08 | $\frac{7}{100}$ .07 | $\frac{70}{100}$ .70 | $\frac{84}{100}$ .84 |
| 3. | $\frac{60}{100}$ .60 | $\frac{1}{100}$ .01 | $\frac{50}{100}$ .50 | $\frac{23}{100}$ .23 | $\frac{6}{100}$ .06 |

Write each of these decimals as a fraction.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 4. | .4 $\frac{4}{10}$ | .28 $\frac{28}{100}$ | .60 $\frac{60}{100}$ | .40 $\frac{40}{100}$ | .5 $\frac{5}{10}$ |
| 5. | .59 $\frac{59}{100}$ | .08 $\frac{8}{100}$ | .8 $\frac{8}{10}$ | .2 $\frac{2}{10}$ | .31 $\frac{31}{100}$ |
| 6. | .09 $\frac{9}{100}$ | .44 $\frac{44}{100}$ | .9 $\frac{9}{10}$ | .84 $\frac{84}{100}$ | .90 $\frac{90}{100}$ |
| 7. | .04 $\frac{4}{100}$ | .1 $\frac{1}{10}$ | .01 $\frac{1}{100}$ | .10 $\frac{10}{100}$ | .11 $\frac{11}{100}$ |

78

ou can *explain* decimals to someone by reading them this way:

**a** 0.8 is "8 tenths"

**b** 0.65 is "65 hundredths"

**c** 0.437 is "437 thousandths"

. How do you know that **a** is tenths, **b** is hundredths, and **c** is thousandths?
a. The place value of the 8 is tenths, one place to the right of the decimal point.
b. Two places to the right of the decimal point is hundredths.
c. Three places to the right of the decimal point is thousandths.

hen you *read* decimals aloud, you name each digit one at a time.

14.2 is "fourteen point two or fourteen decimal two".

8.53 is "eight point five three or eight decimal five three".

11.664 is "eleven point six six four or eleven decimal six six four".

is not correct to read a decimal fraction like a whole number. 1.664 should not be read as "eleven point six hundred sixty-four".

. Read each of these decimals aloud.
Zero decimal six eight or zero point six eight and so on.

**a** 0.68 **b** 0.9 **c** 0.47 **d** 0.297 **e** 0.2

**f** 0.707 **g** 0.5 **h** 0.099 **i** 0.38 **j** 0.164

Zero decimal seven zero seven or zero point seven zero seven, and so on

. Read each of these numbers aloud.
Six decimal four five or six point four five and so on

**a** 6.45 **b** 10.6 **c** 9.304 **d** 2.497

**e** 10.051 **f** 6.33 **g** 22.3 **h** 4.002

Ten decimal zero five one or ten point zero five one, and so on



**goal** Practice in reading and writing decimal notation

**memo** Use this page with pupils who had trouble with rows 6 through 11 on page 77.

**page 79** When learning to read whole numbers, the pupil was told not to say "and" between the hundreds and tens. For example: 1346 is read, 1 thousand 3 hundred 46, not "and" 46. Emphasize the point made in the text, that decimal fractions are not read like whole numbers. 11.664 is not read as "eleven point six hundred sixty-four but as "eleven point six six four" (again no "and").

Problems 2 and 3 should be completed orally—giving you the opportunity to hear whether a mistake is being made.

**lesson** Pages 80, 81, 82

**goal** Introduction to ordering decimals

**page 80** Careful of problems **1d** and **1e**. These may be curve balls for some pupils. We're back to an age-old problem. When writing a numeral for a fractional part, the greater number **does not** signal the greater part. When naming a fractional part, the greater number indicates **more parts**. The more parts, the smaller the size of the parts. If all else fails, relating to money in problem 3 should help.

**1.** The numbers go on and on.

Answers will vary.

**a** Name a number between 0 and 100.

**b** Name a number between 0 and 10.

**c** Name a number between 0 and 1.

**d** Name a number between 0 and 0.1.

**e** Name a number between 0 and 0.01.

**And more and more numbers can go between any two numbers.**

**2.** A fraction can be written as a decimal.

Which fraction is the largest — 0.55, $\frac{99}{100}$, $\frac{9}{10}$, or 0.95?

**3.** Canadian and Australian coins represent fractional parts of one dollar.

**a** 10 cents is what fractional part of one dollar? $\frac{1}{10}$ or 0.1

**b** One cent is what fractional part of one dollar? $\frac{1}{100}$ or 0.01

**c** How many cents in ten dollars? 1000
in twenty dollars? 2000
in thirty dollars? 3000
in fifty dollars? 5000

**4.** There are all sorts of ways to write a number.

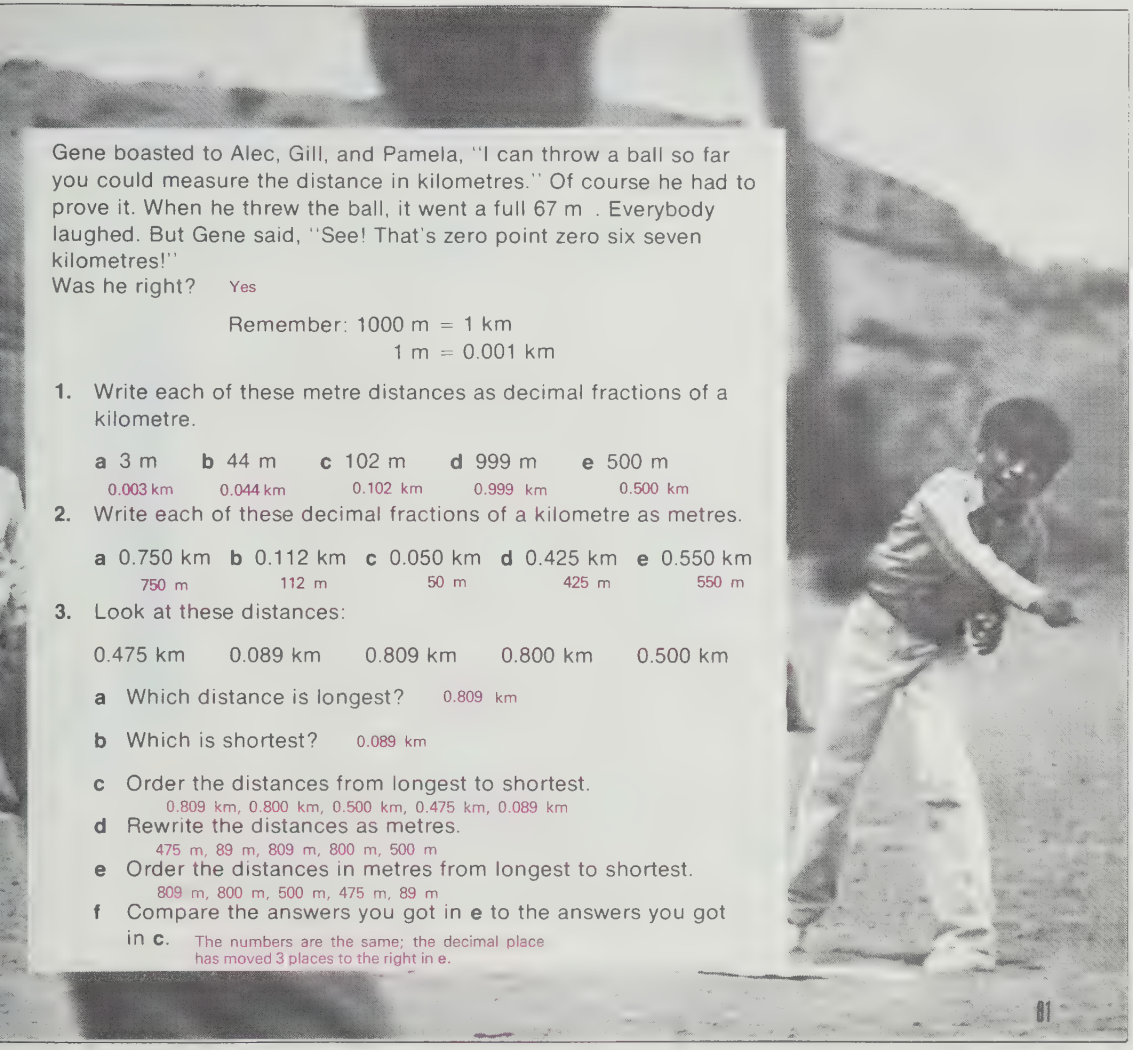
**a** Which of these fractions does not name the number 1?

$\frac{10}{10}$ 1.0 $\frac{1.0}{10}$ $\frac{1}{1}$ 1.00 $\frac{100}{100}$ $\frac{1000}{1000}$

**b** Write two more names for 1.

Answers will vary. Examples: 1.000, $\frac{10\,000}{10\,000}$

80



Gene boasted to Alec, Gill, and Pamela, "I can throw a ball so far you could measure the distance in kilometres." Of course he had to prove it. When he threw the ball, it went a full 67 m . Everybody laughed. But Gene said, "See! That's zero point zero six seven kilometres!"
Was he right? Yes

Remember: 1000 m = 1 km
1 m = 0.001 km

1. Write each of these metre distances as decimal fractions of a kilometre.

   **a** 3 m (0.003 km) **b** 44 m (0.044 km) **c** 102 m (0.102 km) **d** 999 m (0.999 km) **e** 500 m (0.500 km)

2. Write each of these decimal fractions of a kilometre as metres.

   **a** 0.750 km (750 m) **b** 0.112 km (112 m) **c** 0.050 km (50 m) **d** 0.425 km (425 m) **e** 0.550 km (550 m)

3. Look at these distances:

   0.475 km 0.089 km 0.809 km 0.800 km 0.500 km

   **a** Which distance is longest? 0.809 km

   **b** Which is shortest? 0.089 km

   **c** Order the distances from longest to shortest.
   0.809 km, 0.800 km, 0.500 km, 0.475 km, 0.089 km

   **d** Rewrite the distances as metres.
   475 m, 89 m, 809 m, 800 m, 500 m

   **e** Order the distances in metres from longest to shortest.
   809 m, 800 m, 500 m, 475 m, 89 m

   **f** Compare the answers you got in **e** to the answers you got in **c**. The numbers are the same; the decimal place has moved 3 places to the right in **e**.

**goal** Writing, comparing, and ordering decimal fractions

**page 81** Exercises 1 and 2 on this page reinforce the children's grasp of decimals, as most of them are by now fully familiar with a metre as one one-thousandth part of a kilometre.

The children will now begin to appreciate the simplicity of working in the decimal system. There is no "age-old problem" related to larger numbers: with decimals the larger number does indicate the larger fraction. The place value is the key to ordering in problem 2.

Some pupils may need extra help in grasping that 0.089 is the smallest distance.

Be sure that everyone is writing a zero to the left of the decimal point when there is no other number there.

**goal** Practice in comparing and ordering decimals

**page 82** Independent work for everyone. Look for error patterns. It may help some students to rename the decimals so that they have the same place value. For example:

**1c** $.63 \bigcirc ? \bigcirc .629 \qquad .630 \bigcirc ? \bigcirc .629$

Annexing zeros to the right won't change the value of the decimal. This may need a bit of explanation. A quick look at the common fraction should clear things up.

Is $\frac{63}{100}$ equal to $\frac{630}{1000}$?

Then is .63 equal to .630? Problems 3 through 5 will reinforce this idea.

**1.** Write >, <, or = for each ?.

| | | | | | |
|---|---|---|---|---|---|
| **a** | .435 ? .433 | **b** | .025 ? .032 | **c** | .63 ? .629 |
| **d** | .077 ? .66 | **e** | .5 ? .501 | **f** | .71 ? .74 |
| **g** | .3 ? .077 | **h** | .08 ? .009 | **i** | .05 ? .4 |
| **j** | .34 ? .340 | **k** | .3 ? .009 | **l** | .98 ? .981 |

**2.** Write each set of decimals in order from least to greatest.

**a** .62 .619 .089 .09 .6 .621
.089, .09, .6, .619, .62, .621

**b** .3 .92 .093 .34 .339 .4
.093, .3, .339, .34, .4, .92

**c** 2.3 .24 .241 2.31 23.1 2.031
.24, .241, 2.031, 2.3, 2.31, 23.1

**3.** Use a dollar sign and a decimal point.
Write 1 cent. Write 10 cents. Write 100 cents. $.01 $.10
Write 1 dollar. Write 10 dollars. Write 100 dollars.
$1.00; $1.00 $10.00 $100.00

**4.** Think pennies.
Does .01 = .10 = 1.00? No Then does $\frac{1}{100} = \frac{10}{100} = \frac{100}{100}$? No
Does .1 = .10 = .100? Yes Then does $\frac{1}{10} = \frac{10}{100} = \frac{100}{1000}$? Yes

**5.** Think pennies.
Does .05 = .50 = 5.00? No Then does $\frac{5}{100} = \frac{50}{100} = \frac{500}{100}$? No
Does .5 = .50 = .500? Yes Then does $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$? Yes

Form teams of 3 or 4 players. Write two 2- or 3-digit decimal fractions on the chalkboard. Set a time limit. Challenge each team to write as many decimals as possible that are between the two decimals written on the board. A team's score is the number of **different** decimals written. The team with the most points wins.

A team can double points if players can order the decimals they wrote from smallest to largest.

# PROGRESS CHECK

Skill: Telling the value of each digit

Complete.

1. .516 = 5 tenths 1 hundredth 6 thousandths
2. .492 = ? tenths ? hundredths ? thousandths 4 9 2
3. .704 = ? tenths ? hundredths ? thousandths 7 0 4
4. 3.295 = ? ones ? tenths ? hundredths ? thousandths 3 2 9 5
5. .349 = ?*
6. .650 = ? 6 tenths, 5 hundredths, 0 thousandths
7. 14.227 = ?**
8. 5.008 = ? 5 ones, 0 tenths, 0 hundredths, 8 thousandths

Answer the questions. Skill: Naming a decimal between two decimals Answers will vary Examples given.

9. Name two decimals between .210 and .220. .215, .219
10. Name two decimals between .480 and .490. .482, .489
11. Name two decimals between .350 and .360. .351, .357
12. Name two decimals between .35 and .36. .352, .359
13. Name two decimals between .09 and .12. .1, .115
14. Name two decimals between .036 and .046. .038, .044

Skill: Writing common fractions as decimals

Write each as a decimal.

15. $\frac{3}{10}$ .3
16. $\frac{20}{100}$ .20 or .2
17. $\frac{13}{100}$ .13
18. $\frac{13}{1000}$ .013
19. $\frac{3}{100}$ .03
20. $\frac{100}{1000}$ .100 or .1
21. $\frac{10}{1000}$ .010 or .01
22. $\frac{1}{1000}$ .001

Skill: Writing decimals as fractions

Write each as a fraction.

23. .13 $\frac{13}{100}$
24. .6 $\frac{6}{10}$
25. .07 $\frac{7}{100}$
26. .149 $\frac{149}{1000}$
27. .40 $\frac{40}{100}$ or $\frac{4}{10}$
28. .040 $\frac{40}{1000}$ or $\frac{4}{100}$
29. .065 $\frac{65}{1000}$
30. .106 $\frac{106}{1000}$

*3 tenths, 4 hundredths, 9 thousandths
** 1 ten, 4 ones, 2 tenths, 2 hundredths, 7 thousandths

83

See activity 2, page 100b.

See activity 3, page 100b.

**lesson** Page 83

**goal** **Progress Check** – reading, writing, and ordering decimals

**page 83** Problems 1 through 8 require quite a bit of writing. You may want to agree upon abbreviations for the place-value positions.

Pupils who have trouble with problems 1 through 8 will need additional practice with a place-value chart. You may want to duplicate these charts or have the pupil turn lined paper sideways to make his own chart. If page 79 was skipped earlier, you will want to go back now.

Trouble with problems 9 through 14? Then it's back to more work with a number line. Get strips of adding-machine tape. Have the pupils make their own number lines, showing only tenths and hundredths. If page 78 was skipped, use it now with pupils who had trouble with problems 15 through 30. Place-value charts will also prove helpful.

**lesson** Pages 84, 85, 86

**goal** Introduction to rounding decimal fractions on a number line

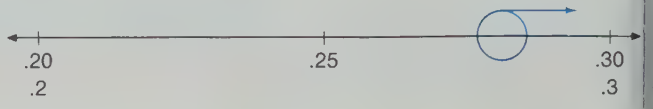
**memo** Rounding is a prerequisite skill for making an estimate. Estimating answers to verify the reasonableness of a computed answer continues to be a major thrust of this program. The prime objective is to achieve computational excellence.

**page 84** Rounding fractions can be difficult in common-fraction form. Once in decimal form, the same basic rules apply as with whole numbers, and the job isn't bad at all.

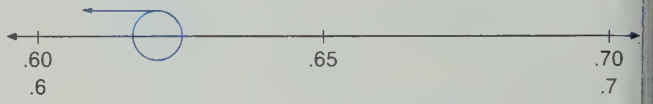
There's lots to talk about in problem 1. Encourage your pupils when in doubt to sketch a number line for the remaining problems.

**1.** More about rounding.

**a** Round .28 to the nearest *tenth*.
.28 is between .20 and .30.
.28 is closer to .30.
.30 = .3, so round .28 to .3.

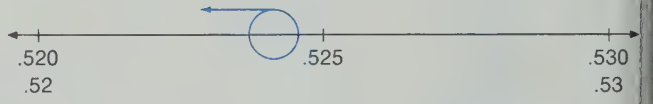



**b** Round .62 to the nearest *tenth*.
.62 rounds to ? . 6

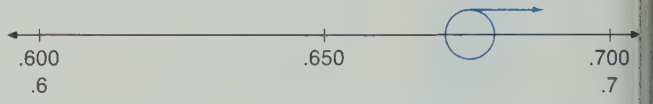



**c** Round .466 to the nearest *hundredth*. .466 rounds to ? . 47

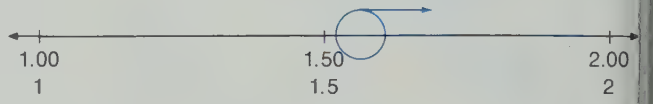
**d** Round .524 to the nearest *hundredth*. .524 rounds to ? . 52




**e** Round .674 to the nearest *tenth*.
.674 rounds to ? . .7




**f** Round 1.56 to the nearest *whole number*. 1.56 rounds to ? . 2




**2.** Round these to tenths.

**a** .75 8 **b** .56 6
**c** .44 4 **d** .567 6
**e** .405 4 **f** .095 1

**3.** Round these to hundredths.

**a** .463 46 **b** .628 63
**c** .456 46 **d** .609 61
**e** .249 25 **f** .048 05

**4.** Round these to a whole number.

**a** 2.7 3 **b** 6.45 6
**c** 10.56 11 **d** 4.09 4
**e** 5.5 6 **f** 3.49 3

84

## Here's another way to round.

Round .76 to the nearest *tenth*. You want one decimal place.
.76 – look at this digit. Five or more? Round to .8.
Yes

Round .845 to the nearest *tenth*. You want one decimal place.
.845 – look at these digits. Fifty or more? Round to .8.
No

Round 1.56 to the nearest *tenth*. You want one decimal place.
1.56 – look at this digit. Five or more? Round to 1.6.
Yes

## Here are more examples.

Round .578 to the nearest *hundredth*. You want two decimal places.
.578 – look at this digit. Five or more? Round to .58.
Yes

Round 1.056 to the nearest *tenth*. You want one decimal place.
1.056 – look at these digits. Fifty or more? Round to 1.1.
Yes

## What will happen now?

Round 1.056 to the nearest whole number or unit. You don't want any decimal place.
1.056 – look at these digits. Five hundred or more? Round to 1.
No

You try these. Think about them in any way you want. BUT round each of these numbers.

1. Round to the nearest tenth.
   **a** .34 .3 **b** .278 .3 **c** 1.346 1.3 **d** 2.45 2.5
2. Round to the nearest hundredth.
   **a** .868 .87 **b** .323 .32 **c** 3.455 3.46 **d** 1.184 1.18
3. Round to the nearest whole number.
   **a** 2.7 3 **b** 1.642 2 **c** 3.584 4 **d** 1.032 1

85

**goal** Examining another method for rounding decimals

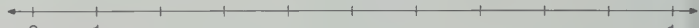
**page 85** The method developed is similar to one used for rounding whole numbers. Some children find it easier than using the number line. The direction for rounding is dependent on thinking greater than or less than halfway.

Encourage pupils to refer back to the top of the page or to a number line if they are unsure as they complete problems 1 through 3.

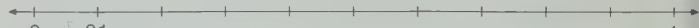
**goal** Practice in rounding decimal fractions to the nearest tenth, hundredth, or whole number

**page 86** Note that each number line shows a different unit. Everyone should be able to work independently.

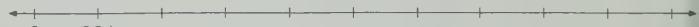
1. **a** Is .6 closer to 0 or 1?
   **b** Is .3 closer to 0 or 1?
   **c** Is .5 closer to 0 or 1? Same distance from both
   Agree that you will round .5 up to the nearest unit.




2. **a** Is .04 closer to 0 or .1?
   **b** Is .09 closer to 0 or .1?
   **c** Is .05 closer to 0 or .1?
   Same distance from both

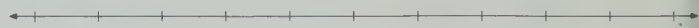



3. **a** Is .007 closer to 0 or .01?
   **b** Is .001 closer to 0 or .01?
   **c** Is .005 closer to 0 or .01?
   Same distance from both



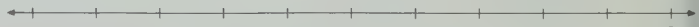

WATCH OUT!

4. **a** Is .026 closer to .02 or .03?
   **b** Is .023 closer to .02 or .03?
   **c** Is .025 closer to .02 or .03?
   Same distance from both




KEEP YOUR EYES OPEN.

5. **a** Is .59 closer to .5 or .6?
   **b** Is .50 closer to .5 or .6?
   **c** Is .55 closer to .5 or .6? Same distance from both




4.876 rounded to the nearest hundredth is 4.88.
4.876 rounded to the nearest tenth is 4.9.
4.876 rounded to the nearest whole number is 5.

6. Round each of these numbers to the nearest hundredth, then to the nearest tenth, and finally to the nearest whole number.

| | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|
| | 14.651 | 20.268 | 14.449 | 9.945 | 9.894 | 8.493 | 6.065 | 99.445 |
| | 14.65 | 20.27 | 14.45 | 9.95 | 9.89 | 8.49 | 6.07 | 99.45 |
| 86 | 14.7 | 20.3 | 14.4 | 9.9 | 9.9 | 8.5 | 6.1 | 99.4 |
| | 15 | 20 | 14 | 10 | 10 | 8 | 6 | 99 |

# PROGRESS CHECK

Round to the nearest tenth. Skill: Rounding decimals

1. .32 .3
2. .48 .5
3. .55 .6

Round to the nearest hundredth.

4. .446 .45
5. .359 .36
6. .273 .27

Round to the nearest whole number.

7. 5.56 6
8. 3.09 3
9. 11.603 12

Skill: Comparing decimals

Which is the smaller number?

10. .36 or .43
11. .127 or .163
12. .447 or .53
13. .23 or .139
14. .462 or .07
15. .287 or .37

Skill: Ordering decimals

Write each set of decimals from smallest to largest.

16. 1.0 .1 .01 1.1 .001
.001, .01, .1, 1.0, 1.1
17. 4.6 46. .46 460. .046
.046, .46, 4.6, 46., 460.
18. 3.7 .96 1.74 .99 .704
.704, .96, .99, 1.74, 3.7
19. .201 .02 .021 .210 .200
.02, .021, .200, .201, .210

Skill: Rounding decimals to whole numbers

Round to the nearest whole number and then find the sum.

20. 8.2 + 4.8
8 + 5 = 13
21. 5.15 + 3.25
5 + 3 = 8
22. 10.6 + 4.35
11 + 4 = 15

87

See activity 4, page 100b.

See activity 5, page 100b.

**lesson** Page 87

**goal** **Progress Check** – rounding, comparing, and ordering decimal fractions

**page 87** Everyone on his own. Encourage anyone who is insecure when rounding to use a number line if this technique has proven helpful. Provide additional practice in using a number line for those who have trouble with any of the problems requiring rounding.

Pupils who have trouble comparing and ordering should first rename the decimals for a problem so that they have the same value and then complete the comparing or ordering job. For example:

- Rename problem 12 to read .447 or .530
- Rename problem 16 to read 1.000 .100 .010 1.100 .001

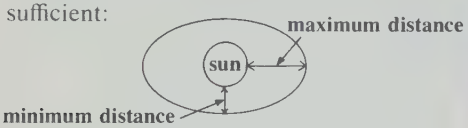
Page 82 may prove helpful with this approach.

**lesson** Pages 88, 89

**goal** Examining numbers having as many as 10 digits

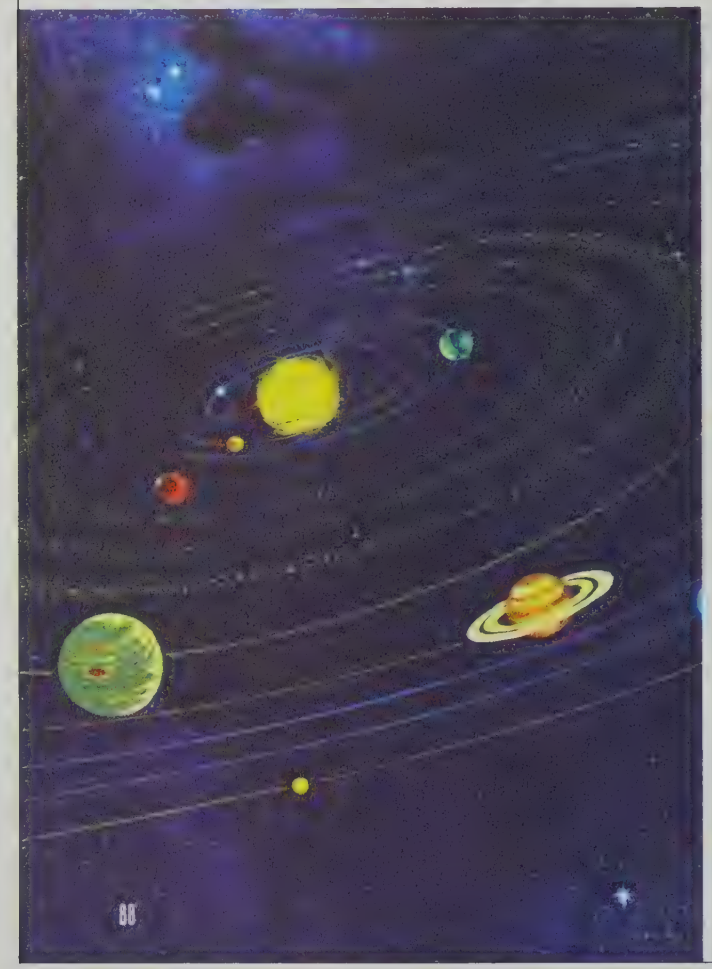
**memo** Pages 88 and 89 should be used together.

**page 88** Maximum and minimum distances are discussed in relation to an elliptical orbit. Although understanding elliptical orbits is not an objective here, pupils must have some understanding of them to do the work meaningfully. A short discussion and a diagram on the board showing the following should be sufficient:




Examine the numbers given on the page together. Are these exact distances? What is the clue that they probably are rounded?

Continue the discussion right on to page 89.

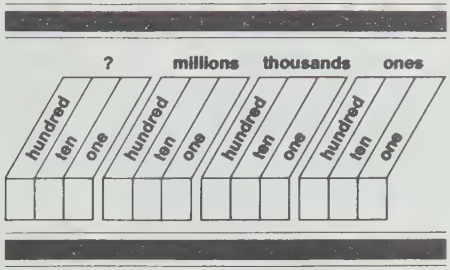



All the planets move around the sun in their own paths. The paths are not circular. These noncircular paths are usually called elliptical orbits. The orbits are curved, but they are longer than they are wide. The distance of a planet from the sun is different from time to time. The range of distances is given below.

**Distances from the Sun in Miles**

| Planet | Maximum distance | Minimum distance |
|---|---|---|
| Mercury | 43,355,000 | 28,566,000 |
| Venus | 67,653,000 | 66,738,000 |
| Earth | 94,452,000 | 91,342,000 |
| Mars | 154,760,000 | 128,830,000 |
| Jupiter | 506,710,000 | 459,940,000 |
| Saturn | 935,570,000 | 836,700,000 |
| Uranus | 1,866,800,000 | 1,698,800,000 |
| Neptune | 2,817,400,000 | 2,769,600,000 |
| Pluto | 4,600,000,000 | 2,760,000,000 |

Mercury's maximum distance from the sun is read "sixty-nine million, eight hundred two thousand kilometres." How would you read the maximum distance of Venus? of Earth? Mars? Jupiter? Listen for the incorrect use of "and."




The maximum distance of Saturn is read "one *billion*, five hundred six million, two hundred sixty-eight thousand kilometres."

**a** How would you read the maximum distance of Uranus? of Neptune? of Pluto?

**b** How would you read each of the minimum distances?

**3.** How many billions are there in 63 521 740 609? 63
How many millions? 521 How many thousands? 740 How many ones? 609

**4.** How many billions are there in 935 004 078 333? 935
How many millions? 4 How many thousands? 78 How many ones? 333

**5.** How many billions are there in 5 672 000 984? 5
How many millions? 672 How many thousands? 0 How many ones? 984

**6.** Find the missing numbers.

**a** 43 764 271 909 = 43 billions 764 millions 271 thousands 909 ones

**b** 833 000 891 754 = 833 billions 0 millions 891 thousands 754 ones

**c** 1 934 244 700 = 1 billions 934 millions 244 thousands 700 ones



**goal** Reading numbers having as many as 12 digits

**page 89** The ability to read numbers orally is an important communication skill and requires practice to achieve. The place-value chart should prove helpful.

The answers to questions 3 through 6 should be written before they are discussed. Agree on a set of abbreviations for the place-value positions.

**things** newspapers and magazines

Newspapers and magazines are a rich source for showing real uses of large numbers in contemporary society. Start a bulletin board or encourage personal collections of large numbers used to communicate information. You may want to have these examples

Numbers in Science
Numbers in Business
Numbers in Politics
Numbers in Sports

Students need to know that there is a reason for learning what these large numbers mean and for learning how to read them.

**lesson** Page 90

**goal** Writing numerals having as many as 12 digits

**memo** Problem 2 requires research work in an encyclopedia or atlas.

**page 90** Consider dividing the research projects in problem 2 between small groups and individual pupils. The results can be reported in a discussion. Beware—there are no universal rules for rounding numbers. Different sources may provide different figures for the same item. The reliability of the source is also important—especially when dealing with population and distance figures.

Problem 3 should lead to a rich discussion—no pun intended.

Why do all the distances to faraway places such as the planets usually end in three or more zeros?
Tell to which place each distance has been rounded on page 88.
Can you always tell? Why?

**1** Write a numeral for each.

**a** one hundred million
100 000 000

**b** six hundred billion
600 000 000 000

**c** one billion
1 000 000 000

**d** fifty-eight billion
58 000 000 000

**e** ten million
10 000 000

**f** nine billion, two hundred thousand
9 000 200 000

**g** one hundred thousand
100 000

**h** thirty billion, five hundred thousand
30 000 500 000

**i** ten billion
10 000 000 000

**j** ten billion, four hundred thousand
10 000 400 000

**2** Do some research. Answers will vary with time an

**a** How many people live in your town?

**b** What is the population of the largest city in your state or province?

**c** What is the population of the largest city in the world? Tokyo: 11 350 000

**d** What is the population of your country?

**e** What is the population of the largest country in the world? China: 740 000 000

**f** What does it mean to have someone or something insured?
Money guarantee against a loss

**g** What amount of life insurance could a person have?
Between $10 000 and $1 000 000 usual

**h** How much could you save in a bank and have the amount insured?
Up to $20 000 on a single account

**3** What is the difference between a millionaire and a billionaire? How much difference might there be?

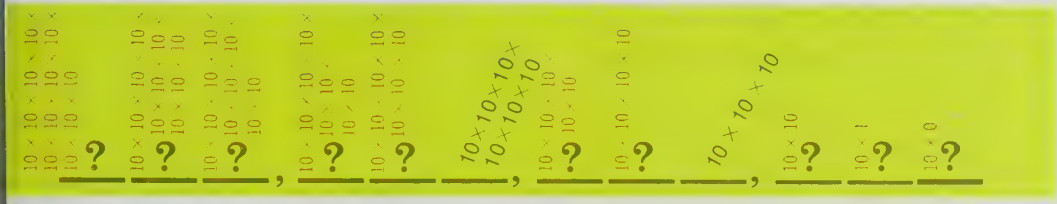
A billionaire could be 1000 times richer than the millionaire. The difference can be from $1 000 000 to $999 998 999 999.

90

$1000 = 10 \times 10 \times 10$

$1{,}000{,}000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

What other numbers can be written as a product of tens?



A number is a *power* of 10 if it can be written as a product with *only* 10s as factors.

Which of the following are powers of 10? See underscored numbers.

2100

10 as a factor 2 times: 100

101,000

10 as a factor 9 times: 1,000,000,000

37,000,000,000

25

49

100,000 — 10 as a factor 5 times

50,000,000

10,000 — 10 as a factor 4 times

100,000,000 — 10 as a factor 8 times

100,100

100,000,000,000 — 10 as a factor 11 times

36

10,000,000 — 10 as a factor 7 times

10,000,000,000 — 10 as a factor 10 times

11,000

125

Write all the powers of 10 as products. Use only 10s as factors.

Can you find any powers of 5 in the list above? 25, 125

any powers of 6? any other powers? 49 is a power of 7 (7 as a factor 2 times).

36

Find the products.

1. $10 \times 10$ — 100
2. $10 \times 10 \times 10$ — 1000
3. $10 \times 10 \times 10 \times 10$ — 10,000

Write the numbers as products of 10.

4. 100 — $10 \times 10$
5. 1000 — $10 \times 10 \times 10$
6. 100,000 — $10 \times 10 \times 10 \times 10 \times 10$



**lesson** Pages 91, 92, 93

**goal** Introduction to the concept of powers of 10

**memo** The remainder of this chapter is purely exploratory. The ideas will be new and sophisticated, but they are presented simply. Mastery is not intended. The goal is to lay a foundation for level 7. Have fun exploring these ideas together.

**page 91** The emphasis is on exploring and learning by doing—no fair telling the shortcuts. You know that the number of zeros tells how many times 10 is used as a factor. Let the youngsters find this relationship after doing the factoring themselves. Some pupils may need help with this step. For example:

$1000 = 10 \times 100$

$= 10 \times 10 \times 10$

Use parentheses if they help.

$100 = (10 \times 10)$

Watch for pupils who naturally want to add rather than multiply for problems 1 through 3. Suddenly $10 \times 10$ becomes 20, rather than 100.

**goal** Introduction to writing powers of 10 as exponents

**page 92** There's lots to talk about on this page. Nice and easy. Your pupils will like these grownup ideas, but they can easily be confused. Squaring and cubing numbers will be explored further on pages 94 and 95.

## 10 × 10 × 10 × 10 × 10 is another way of writing 100,000

It shows that 100,000 can be written as a product using 10 as a factor 5 times. Can you think up a shorter way of writing 10 × 10 × 10 × 10 × 10, or 100,000?

### EXPLAIN YOUR WAY

Products of 10 can be written in shortened form by using *exponents*. For example:

$$10 \times 10 = 10^2$$

The small, raised 2 is the exponent. It tells how many times 10 is used as a factor in the product.

## Consider these exponents

$10^4 = 10 \times 10 \times 10 \times 10$
$10^3 = 10 \times 10 \times 10$
$10^2 = 10 \times 10$
$10^1 =$ ? 10

## What does the exponent 1 mean?

We can think of 10 as a power of 10, even though 10 is a factor only once.
$10^1$ is the first power of 10,
$10^2$ is the second power of 10,
$10^3$ is the third power of 10, and so on.

$10^1$ is read "ten to the first power."
$10^2$ is read "ten to the second power," or sometimes "ten squared."
$10^3$ is read "ten to the third power," or sometimes "ten cubed."
$10^4$ is read "ten to the fourth power."

How would you read $10^5$? $10^6$? Ten to the fifth power. Ten to the sixth power.

92

10,000 is a standard numeral.
10,000 can be written in factor form, $10 \times 10 \times 10 \times 10$.
And $10 \times 10 \times 10 \times 10$ can be written in exponent form, $10^4$.
Similarly, $10^4$ can be written in factor form or as a standard numeral.

**1.** Write each standard numeral in factor form.

**a** 1000
10 as a factor 3 times

**b** 1,000,000
10 as a factor 6 times

**c** 100,000,000
10 as a factor 8 times

**d** 10,000,000,000
10 as a factor 10 times

**e** 100,000
10 as a factor 5 times

**f** 1,000,000,000
10 as a factor 9 times

**2.** Write each factor in exponent form.

**a** $10 \times 10 \times 10 \times 10 \times 10$ $10^5$

**b** $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ $10^7$

**c** $10 \times 10$ $10^2$

**d** $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ $10^8$

**e** $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ $10^{11}$

**3.** Write each standard numeral in exponent form.

**a** 1000 $10^3$

**b** 10,000,000 $10^7$

**c** 100,000,000 $10^8$

**d** 10,000,000,000 $10^{10}$

**e** 100,000 $10^5$

**f** 100,000,000,000 $10^{11}$

Can you find a shortcut?
The exponent is equal to the number of zeros in the standard numeral.

**4.** Write each numeral in factor form.

**a** $10^3$ **b** $10^6$ **c** $10^5$ **d** $10^{11}$ **e** $10^9$ **f** $10^{10}$

The exponent tells number of times 10 is used as a factor.

**5.** Write each numeral in standard-numeral form. To save time, count the number of zeros.

**a** $10^3$ **b** $10^6$ **c** $10^7$ **d** $10^8$ **e** $10^5$ **f** $10^{11}$

Standard numeral should show:
a thousand b million
c ten million d hundred million
e hundred thousand
f hundred billion

93

**goal** Practice writing powers of 10 in factor, exponent, and standard numeral forms

**page 93** The words **standard–numeral form**, **factor form**, and **exponent form** often appear on standardized tests. You may do well to write these terms on the board with an example of each form and talk about them.

Stress that an exponent tells the number of times 10 is used as a factor. Now is the time to challenge everyone to find a shortcut for figuring exponents. Anyone who thinks he has found the shortcut should share it only with you for the present.

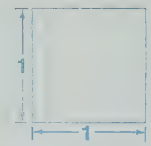
**lesson** Pages 94, 95

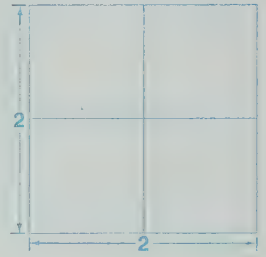
**goal** Exploring the concept of square numbers

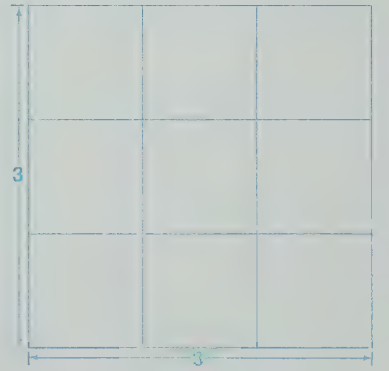
**things** wood cubes or sugar cubes
graph paper

**page 94** A little hands-on experience will go a long, long way. Group pupils and provide each group with 25 to 36 cubes. Sugar cubes make a good substitute for the wood ones. Have each group experiment with their cubes to find all the possible **square** buildings they could make. The floor plans can be outlined on graph paper and the information used to complete the chart.

Graph paper will prove helpful for problems 1 and 3. Watch out for problem **2a**. For some unknown reason pupils have a tendency to misname $9^2$ as 18 rather than 81. They seem to think $9 \times 2$ rather than $9 \times 9$.




The Bigshot Construction Company puts up buildings with square floor plans. There are always exactly as many rooms going east-west as there are going north-south. Some typical floor plans are here.

| Number of rooms expressed as a product | Number of rooms expressed with an exponent | Number of rooms expressed with a standard numeral |
|---|---|---|
| $1 \times 1$ | $1^2$ | 1 |
| $2 \times 2$ | $2^2$ | 4 |
| $3 \times 3$ | $3^2$ | 9 |
| ? $4 \times 4$ | $4^2$ | ? 16 |
| $6 \times 6$ | ? $6^2$ | ? 36 |

1. Draw the floor plan for a building with 7 rooms on a side. Express the number of rooms as a product, with an exponent, and with a standard numeral. $7 \times 7$; $7^2$; 4

2. The following numbers are called *square numbers*. Can you think of a reason for this name? Write the standard numeral for each of these square numbers.

   **a** $9^2$ 81 **b** $15^2$ 225 **c** $10^2$ 100

   **d** $13^2$ 169 **e** $17^2$ 289 **f** $25^2$ 625

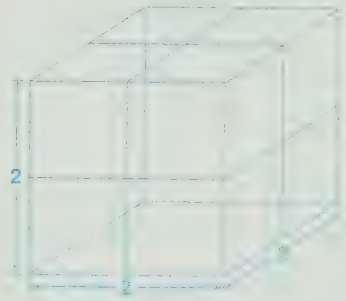
3. Which of these numbers are square numbers?

   64 56 121 45 110 36

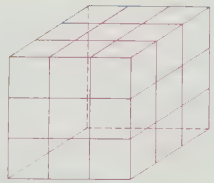
94

The buildings the Bigshot Construction Company puts up always have as many floors as there are rooms along one side of a floor. The buildings are cubical. There are exactly as many rooms going up-down as there are going east-west or north-south.

| | Number of rooms on each floor | Number of floors | Total number of rooms expressed | | |
|---|---|---|---|---|---|
| | | | as a product | with an exponent | in standard notation |
| 1. | $1 \times 1$ | 1 | $1 \times 1 \times 1$ | $1^3$ | 1 |
| 2. | $2 \times 2$ | 2 | **a** $2 \times 2 \times 2$ | $2^3$ | **b** 8 |
| 3. | $3 \times 3$ | **a** 3 | **b** $3 \times 3 \times 3$ | $3^3$ | **c** 27 |
| 4. | $4 \times 4$ | 4 | **a** $4 \times 4 \times 4$ | **b** $4^3$ | 64 |
| 5. | $6 \times 6$ | 6 | $6 \times 6 \times 6$ | $6^3$ | **a** 216 |




6. Draw a sketch of a building with 9 rooms on each floor. How many rooms are in the building all together? 27
Express the total number of rooms as a product, with an exponent, and with a standard numeral. $3 \times 3 \times 3$; $3^3$; 27



7. The following numbers are called *cubic numbers*. Can you think of a reason for this name? Write the standard numeral for each of these cubic numbers.
The same factor is used three times. There are three dimensions to a cube.
**a** $5^3$ 125 **b** $8^3$ 512 **c** $7^3$ 343 **d** $10^3$ 1000 **e** $12^3$ 1728 **f** $15^3$ 3375



**goal** Exploring the concept of cubic numbers

**things** wood cubes or sugar cubes

**page 95** Again group your pupils. Supply each group with 27 cubes. This will allow construction of a two-story (2-by-2) building and a three-story (3-by-3) building. If you have more cubes available, let them construct as many examples as possible. Each example will help develop a feeling for the rapid increase in quantity. The tendency to add rather than multiply is not so great after this experience.

Have the groups complete the chart together. The remainder of the page can be completed independently.

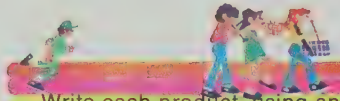
**lesson** Page 96

**goal** **Progress Check**—writing numbers in factor, exponent, and standard numeral forms

**page 96** There are all kinds of errors to watch for:

- Problems 1 through 10—just plain careless counting
- Problems 11 through 18—$3^5$ written as $3 \times 5$ rather than $3 \times 3 \times 3 \times 3 \times 3$
- Problems 19 through 23—$2^4 = 8$ rather than $2^4 = 16$
  The pupil multiplied $2 \times 4$ or added $2 + 2 + 2 + 2$ rather than $2 \times 2 \times 2 \times 2$.

Remember, mastery is not an objective. Yet you don't want to leave a pupil confused. More experiences with cubes will help.



# PROGRESS CHECK

Write each product, using an exponent.
Skill: Writing exponents

1. $10 \times 10 \times 10 = \underline{?}$ $10^3$
2. $7 \times 7 \times 7 \times 7 = \underline{?}$ $7^4$
3. $6 \times 6 \times 6 \times 6 \times 6 = \underline{?}$ $6^5$
4. $2 \times 2 \times 2 \times 2 = \underline{?}$ $2^4$
5. $3 \times 3 \times 3 \times 3 \times 3 \times 3 = \underline{?}$ $3^6$
6. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = \underline{?}$ $4^{10}$
7. $12 \times 12 \times 12 = \underline{?}$ $12^3$
8. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = \underline{?}$ $7^7$
9. $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = \underline{?}$ $5^9$
10. $8 \times 8 \times 8 \times 8 \times 8 = \underline{?}$ $8^5$

Skill: Writing products of equal factors
Write each number as the product of equal factors.

11. $3^5$
12. $6^2$
13. $7^5$
14. $8^4$
15. $5^3$
16. $3^4$
17. $4^8$
18. $2^{10}$

Write each number in standard-numeral form.
Skill: Writing numerals in standard form

19. $2^4$ 16
20. $4^2$ 16
21. $4^3$ 64
22. $3^4$ 81
23. 4 2

*24. Copy and complete the table. Skill: Renaming numb

| Standard numeral | Product of equal factors | Written with an exponent |
|---|---|---|
| 36 | 6 x 6 | $6^2$ |
| a 125 | b $5 \times 5 \times 5$ | $5^3$ |
| c 64 | 4 x 4 x 4 | d $4^3$ |
| e 81 | f $3 \times 3 \times 3 \times 3$ | $3^4$ |
| 100 | 10 x 10 | g $10^2$ |
| 10,000 | h $10 \times 10 \times 10 \times 10$ | i $10^4$ |
| j 100,000,000 | k $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ | $10^8$ |

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11. $3 \times 3 \times 3 \times 3 \times 3$
12. $6 \times 6$
13. $7 \times 7 \times 7 \times 7 \times 7$
14. $8 \times 8 \times 8 \times 8$
15. $5 \times 5 \times 5$
16. $3 \times 3 \times 3 \times 3$
17. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
18. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

**things** wood cubes or sugar cubes

No wood cubes? Sugar cubes sprayed with enamel last a long time. Pupils who tend to add rather than multiply when finding the standard numeral need the hands-on experience of actually making 2 or 3 models.

See activity 6, page 100c.

Look at another way of naming numbers.
Tell how many full hands (all 5 fingers up) and how many fingers extra.

1. $\frac{?}{1}$ hands $\frac{?}{2}$ fingers

2. $\frac{?}{4}$ hands $\frac{?}{1}$ fingers

3. $\frac{?}{3}$ hands $\frac{?}{4}$ fingers

4. $\frac{?}{2}$ hands $\frac{?}{0}$ fingers

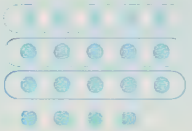
5. $\frac{?}{3}$ hands $\frac{?}{3}$ fingers

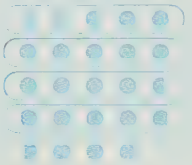
6. $\frac{?}{4}$ hands $\frac{?}{0}$ fingers

Tell how many fives and how many ones.

7. $\frac{2}{?}$ fives $\frac{3}{?}$ ones

8. $\frac{1}{?}$ fives $\frac{1}{?}$ ones

9.  $\frac{3}{?}$ fives $\frac{4}{?}$ ones

10.  $\frac{4}{?}$ fives $\frac{4}{?}$ ones

11. $\frac{3}{?}$ fives $\frac{0}{?}$ ones

12. $\frac{2}{?}$ fives $\frac{2}{?}$ ones

97

The origin of the base-ten system of numeration is a fascinating topic. You may wish to correlate this topic to a social studies project. Pupils should understand that there is nothing sacred about the base-ten system.

**lesson** Pages 97, 98, 99

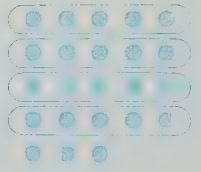
**goal** Exploring naming numbers in base five

**memo** Pages 97, 98, and 99 are extension pages. Base five should **not** be taught for mastery. This should be a pleasant exploration to give further insight into our own base-ten system. You may not want to use these pages with everyone.

**page 97** Why do you suppose we use the base-ten system for writing numerals? If anyone mentions counting the fingers on both hands, he is probably on the right track. Suppose we use only one hand, rather than two, to count on. That's the whole idea of the hand problems at the left of the page.

**goal** Exploring writing numbers in base five

**page 98** Talk about how to write base-five numerals together. You may even want to try an example or two. As pupils feel secure let them continue independently. Discuss everyone's answers as a group.



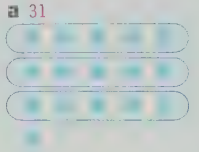
We can describe the number of dots like this: $43_{five}$

The first digit (4) tells how many fives. The second digit tells how many ones. The "five" says that the dots are grouped in fives.

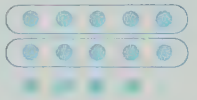
$43_{five}$ is called a *base-five numeral.*

**1.** Write a base-five numeral to tell how many dots.

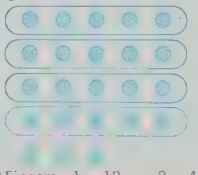
**a** 31 **b** 10



**c** 24 **d** 21



**e** 43 **f** 32



**2.** Write a base-five numeral to tell how many for each set of fingers and each set of dots shown on the page before. *See below.

**3.** Draw dots to show how many.

**a** $24_{five}$ **b** $31_{five}$ **c** $33_{five}$

**d** $20_{five}$ **e** $14_{five}$ **f** $1_{five}$ **See below.

**4.** Which digits do base-five numerals use? 0, 1, 2, 3, 4 How many different digits is this? 5

**5.** The numerals we usually use are called *base-ten numerals*. Which digits do base-ten numerals use? How many different digits is this?
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

**6.** Draw dots and then write a base-five numeral to tell how many. ***See below.

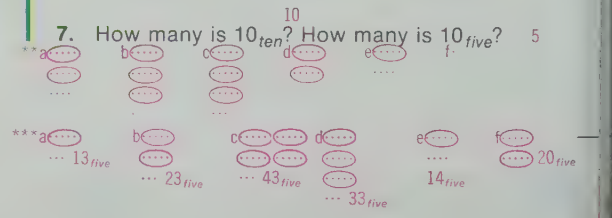
**a** $8_{ten}$ **b** $13_{ten}$ **c** $23_{ten}$

**d** $18_{ten}$ **e** $9_{ten}$ **f** $10_{ten}$

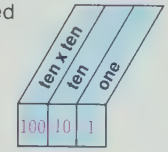
**7.** How many is $10_{ten}$? How many is $10_{five}$? 5

*Fingers: 1. $12_{five}$ 2. $41_{five}$ 3. $34_{five}$ 4. $20_{five}$ 5. $33_{five}$ 6. $40_{five}$

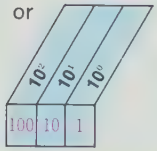
*Dots: 7. $23_{five}$ 8. $11_{five}$ 9. $34_{five}$ 10. $44_{five}$ 11. $30_{five}$ 12. $22_{five}$



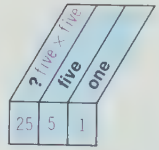
Our base-ten system is called the *decimal system*. In the decimal system a place value is written with a 10 and an exponent or as a product of tens.



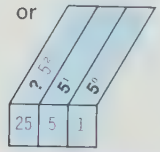

or




What do you think the next place value is in base five?




or




The base-five system can be compared to a money system with only pennies (ones), nickels (fives), and quarters (five times five). Whenever possible, five pennies must be exchanged for a nickel, and five nickels must be exchanged for a quarter. Use this system to complete the following table.

| | Dollars and cents | Quarters | Nickels | Pennies |
|---|---|---|---|---|
| 1. | $ .58 | **a** 2 | **b** 1 | **c** 3 |
| 2. | .44 | **a** 1 | **b** 3 | **c** 4 |
| 3. | .91 | **a** 3 | **b** 3 | **c** 1 |
| 4. | 1.22 | **a** 4 | **b** 4 | **c** 2 |
| 5. | .73 | **a** 2 | **b** 4 | **c** 3 |
| 6. | 1.19 | **a** 4 | **b** 3 | **c** 4 |
| 7. | .24 | **a** 0 | **b** 4 | **c** 4 |
| 8. | .85 | **a** 3 | **b** 2 | **c** 0 |
| 9. | .69 | **a** 2 | **b** 3 | **c** 4 |
| 10. | 1.00 | **a** 4 | **b** 0 | **c** 0 |

99

Challenge the youngsters to write base-ten numerals such as 455, 1055, and 989 as base-five numerals. ($3310_{five}$, $13210_{five}$, $12424_{five}$)

**goal** Relating place value in base ten to place value in base five

**page 99** The focus is on place-value positions. Money is a help in base five as well as in base ten. Examine the differences though:

Base ten—pennies, dimes, dollars

Base five—pennies, nickels, quarters

Once place value is understood, the completion of the chart will be easy. The easiest way to begin is to ask, "How many twenty-fives are there in fifty-eight?" Then ask, "How many fives in the remainder?" Finally, "How many ones remain?"

**lesson** Page 100

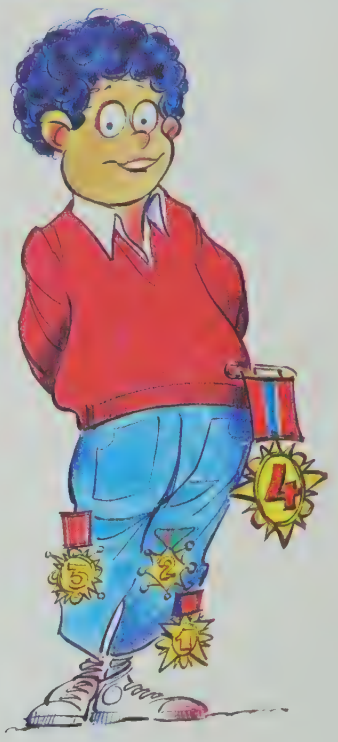
**goal** **Checkout**—reading, writing, and rounding decimals; writing exponents as standard numerals

**page 100** Encourage thinking—then recording only answers.

Ideas for helping with the various skills were given earlier in the chapter. The following chart will help you locate these ideas quickly.

| Problem | Refer to |
|---|---|
| 1 | page 79 (Check for reading problem first.) |
| 2 | pages 78 and 83 |
| 3 | pages 78 and 83 |
| 4 | page 87 |
| 5 | page 96 (NOTE: Not a mastery objective for the year) |

## CHECKOUT



Skill: Writing decimals given word names

**1.** Write a decimal for each.

**a** 6 and 34 hundredths 6.34

**b** 26 and 5 tenths 26.5

**c** 1 and 365 thousandths 1.365

**d** 7 and 97 thousandths 7.097

**e** 24 thousandths .024

**f** six and twenty-four hundredths 6.24

**g** one hundred sixty-three thousandths .163

**h** four hundred twenty-one and seven-tenths 421.7

Skill: Rewriting fractions as decimals

**2.** Write a decimal for each.

**a** $\frac{6}{10}$ .6 **b** $\frac{23}{100}$ .23 **c** $\frac{2}{100}$ .02 **d** $\frac{48}{1000}$ .048

Skill: Rewriting decimals as fractions

**3.** Write a fraction for each.

**a** .07 $\frac{7}{100}$ **b** .6 $\frac{6}{10}$ **c** .146 $\frac{146}{1000}$ **d** .29 $\frac{29}{100}$

Skill: Rounding decimals

**4.** Round each to the nearest hundredth, tenth, and whole number.

**a** 4.638 4.64; 4.6; 5 **b** 27.134 27.13; 27.1; 27 **c** 1.975 1.98; 2.0; 2 **d** .847 .85; .8; 1

**e** 2.603 2.60; 2.6; 3 **f** 10.738 10.74; 10.7; 11 **g** 4.455 4.46; 4.5; 4 **h** 3.505 3.51; 3.5; 4

Skill: Multiplying exponents to find standard numeral

**5.** Write the standard numeral for each.

**a** $10^3$ 1000 **b** $4^2$ 16 **c** $2^3$ 8 **d** $10^4$ 10,000

100

See activity 7, page 100c.

See activity 8, page 100c.

# RESOURCES

## another form of evaluation

**for progress check** – page 83

Complete.

1. .625 = 6 tenths 2 hundredths 5 thousandths
2. .273 = 2 tenths 7 hundredths 3 thousandths
3. .460 = 4 tenths 6 hundredths 0 thousandths
4. 5.641 = 5 ones 6 tenths 4 hundredths 1 thousandths
5. .974 = ?

9 tenths 7 hundredths 4 thousandths

6. .308 = ?

3 tenths 0 hundredths 8 thousandths

7. 4.529 = ?

4 ones 5 tenths 2 hundredths 9 thousandths

8. 23.006 = ?

2 tens 3 ones 0 tenths 0 hundredths 6 thousandths

Name two decimals between –

Answers will vary. Examples shown.

9. .320 and .330 .321, .325
10. .560 and .570 .564, .569
11. .840 and .850 .842, .843
12. .84 and .85 .841, .846
13. .28 and .31 .284, .30
14. .054 and .064 .056, .06

Write each as a decimal.

15. $\frac{6}{10}$ .6
16. $\frac{70}{100}$ .70 or .7
17. $\frac{26}{100}$ .26
18. $\frac{26}{1000}$ .026
19. $\frac{2}{100}$ .02
20. $\frac{2}{1000}$ .002
21. $\frac{20}{1000}$ .020 or .02
22. $\frac{200}{1000}$ .200 or .2

Write each as a fraction.

23. .46 $\frac{46}{100}$
24. .8 $\frac{8}{10}$
25. .06 $\frac{6}{100}$
26. .253 $\frac{253}{1000}$
27. .30 $\frac{30}{100}$ or $\frac{3}{10}$
28. .030 $\frac{30}{1000}$ or $\frac{3}{100}$
29. .027 $\frac{27}{1000}$
30. .409 $\frac{409}{1000}$

**for progress check** – page 87

Round to the nearest tenth.

1. .57 .6
2. .92 .9
3. .45 .5

Round to the nearest hundredth.

4. .386 .39
5. .423 .42
6. .975 .98

Round to the nearest whole number.

7. 4.564 5
8. 2.006 2
9. 8.641 9

Which is the smaller number?

10. .54 or .72
11. .362 or .263
12. .38 or .524
13. .436 or .73
14. .027 or .43
15. .79 or .628

Write each set from smallest to largest.

16. 3.0 .3 .03 .330 3.3 .03, .3, .330, 3.0, 3.3
17. 2.5 .025 25. 2.05 20.5 .025, 2.05, 2.5, 20.5, 25.
18. .326 .12 8.06 21.3 1.25 .12, .326, 1.25, 8.06, 21.3
19. .76 .076 .706 .761 .7 .076, .7, .706, .76, .761

Round to the nearest whole number and then find the sum.

20. 5.7 + 3.4 6 + 3 = 9
21. 6.34 + 2.75 6 + 3 = 9
22. 14.8 + 2.36 15 + 2 = 17

**for progress check** – page 96

Write each product, using an exponent.

1. $5 \times 5 \times 5 \times 5 = ?$ $5^4$
2. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = ?$ $10^7$
3. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = ?$ $2^{10}$
4. $12 \times 12 = ?$ $12^2$
5. $4 \times 4 \times 4 \times 4 \times 4 \times 4 = ?$ $4^6$
6. $9 \times 9 \times 9 = ?$ $9^3$
7. $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = ?$ $6^8$
8. $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = ?$ $8^9$
9. $7 \times 7 \times 7 \times 7 \times 7 = ?$ $7^5$
10. $3 \times 3 \times 3 = ?$ $3^3$

Write each number as the product of equal factors. Answers should be in this form: $2 \times 2 \times 2 \times 2 \times 2 \times 2$

11. $2^6$
12. $8^5$
13. $10^4$
14. $6^9$
15. $7^7$
16. $3^2$
17. $5^8$
18. $4^3$

Write each number in standard-numeral form.

19. $3^5$ 243
20. $5^3$ 125
21. $2^6$ 64
22. $6^2$ 36
23. $4^4$ 256

24. Complete the table.

| Standard numeral | Product of equal factors | Written with an exponent |
|---|---|---|
| 25 | $5 \times 5$ | $5^2$ |
| a) 216 | $6 \times 6 \times 6$ | b) $6^3$ |
| c) 144 | d) $12 \times 12$ | $12^2$ |
| e) 32 | f) $2 \times 2 \times 2 \times 2 \times 2$ | $2^5$ |
| 1000 | $10 \times 10 \times 10$ | g) $10^3$ |
| h) 100,000 | i) $10 \times 10 \times 10 \times 10 \times 10$ | $10^5$ |
| j) 10,000 | $10 \times 10 \times 10 \times 10$ | k) $10^4$ |

**for checkout** – page 100

1. Write a decimal for each.
   a) 6 and 4 tenths 6.4
   b) 12 and 36 hundredths 12.36
   c) 5 and 133 thousandths 5.133
   d) 2 hundredths .02
   e) 4 and 26 thousandths 4.026
   f) three and two hundred sixty-three thousandths 3.263
   g) one and four thousandths 1.004
   h) six hundred twenty-five and forty-four hundredths 625.44
2. Write a decimal for each.
   a) $\frac{4}{10}$ .4
   b) $\frac{58}{100}$ .58
   c) $\frac{735}{1000}$ .735
   d) $\frac{29}{1000}$ .029
3. Write a fraction for each.
   a) .643 $\frac{643}{1000}$
   b) .8 $\frac{8}{10}$
   c) .47 $\frac{47}{100}$
   d) .036 $\frac{36}{1000}$

4. Round each to the nearest hundredth, tenth, and whole number.

a) 3.756 b) 4.382 c) 1.861

3.76, 3.8, 4 4.38, 4.4, 4 1.86, 1.9, 2

d) 2.439 e) .628 f) 4.206

2.44, 2.4, 2 .63, .6, 1 4.21, 4.2, 4

g) 8.953 h) 3.082

8.95, 9.0, 9 3.08, 3.1, 3

5. Write the standard numeral for each.

a) $5^3$ 125 b) $10^5$ 100,000 c) $2^4$ 16 d) $10^6$ 1,000,000

## activities

**1. things** small cards

Prepare three sets of cards:
- Digit cards for 0 through 9
- Place-value names for decimal positions—tenths, hundredths, thousandths
- 1-, 2-, and 3-digit decimals (.3, .25, .125) less than 1 whole

An equal number of digit cards and place-value cards are dealt to each player. The decimal cards are shuffled and placed facedown in a stack. The top card is turned faceup. The first player examines his cards and attempts to find a digit card **and** a place-value card that together correspond to one of the digits in the decimal. If he can form such a pair, he lays the cards down faceup. If he cannot form such a pair, he passes. The next player repeats the procedure, looking for a pair of cards that correspond to a **different** digit of the decimal. Play continues until each digit of the decimal has been matched or until all the players must pass. All cards laid down by players are stacked to one side. The next decimal card is turned over and play resumes.

The winner is the first player to use all his cards. If further play is impossible, the player with the fewest unplayed cards is the winner.

**2. things** small cards; pencil; paper

Prepare two sets of cards:
- 4 cards for each digit 0 through 9
- 4 place-value cards for ones, tenths, hundredths, thousandths

Each player writes a 4-digit decimal less than 10 on his paper and is dealt 5 cards from each set. The remaining cards are placed facedown in separate stacks. The top card in each stack is turned faceup to begin two discard stacks.

Players alternate drawing the top card from either set, or the discard stack for either set, attempting to show the expanded form of their decimal. Only one card can be laid down per turn and one card discarded. The first player to complete the expanded form of his numeral is the winner.

**3. things** small cards

Have each player write three pairs of fractions—a decimal and its equivalent common fraction. These pairs are used to prepare two sets of cards—a set of decimals and a set of equivalent common fractions. The sets are shuffled separately and placed facedown to form two arrays.

The first player turns over a card in each array. If the cards match (name the same number), he keeps the cards and takes another turn. If the cards do not match, they are again turned facedown and play passes to the next player. When all the cards are matched, the player with the most cards wins.

**4. things** small cards

Prepare a set of cards (30 for 2 players, 32 for 3 players, 34 for 4 players) by writing a decimal on each card. These decimals should be within the limits used on the page.

The dealer lays down 2 cards faceup in the center of the table and deals the remaining cards equally to all players. The first player examines his cards, looking for a decimal between the two decimals shown on the cards on the table. If he does not have one, he passes. If he has such a card, he places it on the table, takes the original 2 cards and places them facedown in front of him. Then he lays down another card faceup from his hand to form a new pair of decimals for the next player. Play continues until one player has used all his cards or until everyone must pass. The winner is the player who has the most facedown cards in front of him.

**5. things** small cards

Prepare a set of cards by writing a decimal on each card. Let the decimals used on the page serve as your guide.

Five cards are dealt to each player. Two cards are placed faceup on the table and ordered. The remaining cards are placed facedown in a stack. The first player attempts to place a card from his hand that contains a decimal that comes between the two showing on the table. If he is unable to do so, he draws a card and either lays it down or adds it to the cards in his hand. The next player attempts to lay down a card from his hand that comes **between** any two of the cards faceup on the table. The cards are always placed to form an ordered row. A player may draw only one card per turn.

The first player to use all his cards wins. If all the undealt cards are drawn and play cannot continue, the player with the fewest cards remaining in his hand is declared the winner.

**6.** Individual activity. (Provide the pupil with the following directions.)

Use square numbers and the operations of only addition and subtraction to rename any number less than 100. There may be more than one way to rename a number.
Example: $29 = 6^2 - 4^2 + 3^2$

or

$29 = 4^2 + 2^2 + 3^2$

Try these:

$46 = (6^2 + 3^2 + 1^2)$
$60 = (8^2 - 2^2)$
$84 = (8^2 + 4^2 + 2^2)$

**7. things** for each pair of pupils: 4-by-4 square array; 4 markers each of 2 colors; small cards

Pair pupils who need the same kind of practice. Prepare a set of problem cards to meet their needs. All computation should be of the type that pupils can complete mentally.

The players place their markers on the game board as shown by the **X**'s and **O**'s.

| | | | |
|---|---|---|---|
| X | | | O |
| O | | | X |
| X | | | O |
| O | | | X |

The cards are shuffled and placed in a stack facedown. In turn, each player selects a card, states the answer, and moves one of his markers one space either horizontally or vertically. Jumping and diagonal moves are not permitted. If a player gives an incorrect answer, he loses a turn. The first player to arrange his 4 markers in a row (horizontally, vertically, or diagonally) is the winner.

**8. things** wood cubes of 2 colors

Number each face of 2 cubes of the same color (red) from 1 through 6. Number the faces of 1 cube of another color (yellow) as follows:

- 2 faces – 1
- 2 faces – 2
- 2 faces – 3

In turn, each player rolls the 3 cubes. The player decides whether to use the sum or the product of the numbers on the red cubes that land faceup and raises it to the power shown by the faceup number on the yellow cube. This product is the player's score.

Play continues for a predetermined number of rounds. The player's score for each round is added to his preceding score. The player with the highest total wins.

This game involves strategy. Suppose 1 and 6 land faceup on the red cubes. $1 + 6 = 7$; $1 \times 6 = 6$. Raising the sum rather than the product will increase the player's score.

"I like everything about this place except the placing of the decimal points."

The Better Half by Barnes, reprinted courtesy of The Register and Tribune Syndicate

## additional learning aids

**notation**—chapter objectives 1, 2, 3, 4

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: F-20, 21, 22
*Computapes*, SRA (1972)
Module 6, Lesson: DP 2
*Computational Skills Development Kit*, SRA (1965)
Use of Decimal Numerals cards: 1, 2, 3
*Diagnosis: an instructional aid—Mathematics Level B*, SRA (1972)
Probe: M-18
*Mathematics Involvement Program*, SRA (1971)
Card: 365
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 44, 45, 46, 47, 48, 49, 50, 51
*Visual Approach to Mathematics, Rational Numbers*, SRA (1967)
Visual: 14

**other learning aids** (described on page 144j)—Decimal Fraction Dominoes, Decimal/Fraction Matching Cards

Here are more ideas to add to the Super Sleuth Box.

Multiply any number by 5.
Add 25 to the product.
Divide the sum by 5.
Subtract the number you started with.
Multiply that number by 3.
The result will always be 15.

**WHY?**

Think of a number 1 through 8.
Add 9 to get a sum.
Double the sum.
Divide by 2.
Subtract the number you picked.
The last number is always 9.

**WHY?**

Think of a number 1 through 9.
Add 1 and multiply by 3.
Add 2 and multiply by 4.
Add 1 and divide by 3.
Add 1 and divide by 4.
Subtract the number you picked.
The answer is always 2.

**WHY?**

Take any number. Write it down.
Reverse the order of the digits
to get a second number.
Subtract the smaller number
from the larger number.
That answer will *always* be
exactly divisible by 9.

***You don't have to figure out why on this one.***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# 5 OPERATIONS WITH DECIMALS

**before this chapter, the learner has –**

1. Read, written, and told the value of each digit of a decimal expressed to thousandths
2. Compared and ordered decimals expressed to thousandths
3. Renamed a common fraction as a decimal fraction and a decimal fraction as a common fraction
4. Computed with whole numbers using all four number operations
5. Found the product for two common fractions

**in chapter 5, the learner is –**

1. Mastering renaming a decimal as a common fraction
2. Mastering reading, writing, and telling the value of each digit of a decimal expressed to thousandths
3. Mastering comparing and ordering a set of decimals expressed in tenths, hundredths, and thousandths
4. Finding the sum or difference for two mixed decimals
5. Finding the product for two decimals expressed in tenths or hundredths
6. Finding the quotient for a division example having a decimal as the divisor
7. Using decimals with the metric system
8. Exploring the concept of percent

**in later chapters, the learner will –**

Master dividing by a 2-digit divisor

# Notes & Things

All four operations are presented in this chapter. The amount of time and effort needed for this chapter depends on some very basic things. If the youngsters have no trouble with the addition, subtraction, multiplication, and division of whole numbers and if they have a good sense of place value, this chapter will be a snap. Each algorithm for computation with decimals parallels the same algorithm for whole numbers. There are no curve balls for you or the pupils. In fact, all the applications of decimal computation should make this chapter's study a delight. But have compassion for the poor soul who is still having trouble with computing whole numbers. That youngster will probably have trouble here too. There is one bright ray of hope, however. Maybe the real situation with money, for example, will be new motivation to try again. It is surely worth a try.

The metric system is a perfect application of decimals. If any of your pupils have had little or no experience with the metric units of measure, you will want to take time out, borrow a pupil book from the teacher who is responsible for level 5, and review chapter 9 in that book. It will be time well spent.

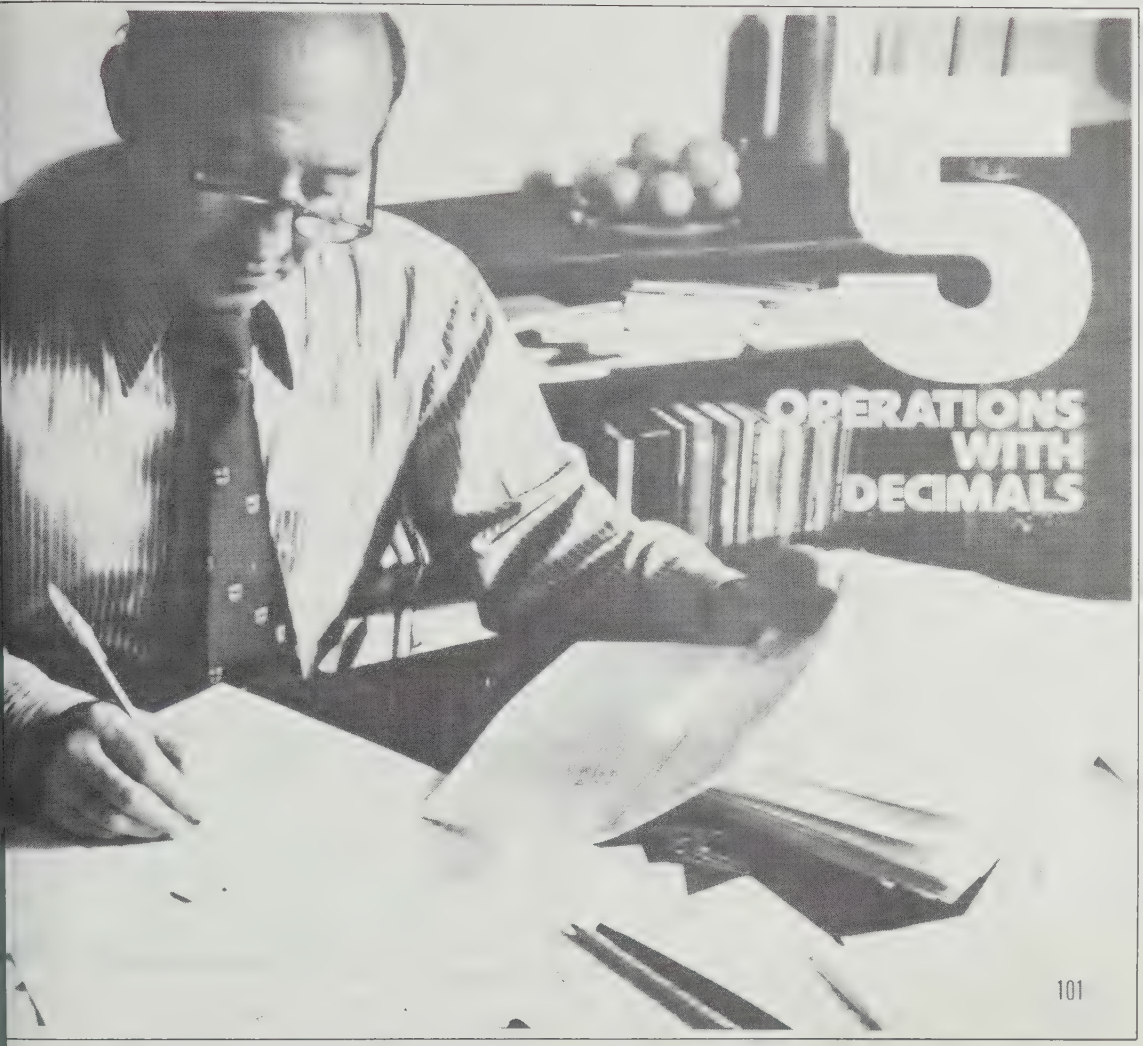
There is a heavy emphasis on place value. And you will find that stressing estimation will be a big help. The models for multiplication will help some pupils, but others may not see their worth at all. Be watchful for those who do not understand the models, for they may need quite a different approach.

The idea of percent is introduced toward the end of the chapter. The youngsters have heard the word **percent** many times on the news and in newspaper, radio, and TV advertising. The emphasis in this chapter is not on computation with percent but rather the study of the idea itself. If each learner comes to understand that it means parts of one hundred, and that percent is a notation for a decimal, then he has reached the desired goal.

## things

place-value chart (spirit master or transparency)
dictionaries
almanac

For the extra activities you will want to have these things available:
- for each pair of pupils: 4 dice; 2 place-value charts (as shown on page 104); 5 markers each of two colors
- newspapers and magazines
- oaktag strips
- paper clips
- adding-machine tape




**goal** Think about and explore ideas through a picture clue

**page 101** It is quite possible that income tax is meaningful only to adults, but why not take some time to find out who knows what about this annual chore?

What is tax? What is it used for? What kind of tax is this man probably trying to compute? What things are considered in preparing an income tax form? Who has to do it? How often?

Do you ever pay a tax? What kind? When? Even if there is no sales tax in your locality, the youngsters may have seen a telephone bill with tax. Or maybe they have noticed the tax added to the cost of gasoline. Here is another choice subject for independent research: "Taxes—who pays them—what do they pay for?"

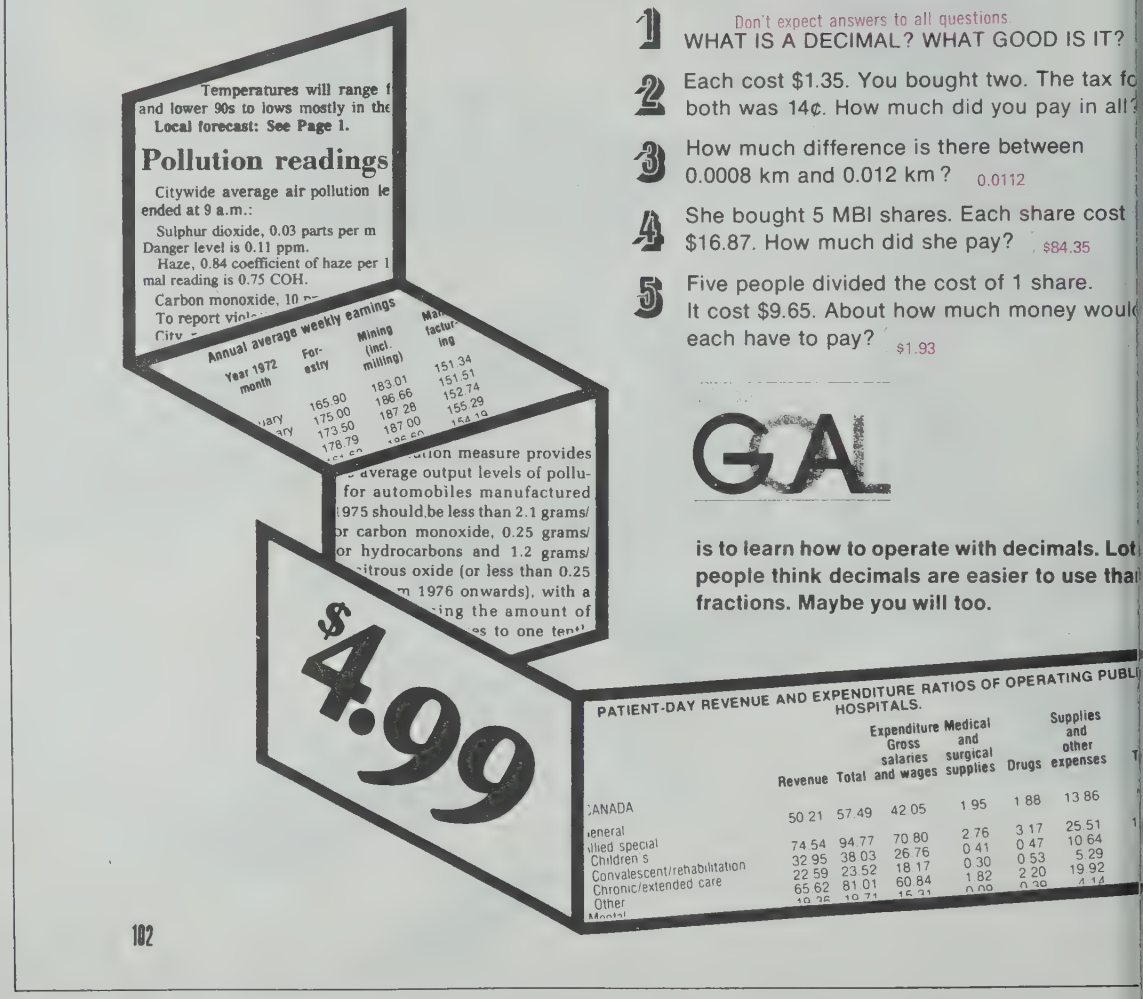
Taxes certainly are one of the most obvious things in which decimals are used in computation. There are others. The chapter will reveal some of them.

**lesson** Page 102

**goal** **Survey**—ability to operate with decimals

**page 102** The page illustrates some very real uses for decimals. Problem 1 may be the source of some ideas well worth sharing.

Problems 2 through 5 vary in level of difficulty. Assure everyone that if they cannot compute each one of these problems, there is no need to worry—that's the learning goal of this chapter.


Temperatures will range f
and lower 90s to lows mostly in the
Local forecast: See Page 1.
Pollution readings
Citywide average air pollution le
ended at 9 a.m.:
Sulphur dioxide, 0.03 parts per m
Danger level is 0.11 ppm.
Haze, 0.84 coefficient of haze per 1
mal reading is 0.75 COH.
Carbon monoxide, 10 p
To report viol
City
Annual average weekly earnings
Year 1972 month
For-estry
Mining (incl. milling)
Manufacturing
165.90 183.01 151.34
175.00 186.66 151.51
173.50 187.28 152.74
178.79 187.00 155.29
average output levels of pollu-
for automobiles manufactured
975 should be less than 2.1 grams/
or carbon monoxide, 0.25 grams/
or hydrocarbons and 1.2 grams/
itrous oxide (or less than 0.25
m 1976 onwards), with a
ing the amount of
es to one ten
$4.99
PATIENT-DAY REVENUE AND EXPENDITURE RATIOS OF OPERATING PUBLI
HOSPITALS.
Revenue
Expenditure Total
Gross salaries and wages
Medical and surgical supplies
Drugs
Supplies and other expenses
CANADA 50 21 57.49 42 05 1 95 1 88 13 86
General 74 54 94 77 70 80 2 76 3 17 25 51
Allied special 32 95 38 03 26 76 0 41 0 47 10 64
Children's 22 59 23 52 18 17 0 30 0 53 5 29
Convalescent/rehabilitation 65 62 81 01 60 84 1 82 2 20 19 92
Chronic/extended care
Other
Mental
102

1 Don't expect answers to all questions
WHAT IS A DECIMAL? WHAT GOOD IS IT?

2 Each cost $1.35. You bought two. The tax fo
both was 14¢. How much did you pay in all?

3 How much difference is there between
0.0008 km and 0.012 km? 0.0112

4 She bought 5 MBI shares. Each share cost
$16.87. How much did she pay? $84.35

5 Five people divided the cost of 1 share.
It cost $9.65. About how much money would
each have to pay? $1.93

GOAL

**is to learn how to operate with decimals. Lot
people think decimals are easier to use tha
fractions. Maybe you will too.**

**things** newspapers and magazines

Put your pupils to work looking through newspapers and magazines to find more clippings similar to those on the page. Start a bulletin board that can grow over the next several weeks.

Back in the year 1585 a man named Simon Stevin published a book on the decimal numeration system. Stevin didn't invent decimals. And he didn't write them the way we do now. But many people first learned about decimals in his book.

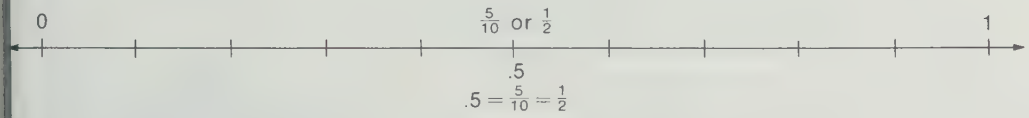
Did you know that even today people have different ways to write a decimal?

In England they write 1.1 as 1·1.

In other European countries they write 1.1 as $1{,}1$ or $1_{,}1$.

Maybe someday everyone will write them the same.

The same number can be named with different numerals. The decimal .5 means five-tenths of one. It can be put on a number line. In fact, .5 is another name for $\frac{1}{2}$.




$.5 = \frac{5}{10} = \frac{1}{2}$

Do you suppose *every* fraction can be written as a decimal?

Say each of these decimals to yourself: .5 .50 .05

Do you hear yourself saying the "denominator"?

.5 or five-tenths or $\frac{5}{10}$

.50 or fifty-hundredths or $\frac{50}{100}$

.05 or five-hundredths or $\frac{5}{100}$

Go on. Say these fractions.

Listen to yourself. .2 .20 .25 .75 .40

Write each of these decimals as a fraction with a numerator and denominator.

$\frac{2}{10}, \frac{20}{100}, \frac{25}{100}, \frac{75}{100}, \frac{40}{100}$



**lesson** Pages 103, 104, 105, 106

**goal** Review of reading and writing decimal fractions

**memo** Pages 103 through 106 provide a review of the concepts and skills developed in chapter 4. You will want to use all the pages with pupils who were insecure working with decimals before. With others you will want to use specific assignments for reinforcement of a weak skill. Pupils who did extremely well in chapter 4 may go directly to the Progress Check on page 106. If they do a good job, they may need no review at all.

**page 103** This is purely a discussion page. Finding out that notation is not uniform the world over should arouse curiosity. Reference books will provide further information.

Seeing the decimal .5 renamed as $\frac{1}{2}$ will be new. Renaming decimals is one of the major thrusts of this chapter.

Have pupils take turns reading the decimals at the bottom of the page. This will help you identify those who still have a problem with place value.

**goal** Review of place value and reading decimal numerals

**things** place-value chart (spirit master or transparency)

**page 104** Pupil responses will indicate whether more practice with a place-value chart is needed. If so, prepare a spirit master or transparency of a chart similar to the one on the page.

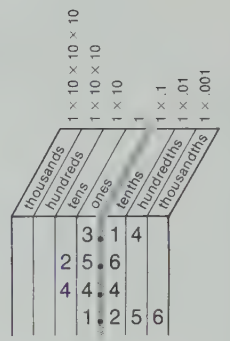
Renaming a decimal fraction by annexing zeros is hit head-on in problems 3 through 6. Rely on the place-value chart together with common fraction concepts to help the confused. For example:

Does $.1 = .10$? Examine: $\frac{1 \times ?}{10 \times ?} = \frac{10}{100}$

Try several more examples if necessary.

Emphasize reading the decimal point as **and** if anyone forgets.

The place-value chart shows that the number 3.14 can be thought of as $3 + .1 + .04$. The value of each digit depends on its place in the numeral.
Another way of thinking about the number 3.14 is
$(3 \times 1) + (1 \times .1) + (4 \times .01)$

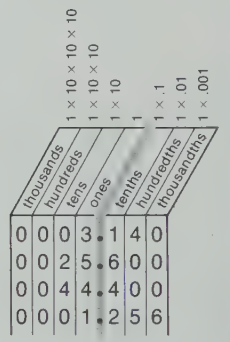



1. This example shows the place value of each digit in 25.6. What is the 2 multiplied by? the 5? the 6?
   10 1 .1

   $2 \times \blacksquare = 20.$
   $5 \times \blacksquare = 5.$
   $6 \times \blacksquare = .6$
   $25.6$

2. How many times larger is 4 tens than 4 ones? 10
   4 tenths is what fraction of 4 ones? $\frac{1}{10}$
   Complete the following sentences.
   $4 \times 10 = \underline{?}$ 40 $4 \times 1 = \underline{?}$ 4 $4 \times .1 = \underline{?}$ .4

This is the same chart as the one above. But there is a change in the way the numerals look. Now every available place has a zero.




3. Does a zero on the left of a number change the value of the number? No

4. Does a zero on the right change the value of the number? No

5. 3.14 is read "three and fourteen-hundredths."
   How is 3.140 read? Three and one hundred forty thousandths

6. 25.6 is read "twenty-five and six-tenths."
   How is 25.600 read? Twenty-five and six hundred thousandths

104

# 1

If you're not feeling well, someone may put a thermometer in your mouth. Normal body temperature is 37 degrees Celsius. If your temperature is 37.2°C you may or may not be healthy. If your temperature is 37.7°C, something is probably wrong with you or the thermometer. There is a very small range in body temperature. That's why it is measured in tenths. Arrange in order from the lowest temperature to the highest temperature.

| | | | |
|---|---|---|---|
| 38.1 | 37.9 | 37.7 | 38.0 |
| 37.4 | 38.2 | 37.5 | 37.6 |

37.4, 37.5, 37.6, 37.7, 37.9, 38.0, 38.1, 38.2

# 2

| | | |
|---|---|---|
| A. 0.289 km | D. 0.342 kg | G. 0.250 ℓ |
| B. 0.307 km | E. 0.269 kg | H. 0.198 ℓ |
| C. 0.282 km | F. 0.273 kg | I. 0.891 ℓ |

**a** Which is the longer distance — A or B? B
How can you tell? 0.③07 > 0.②89

**b** Which is the greater mass — E or F? F
How can you tell this time? 0.2⑦3 > 0.2⑥9

**c** Which is the longer distance — A or C? A
Once again, how can you tell? 0.28⑨ > 0.28②

**d** Which is the greater volume —
G or H? G
H or I? I

**e** Order A, B, and C from longest distance to shortest distance. 0.307, 0.289, 0.282

**f** Order D, E, and F from greatest mass to least mass. 0.342, 0.273, 0.269

**g** Order G, H, and I from greatest volume to least volume. 0.891, 0.250, 0.198

# 3

Arrange these decimals in order from lowest to highest.

| | | | |
|---|---|---|---|
| 0.344 | 0.367 | 0.358 | 0.342 |
| 0.340 | 0.298 | 0.333 | 0.341 |

0.298, 0.333, 0.340, 0.341,
0.342, 0.344, 0.358, 0.367



**goal** Review of ordering decimals

**page 105** These applications of decimals should stimulate an interesting discussion — especially if anyone has been running a high temperature lately.

Everyone should order the sets of numbers for problems 1 to 3 independently.

**goal** Review of comparing decimals; **Progress Check**—ordering decimals

**page 106** Problem 1 is perfect for a teasing exchange of ideas. Decide what the actual amounts of money for each example would be. This will help those who are unsure to make up their minds quickly.

The Progress Check is not hard. Only problems 5 and 6 have different place values. Hope that after the review and practice on the preceding pages no one is in trouble. If someone is, have a peer tutor use number-line diagrams and place-value charts for special help.

**1** Can numbers with different place values be equivalent? **THINK!**

One tenth = 0.1 = $\frac{1}{10}$ Right?

One hundredth = 0.01 = $\frac{1}{100}$ Right?

One thousandth = 0.001 = $\frac{1}{1000}$ Are you sure that's right?

**a** Does 0.1 = 0.01? No
If you had $\frac{1}{10}$ of one dollar and I had $\frac{1}{100}$ of a dollar, would you trade with me? No

**b** Does 0.10 = 0.01? No
If you had $\frac{10}{100}$ of one dollar and I had $\frac{1}{100}$ of one dollar, would you trade with me? No

**c** Does 0.1 = 0.10? **THINK!** Yes
If you had $\frac{1}{10}$ of one dollar and I had $\frac{100}{1000}$ of one dollar, would we have the same amount of money? Yes

**d** Does 0.01 = 0.001? No

**e** Does 0.010 = 0.001? No

**2** Arrange these in order from smallest to largest.

**a** 12.35, 12.4, 12.355, 12.401
12.35, 12.355, 12.4, 12.401

**b** 11.250, 11.330, 11.255, 11.200
11.200, 11.250, 11.255, 11.330

Which set was easier to do? Can you figure out why?

Probably b; because all the numbers had the same place value

## PROGRESS CHECK

Copy and order each set of decimals from smallest to largest.

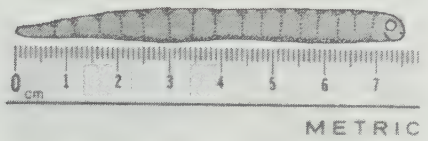
1. 2.1, 4.7, 3.6, 5.1
   2.1, 3.6, 4.7, 5.1
2. 0.9, 0.7, 0.6, 1.2, 0.5
   .5, .6, .7, .9, 1.2
3. 0.106, 0.359, 0.201, 0.275
   .106, .201, .275, .359
4. 3.015, 3.501, 3.105, 1.503
   1.503, 3.015, 3.105, 3.501
5. 0.79, 0.406, 9.3, 1.6
   .406, .79, 1.6, 9.3
6. 2.704, 2.71, 2.695, 2.701
   2.695, 2.701, 2.704, 2.71

106

See activity 1, page 144b.

See activity 2, page 144b.

How long is the worm?




You could say 75 mm . In our system of measurement we never mix units of measure. If you wanted to find the length of the worm in centimetres, you would have to use a decimal fraction. Fractions related to measurement are always written in decimal form. Measuring in centimetres, the worm is 7.5 cm long.

**1.** Measure these lines. Write your answers first in millimetres and then in centimetres.

**a** 65 mm 6.5 cm

**b** 54 mm 5.4 cm

**c** 104 mm 10.4 cm

**d** 27 mm 2.7 cm

**e** 76 mm 7.6 cm

**f** 43 mm 4.3 cm

**2.** Measure the width and length of the following books. Write your answers first in millimetres and then in centimetres.

**a** this book **b** a reader **c** a workbook

**d** a library book **e** a dictionary **f** one other book



**lesson** Pages 107, 108

**goal** Examining the use of decimals with SI measurements

**page 107** Actual measuring of lines and distances in millimetres and centimetres, and seeing the relationship between 65 mm and 6.5 cm, should prove to be a very useful reinforcement of the pupils' understanding of the decimal system.

You may want to assign additional measuring tasks to those who like this activity, and to those who seem to have difficulty with the relationship between millimetres and centimetres.

**goal** Practice in finding equivalent measurements; exact measurements

**page 108** Everyone should do exercises 1 and 2. The THINK section and exercises 3 and 4 can probably be best handled in class discussion, when any gaps in understanding can be identified and handled.

Problem 5 is a research project. You will want to assign it only to those who are interested; then have them report to the group.

**1.** How many:

**a** millimetres in a metre? 1000 **b** millimetres in a centimetre? 10

**c** centimetres in a metre? 100 **d** decimetres in a metre? 10

**2.** Supply the missing word.

**a** Decimetres are ? $\frac{1}{10}$ of a metre. **b** Centimetres are hundredths of a ?. metre

**c** Grams are ? $\frac{1}{1000}$ of a kilogram. **d** ? millimetres are thousandths of a litre.

**e** ? Millimetres are tenths of a centimetre. **f** ? Centimetres are tenths of a decimetre.

Think carefully about how accurately this measurement has been made.

4.50 m THINK:
Two decimal places.
That's hundredths.
Centimetres are hundredths of a metre. The measurement has been made to the nearest centimetre.

**3.** How accurately have these measurements been made?

**a** 7.2 cm To the nearest millimetre **b** 4.500 kg To the nearest gram **c** 8.75 m To the nearest centimetre **d** 15.125 km To the nearest metre

**4.** 4.65 m can be written 465 cm . Rewrite each measurement in question 3 as a smaller measure so that you don't need to use a decimal fraction.

72 mm 4500 g 875 cm 15 125 m

**5.** The Olympic games bring together athletes from more than a hundred nations. They are played every four years. Swimming, running, speed skating, and skiing events are based on lengths measured in metric units. Check an almanac. Find out the last reported records set in the last Olympics.

Answers will vary. Have students compare their answers and discuss any discrepancies.

Two of them decided to go on a backpack mountain-climbing trip. There were several stopping points along the way. The first morning they started at $A$ and climbed .7 of a kilometre to $B$. They rested at $B$, had lunch, and then climbed .2 of a kilometre to $C$. They made camp for the night at $C$. How far up did they climb that day?

$$\begin{array}{r} .7 \\ +.2 \\ \hline .9 \end{array} \quad \begin{array}{r} \frac{7}{10} \\ +\frac{2}{10} \\ \hline \frac{9}{10} \end{array}$$

1. The next morning they climbed .9 of a kilometre to $D$.
   In the afternoon they climbed .6 of a kilometre to $E$.
   How far did they climb the second day?

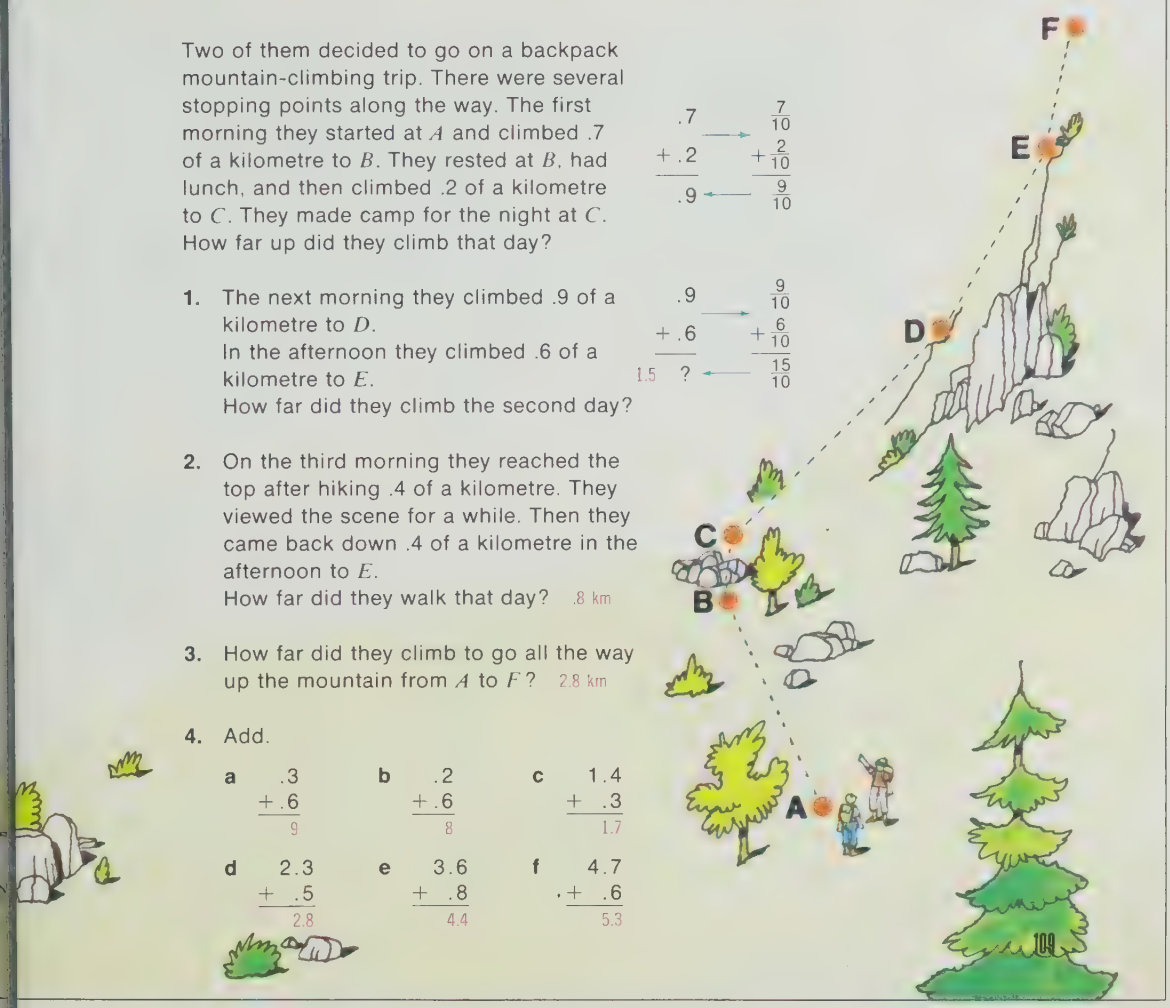
$$\begin{array}{r} .9 \\ +.6 \\ \hline 1.5 \quad ? \end{array} \quad \begin{array}{r} \frac{9}{10} \\ +\frac{6}{10} \\ \hline \frac{15}{10} \end{array}$$

2. On the third morning they reached the top after hiking .4 of a kilometre. They viewed the scene for a while. Then they came back down .4 of a kilometre in the afternoon to $E$.
   How far did they walk that day? .8 km

3. How far did they climb to go all the way up the mountain from $A$ to $F$? 2.8 km

4. Add.

| | | | | | |
|---|---|---|---|---|---|
| **a** | .3 | **b** | .2 | **c** | 1.4 |
| | +.6 | | +.6 | | +.3 |
| | 9 | | 8 | | 1.7 |
| **d** | 2.3 | **e** | 3.6 | **f** | 4.7 |
| | +.5 | | +.8 | | +.6 |
| | 2.8 | | 4.4 | | 5.3 |




**lesson** Pages 109, 110

**goal** Introduction to the addition of decimals

**page 109** Compare adding decimals to adding common fractions. Is one form of the number easier to work with than the other form? How? To add fractions, denominators must be alike. How can you be sure denominators are alike when adding decimals? Point out the importance of place value and of aligning the decimal points. Compare adding decimals to adding whole numbers. The computations are simple. Watch for students who have difficulty with renaming in the addition problems.

**goal** Introduction to the subtraction of decimals

**page 110** Subtraction requiring renaming may need some explanation. If so, use problems **2d** and **2e** for discussion. Remind the students of the steps necessary to subtract mixed numbers in common fraction form. Everyone should really feel grateful to Simon Stevin now.

**1.** Coming down the mountain they went 1.5 kilometres from $E$ to $C$ in one day. In the morning they covered .6 of a kilometre.

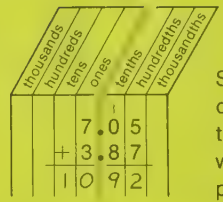
$$\begin{array}{rr} 1.5 & \frac{15}{10} \\ -\ .6 & -\frac{6}{10} \\ \hline .9 & \frac{9}{10} \end{array}$$

How far did they have left to go in the afternoon?

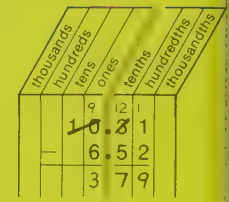
# SUBTRACT

**2.**

| **a** | **b** | **c** | **d** | **e** |
|---|---|---|---|---|
| .9 | 2.4 | 1.8 | 2.3 | 3.0 |
| $-.5$ | $-\ .3$ | $-\ .4$ | $-\ .6$ | $-\ .7$ |
| .4 | 2.1 | 1.4 | 1.7 | 2.3 |




Sorry, but there really isn't much new to learn when it comes to adding and subtracting decimals. You had to pay attention to place value when you operated with whole numbers. You have to pay attention to place value with decimals, too.

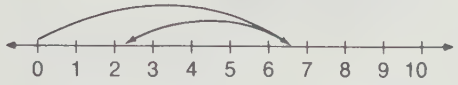



Compute these. Watch the operation symbol!

| | **a** | **b** | **c** | **d** | **e** | **f** | **g** | **h** |
|---|---|---|---|---|---|---|---|---|
| **3.** | 2.3 | 4.7 | 3.7 | 4.9 | 6.2 | 5.4 | 6.3 | 7.60 |
| | $+\ .4$ | $-2.5$ | $+\ .8$ | $+1.6$ | $-1.7$ | $-1.4$ | $+1.7$ | $-4.06$ |
| | 2.7 | 2.2 | 4.5 | 6.5 | 4.5 | 4.0 | 8.0 | 3.54 |
| **4.** | 5.6 | 6.2 | 8.8 | 2.3 | 10.0 | 4.5 | 7.4 | 10.64 |
| | $-\ .6$ | $-\ .9$ | $+7.6$ | $-1.5$ | $-\ 3.2$ | $+5.5$ | $-6.5$ | $+\ 9.36$ |
| | 5.0 | 5.3 | 16.4 | .8 | 6.8 | 10.0 | .9 | 20.00 |

110

This number line shows this subtraction problem.




$$\begin{array}{r} 6.5 \\ -\ 4.25 \\ \hline \end{array} \longrightarrow \begin{array}{r} 6.50 \\ -\ 4.25 \\ \hline 2.25 \end{array}$$

Why do you suppose 6.5 was renamed as 6.50? So you can subtract

**PRACTICE** You may have to rename to subtract.

| a | b | c | d | e |
|---|---|---|---|---|
| 4.6 | 3.35 | 6.8 | 3.62 | 3.07 |
| − 2.14 | − 1.41 | − 2.42 | − 1.91 | − 2.45 |
| 2.46 | 1.94 | 4.38 | 1.71 | 0.62 |

| a | b | c | d | e |
|---|---|---|---|---|
| 6.46 | 4.0 | 7.56 | 2.3 | 6.5 |
| − 4.33 | − 2.03 | − 4.45 | − 1.26 | − 4.36 |
| 2.13 | 1.97 | 3.11 | 1.04 | 2.14 |

These two problems differ in three ways.
What are these differences? Answers will vary

**a**

$$\begin{array}{r} 4.23 \\ +\ 3.45 \\ \hline 7.68 \end{array} \longrightarrow \begin{array}{r} 4\frac{23}{100} \\ +\ 3\frac{45}{100} \\ \hline 7\frac{68}{100} \end{array}$$

**b**

$$\begin{array}{r} 7.643 \\ -\ 2.381 \\ \hline 5.262 \end{array} \longrightarrow \begin{array}{r} 7\frac{643}{1000} \\ -\ 2\frac{381}{1000} \\ \hline 5\frac{262}{1000} \end{array}$$

Examples: a addition
place value to hundredths
no renaming

b subtraction
place value to thousandths
renaming



**lesson** Pages 111, 112, 113, 114

**goal** Practice in subtracting decimals with two places to the right of the decimal point.

**page 111** Talk about the first problem together. Some youngsters may **think** common fractions rather than decimals as they look at the number line and think about the length of the arrows. Challenge them to write $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ as decimals. Need a clue? How is a quarter written in dollars and cents?

**goal** Practice in adding and subtracting decimals requiring and not requiring renaming

**page 112** If any pupils have been having trouble with renaming, you will want to talk with them about this skill before making an assignment. Stress that the operations of addition and subtraction are the same for whole numbers and decimals. The place value of each digit of each number is the only major thing to remember. You may have to remind them that annexing a zero to the right simply renames the number.

# add

Watch for a place-value pattern.

| | a | b | c | d |
|---|---|---|---|---|
| 1. | 0.1 + 0.1 = .2 | 0.3 + 0.2 = .5 | 0.6 + 0.3 = .9 | 0.9 + 0.1 = 1.0 ← How many tenths in one? 10 |
| 2. | 0.5 + 0.5 = 1.0 | 0.9 + 0.9 = 1.8 | 1.9 + 0.1 = 2.0 | 9.1 + 1.9 = 11.0 ← Make sure that answer is corre |

# subtract

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| 3. | 0.8 − 0.1 = .7 | 0.9 − 0.9 = .0 | 1.3 − 0.3 = 1.0 | 1.0 − 0.3 = .7 | 2.5 − 1.6 = .9 | 10.0 − 2.9 = 7.1 |
| 4. | 21.8 − 6.2 = 15.6 | 35.6 − 12.4 = 23.2 | 64.5 − 21.3 = 43.2 | 43.2 − 26.7 = 16.5 | 60.2 − 13.5 = 46.7 | 20.0 − 16.1 = 3.9 |

There is an old rule that says—
*Keep your decimal points in a straight line when you add or subtract.*
Does this rule make sense? Why? Yes; its necessary to maintain place value.

5. Compute these.

**a** 1.5 + 3.25 4.75 **b** 3.5 − 2.6 .9 **c** 17.1 + 6.9 + 3.15 27.15

**d** $15 − $0.15 $14.85 **e** $100 + $25.25 + $0.50 $125.75 **f** $85.50 − $65 $20.50

112

**goal** Practice in estimating differences with decimals; practice in adding and subtracting with decimals

**page 113** Rounding to the nearest multiple of 25 helps when estimating with money, and it may help when estimating decimals written in hundredths also. Stress estimating to verify the reasonableness of a computed answer.

**1.** Have you been using your estimating skills?
Think money. That always helps.
Do not compute. Quickly write your answer.

| | You owe | Bill you pay with | Change you will get back |
|---|---|---|---|
| **a** | 4.78 | 5 | More than 25¢? No |
| **b** | 3.56 | 5 | More than $2.00? No |
| **c** | 7.68 | 10 | More than $2.00? Yes |
| **d** | 15.02 | 20 | More than $5.00? No |
| **e** | 4.49 | 5 | More than 50¢? Yes |
| **f** | 6.50 | 10 | More than $4.00? No |
| **g** | 4.50 | 10 | More than $6.00? No |
| **h** | 9.28 | 20 | More than $11.00? No |

Add. Think money if it helps. Don't forget to estimate.

| | **a** | **b** | **c** | **d** | **e** | **f** |
|---|---|---|---|---|---|---|
| **2.** | 1.50 | 2.65 | 4.34 | 6.72 | 8.25 | 9.61 |
| | +1.25 | +1.15 | +2.62 | +5.28 | +1.75 | +1.29 |
| | 2.75 | 3.80 | 6.96 | 12.00 | 10.00 | 10.90 |

Subtract.

| | | | | | | |
|---|---|---|---|---|---|---|
| **3.** | 4.75 | 3.75 | 5.50 | 2.06 | 4.69 | 7.00 |
| | −1.25 | −1.26 | −3.36 | −1.92 | −3.89 | −6.69 |
| | 3.50 | 2.49 | 2.14 | 14 | .80 | .31 |

**4.** Compute these.

**a** 6.5 + 3.56 10.06 **b** 4 − .50 3.50 **c** 1. + .1 + .01 1.11 **d** 6.01 − 3.9 2.11

**goal** Adding and subtracting decimals with unlike place value

**page 114** The pupils have been introduced to a few problems with unlike place value on the preceding pages. Let their previous experiences serve as clues to the most appropriate way for you to handle the page with them.

Stress that the name of the game is renaming. Sure, it's possible to add .1 + .01 without renaming, but it's far better to rename consistently in both addition and subtraction—especially during these early experiences in computing with decimals. Picky? Not really. If renaming becomes a **think** pattern, the possibility of errors in subtraction will be greatly diminished. The following mistakes are common ones.

```
  1.5
- .43
 1.1③   ← This 3 was simply brought down rather than subtracted.

  5.9
- 1.45
 4.5⑤   ← This looks like the error, but it probably isn't.
  ↑ ↑
  │ └── This 5 was brought down.
  └──── This column was subtracted.
```

Time to take a closer look at addition and subtraction of numbers.

**1.** Sometimes place value is especially important.
A fractional number can be written in either of two ways.
For example:

.1 or $\frac{1}{10}$ .75 or $\frac{75}{100}$ .300 or $\frac{300}{1000}$

**a** Can .8 + .76 be rewritten as a problem with fractions? Would it be easier to compute? No

Yes Rename .8 as .80

```
  .8   What should you do before you solve this one?
+.76   What did you do with the "denominators"?
 1.56  Solve this problem.   Made them the same
```

**b** Can .9 − .29 be written as a problem with fractions? Would it be easier to compute?

Yes Rename .9 as

```
  .9   What must you do before you solve this on
-.29   What did you do with the "denominators"?
 .61   Solve this problem   Made them the same value
```

**2.** Add .4 plus .22. Say the answer to yourself. .62
What place value was said? Hundredths

**3.** Subtract .49 from .567. .077
What place value is indicated in the answer? Thousandths

**4.** Add .9 plus .5. 1.4
What place value is in the answer? Tenths

**5.** Add .7 plus .55. 1.25
What place value is in the answer? Hundredths

**6.** Subtract .56 from .9. .34
What place value is in the answer? Hundredths

**7.** Subtract .35 from 2.2. 1.85
What place value? Hundredths

**8.** Compute. Watch the operation signs.

| a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|
| 1.5 | 2.17 | .6 | 1.31 | 3.1 | 4.05 | 3.65 |
| + .67 | − .67 | +.71 | − .6 | + .92 | − .9 | + .05 |
| 2.17 | 1.50 | 1.31 | .71 | 4.02 | 3.15 | 3.70 |

114

# PROGRESS CHECK

Add. Skills: Adding decimals, renaming hundredths

| | | | | |
|---|---|---|---|---|
| (1.) 2.76 + 3.13 = 5.89 | (2.) 4.01 + 2.54 = 6.55 | (3.) 3.23 + 1.58 = 4.81 | (4.) 3.37 + 2.46 = 5.83 | (5.) 6.85 + 1.16 = 8.01 |

Skills: Adding decimals, renaming tenths and hundredths

| | | | | |
|---|---|---|---|---|
| (6.) 2.87 + 2.57 = 5.44 | (7.) 5.05 + 4.68 = 9.73 | (8.) 4.76 + 8.96 = 13.72 | (9.) 16.32 + 4.79 = 21.11 | (10.) 56.85 + 14.69 = 71.54 |

Subtract. Skills: Subtracting decimals, renaming tenths

| | | | | |
|---|---|---|---|---|
| (11.) 4.89 − 1.43 = 3.46 | (12.) 7.88 − 1.26 = 6.62 | (13.) 6.61 − 3.24 = 3.37 | (14.) 8.71 − 3.46 = 5.25 | (15.) 5.15 − 4.17 = .98 |

Skills: Subtracting decimals, renaming tens, ones, and tenths

| | | | | |
|---|---|---|---|---|
| (16.) 9.65 − 7.96 = 1.69 | (17.) 8.15 − 5.78 = 2.37 | (18.) 15.39 − 9.49 = 5.90 | (19.) 10.00 − 5.74 = 4.26 | (20.) 92.02 − 23.29 = 68.73 |

Compute. Watch out! Skills: Adding and subtracting decimals

21–23: No renaming. 24–30: With renaming

| | | | | |
|---|---|---|---|---|
| (21.) 3.723 + 1.246 = 4.969 | (22.) 5.089 + 2.310 = 7.399 | (23.) 4.681 − 2.150 = 2.531 | (24.) 3.729 + 1.391 = 5.120 | (25.) 5.960 + 2.580 = 8.540 |
| (26.) 4.163 + 5.095 = 9.258 | (27.) 9.278 − 1.709 = 7.569 | (28.) 7.797 + 2.566 = 10.363 | (29.) 6.437 − 3.779 = 2.658 | (30.) 2.076 − 1.109 = .967 |

Skill: Renaming

*(31.) Somebody made some errors. Can you find them?

3.465 + 2.371 = ~~5.736~~ 5.836

6.435 − 2.716 = ~~4.719~~ 3.719

115

See activity 3, page 144b.

See activity 4, page 144b.

**lesson** Page 115

**goal** **Progress Check** – adding and subtracting decimals

**memo** Do not assign the entire page on the same day. There are too many problems.

**page 115** The groups of problems are ordered by level of difficulty. Pupils who are still making errors when renaming is required will be identified. Those who have trouble with problems 1 through 20 will need additional help. Odds are they are weak in adding and subtracting whole numbers. This is a prerequisite skill for operating with decimals.

Very capable pupils may go directly to problems 21 through 31.

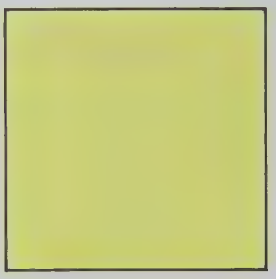
**lesson** Pages 116, 117, 118

**goal** Introduction to the multiplication of decimals

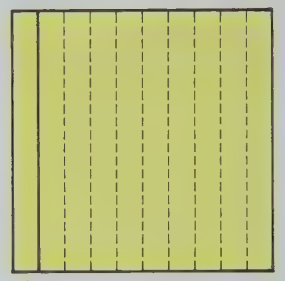
**memo** Pages 116, 117, and 118 should be used together on the same day. These pages literally lead the pupil right into the multiplication algorithm.

**page 116** This page will occupy your discussion group for a very short time, but it is an important first step in understanding the operation of multiplication and the reasonableness of the answer. How were the hundredths formed? (By dividing the region into tenths vertically **and** horizontally)

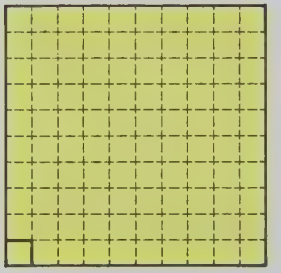
Continue right on to page 117.



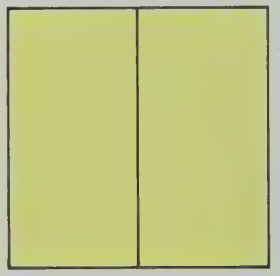
1 whole square region.



1 square divided into 10 parts.
1 of 10 parts. Could be written
1 tenth or $\frac{1}{10}$ or .1.



1 square divided into 100 parts.
1 of 100 parts. Could be written
1 hundredth or $\frac{1}{100}$ or .01.



1 square divided into 2 parts.
1 of 2 parts. Could be written $\frac{1}{2}$.
Could it also be written 5 tenths or .5?
Could it be 50 hundredths or .50? Ye

116

$.1 + .1 + .1 + .1 = .4$

$4 \times .1 = .4$

1. Answer these.

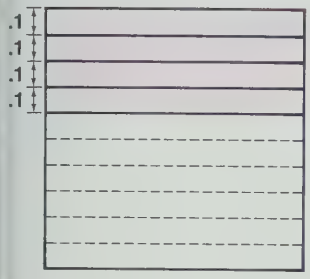
   **a** $6 \times .1 = \underline{?}$ .6

   **b** $2 \times .1 = \underline{?}$ .2

   **c** $7 \times .1 = \underline{?}$ .7

   **d** $9 \times .1 = \underline{?}$ .9

   **e** $10 \times .1 = \underline{?}$ 1.0




$.3 + .3 + .3 = .9$

$3 \times .3 = .9$

2. Answer these.

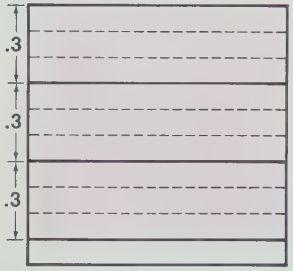
   **a** $1 \times .3 = \underline{?}$ .3

   **b** $2 \times .3 = \underline{?}$ .6

   Think carefully here.

   **c** $4 \times .3 = \underline{?}$ 1.2

   **d** $5 \times .3 = \underline{?}$ 1.5




$.5 + .5 = 1.0$

$2 \times .5 = 1.0$

3. Answer these.

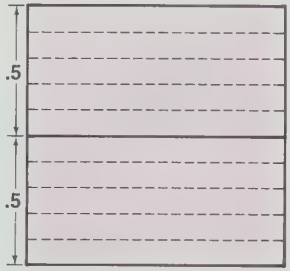
   **a** $1 \times .5 = \underline{?}$ .5

   **b** $2 \times .5 = \underline{?}$ 1.0

   **c** $3 \times .5 = \underline{?}$ 1.5

   **d** $4 \times .5 = \underline{?}$ 2.0

   **e** $5 \times .5 = \underline{?}$ 2.5






**goal** Introduction to the multiplication of decimals (*continued*)

**page 117** Continue with the discussion. Relate repeated addition to multiplication. This technique has been used first for whole numbers, then for fractions, and now for decimals. Check understanding by having your pupils jot down answers to problems 1 through 3 independently. Then compare the answers orally. The words "four times point one equals point four" will be used eventually, but for the present, work for the place-value names instead. The words should be "four times one tenth equals four tenths." Listen to hear how $10 \times .1 = 1.0$ is stated. Settle for that product being read as "one and zero tenths." Just "one" is O.K. too.

Go right on to page 118.

**goal** Practice in multiplying decimals

**page 118** You'll want to talk about the models at the top of the page with your pupils. So far only one square unit has been used at a time. Each unit in turn has been divided to show tenths or hundredths. The first model shows the familiar unit square with 1 tenth shaded. But the questions ask about more than 1 tenth being shaded. It will be worth your effort to show other models on a transparency or on the chalkboard.

$1 \times .2 = \underline{?}$ (Show 2 tenths shaded)

Shade one more tenth strip to illustrate $1 \times .3 = \underline{?}$ and so on.

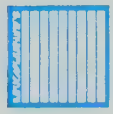
Now look out! The really dangerous spot is when you start on the next column where two unit squares are used. Each unit square has 1 tenth shaded. Draw another model and label.

1 unit square .1 part shaded
1 unit square .1 part shaded

2 (unit squares) × .1 (shaded in each) = .2 (shaded in all)

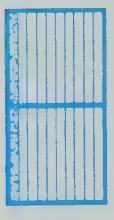
Use your same model to illustrate $2 \times .3$. Shade each unit square so 3 tenths of each square are shaded. Shade the next tenth strip in each unit square to show $2 \times .4$ and so on.

Don't assign problems 11 through 15 to anyone who is insecure. Use those problems only with pupils who have caught on to the concept and are ready to become independent operators.



$1 \times .1 = .1$

What if more tenths were shaded?



$2 \times .1 = .2$
$2 \times .2 = \underline{?}$ .4
$2 \times .3 = \underline{?}$ .6
$2 \times .4 = \underline{?}$ .8
$2 \times .5 = \underline{?}$ 1.0
$2 \times .6 = \underline{?}$ 1.2
$2 \times .7 = \underline{?}$ 1.4
$2 \times .8 = \underline{?}$ 1.6
$2 \times .9 = \underline{?}$ 1.8
$2 \times 1.0 = \underline{?}$ 2.0

$3 \times .1 = .3$
$3 \times .4 = \underline{?}$ 1.2
$3 \times .5 = \underline{?}$ 1.5
$3 \times .6 = \underline{?}$ 1.8
$3 \times .7 = \underline{?}$ 2.1
$3 \times .8 = \underline{?}$ 2.4
$3 \times .9 = \underline{?}$ 2.7
$3 \times 1.0 = \underline{?}$ 3.0

$4 \times .1 = .4$
$4 \times .7 = \underline{?}$ 2.8
$4 \times .8 = \underline{?}$ 3.2
$4 \times .9 = \underline{?}$ 3.6
$4 \times 1.0 = \underline{?}$ 4.0

$5 \times .1 = .5$
$5 \times 1.0 = \underline{?}$

Tenths times any whole number result in tenths in the answer. Prove it.

**TRY THESE**

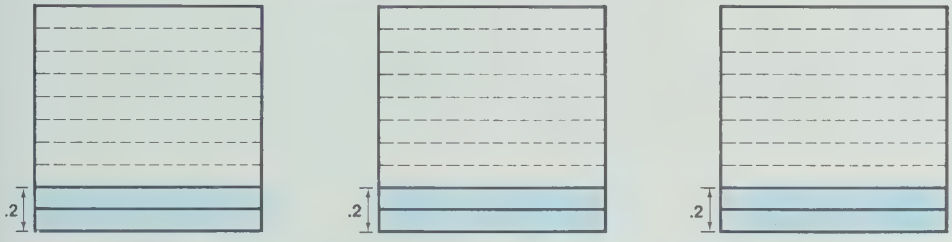
| | | | | | |
|---|---|---|---|---|---|
| **1.** 6 × .9 = 5.4 | **2.** 7 × .2 = 1.4 | **3.** 5 × .5 = 2.5 | **4.** 9 × .1 = .9 | **5.** 8 × .4 = 3.2 |
| **6.** 12 × .2 = 2.4 | **7.** 14 × .1 = 1.4 | **8.** 15 × .3 = 4.5 | **9.** 19 × .6 = 11.4 | **10.** 20 × .1 = 2.0 |
| **11.** 17 × 1.4 = 23.8 | **12.** 55 × 2.3 = 126.5 | **13.** 56 × 3.5 = 196.0 | **14.** 123 × 1.2 = 147.6 | **15.** 514 × 6.7 = 3443.8 |

118

## HERE'S A NEW PROBLEM

How much is .2 of 3?

You know what two-tenths is.
You know what three units are.
This is a multiplication situation.
But you could still use addition.




The pictures show .2 + .2 + .2

But this is 3 × .2. You know that is .6. Does 3 × .2 = .2 × 3? Yes

Answer these.

1. 2 × .2 = ? .4 **2.** 4 × .2 = ? .8 **3.** 5 × .2 = ? 1.0 **4.** 10 × .2 = ? 2.0 **5.** 1 × .2 = ? .2

Take a closer look
at problems like this one.
THINK
How many tenths in 1? 10
This problem deals with
two-tenths—5 of them.

Stack all the 5
two-tenths together.
.2 + .2 + .2 + .2 + .2

That makes ten-tenths
$\left(\frac{10}{10}\right)$, or 1 whole unit.



**lesson** Page 119

**goal** Examining finding a decimal part of a whole

**memo** Don't assume that the three square unit models will be obvious to the learner. Understanding of this page is a prerequisite to success with the next pages. The whole thing looks simple, and maybe it will be.

**page 119** Treat this as an exploratory page. Challenge your pupils to use what they already know to solve this new problem that they are faced with—finding a decimal part of a whole. The solution makes use of the commutative property of multiplication. Whether or not you use this technical vocabulary is your decision.

Examine the products for problems 1, 2, and 5. Strange things are happening! A whole number and a decimal less than one are multiplied. The product is less than one. Can this be??? Examine the diagrams. Remind your pupils that one of the factors is less than a whole.

**lesson** Pages 120, 121, 122, 123

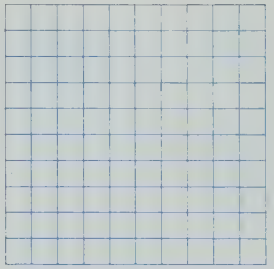
**goal** Exploring multiplying two decimals

**memo** Pages 120, 121, and 122 develop the concept of multiplying two decimals. These pages should be used together on the same day.

**page 120** Enjoy the page together, but don't miss these concepts:

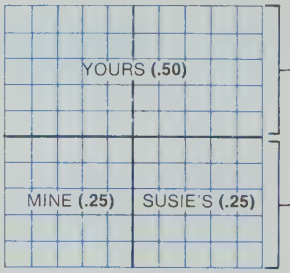
- A hundred pennies are being considered. Each penny is .01 of a dollar.
- A half-dollar can be thought of as .50.
- Half of a half-dollar is a quarter or .25.
- A fifth of a quarter is a nickel or 5 pennies. Either can be written .05.

Go right on to page 121.



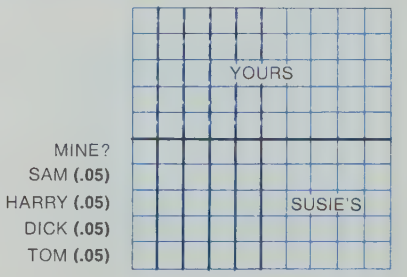
How many pennies in 1 dollar? 100
Pretend we have square pennies.
Here they are all lined up in 10 rows
of 10 square pennies in each row.
Make sure that there are 100 pictured.

The pennies are mine. But you say you
want half of them. You're a nice
person, so I'll give you half of them.
(You can have half of my square
pennies anytime!)




This is yours. How many pennies do you get? 50
What fraction of the dollar do you get? $\frac{1}{2}$
Can your amount be written $.50? Yes

This is mine. Susie says she would like
half of mine. She's nice too. So she
gets half. How many cents do I have left? 25¢




Tom, Dick, Harry, and Sam would like
to share with me too. They would
like me to give them a share. They ar
really nice!

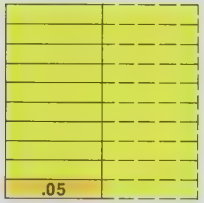
I had $1.00. I gave you .50. Susie got
.25, and then Tom, Dick, Harry, and
Sam came along.
How much do I have left? $.05

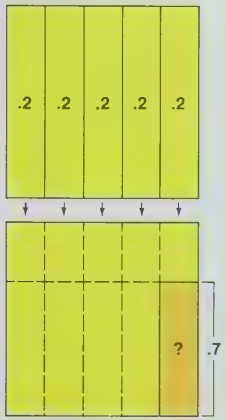
120

I went out and earned another dollar. You guessed it! I found a lot of nice people again. Can you figure out what happened this time from the drawing on the right? It is too painful for me to talk about. You tell the story.

The picture could also be used as a model for decimals. The first picture shows halves. Each part is marked .5, and that is a name for halves.

The region was also marked into ten equal parts. Each of the ten parts represents .1 of the total square region. The shaded part shows .1 of .5 parts. .1 × .5 = ? .05




A picture showing multiplication of decimals probably isn't as much fun as your story. The math picture is just as important.

Now your job is to find the product of .7 × .2. The top picture shows the whole unit in five equal parts. Each part shows .2.

The problem says you want .7 of .2. .7 of .2 is how many hundredths? 14

.7 × .2 = ? 14



**goal** Introduction to multiplying two decimals

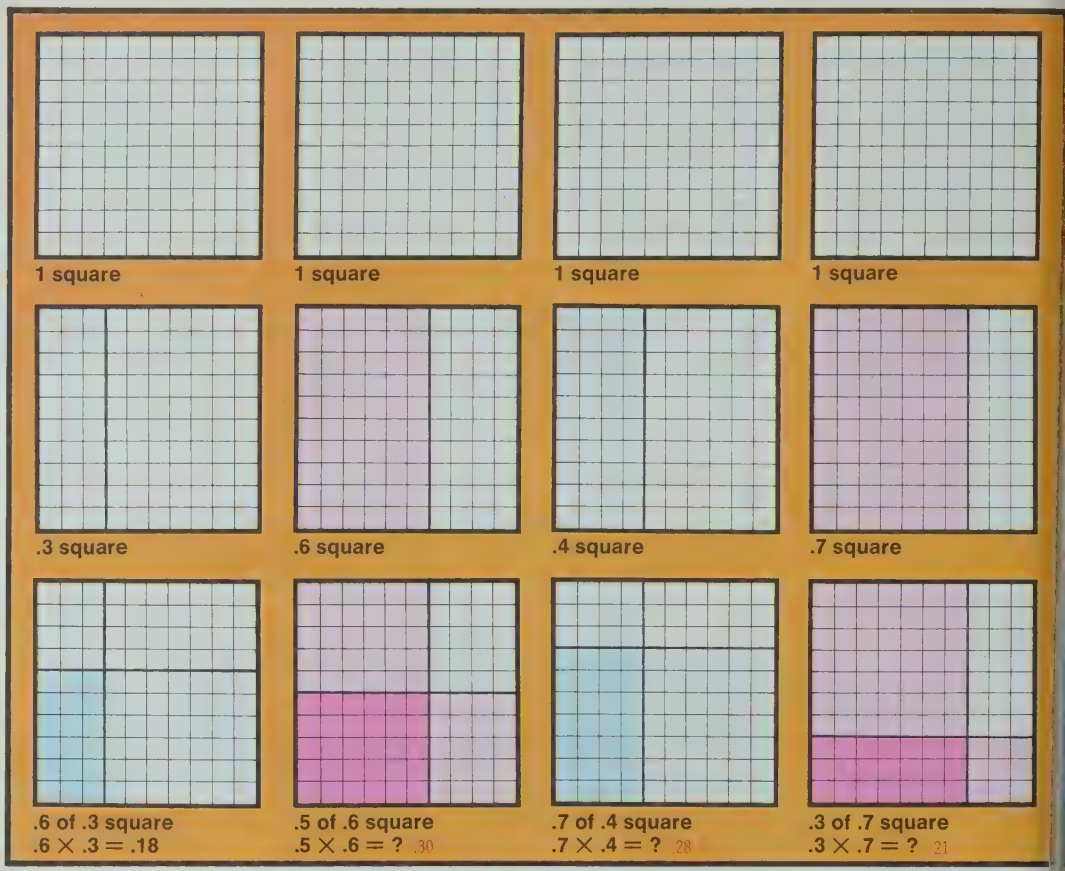
**page 121** More to enjoy together. Encourage imagination. Have the pupils make up several stories for the example at the top of the page. Notice that the halves have each been marked as .5. Act puzzled. Ask for an explanation. Could the model indicate 5 dimes? a half-dollar? 50 pennies?

Examine the models together. Praise the pupils who recall that models like these were used to illustrate the multiplication of fractions. Let them come to this realization—don't tell. Point out that tenths times tenths are hundredths—nice and consistent with our system of place value.

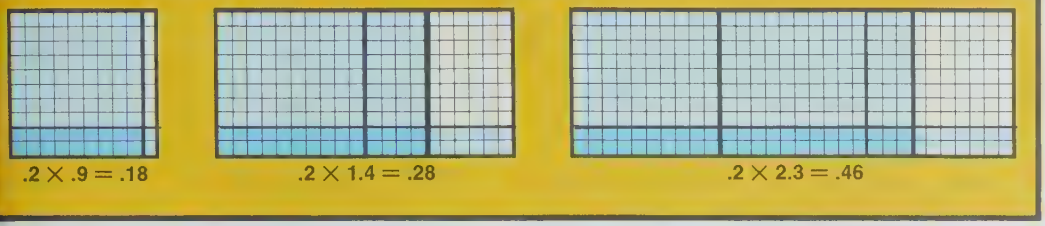
Continue on to page 122.

**goal** Multiplying two decimals, using a model

**page 122** The models should provide enough help for everyone to complete the page independently. After the answers have been recorded by each pupil, take time to compare answers. Identify pupils who need more guidance.




122




What if more tenths were shaded?

| | | |
|---|---|---|
| .3 × .9 = ? .27 | .3 × 1.4 = ? .42 | .3 × 2.3 = ? .69 |
| .6 × .9 = ? .54 | .5 × 1.4 = ? .70 | .4 × 2.3 = ? .92 |
| .8 × .9 = ? .72 | .7 × 1.4 = ? .98 | .6 × 2.3 = ? 1.38 |
| .9 × .9 = ? .81 | .8 × 1.4 = ? 1.12 | 1.0 × 2.3 = ? 2.30 |

Any number with tenths in its place value times another number with tenths in its place value results in hundredths in the answer.

Prove it. Try these.

| | | | | |
|---|---|---|---|---|
| **1.** .2 × .6 = .12 | **2.** .7 × .8 = .56 | **3.** .9 × .6 = .54 | **4.** .5 × .2 = .10 | **5.** .3 × .3 = .09 |
| **6.** 1.2 × .2 = .24 | **7.** 2.1 × .6 = 1.26 | **8.** 1.5 × .3 = .45 | **9.** 1.7 × .8 = 1.36 | **10.** 2.5 × .7 = 1.75 |
| **11.** 21.3 × .2 = 4.26 | **12.** 14.1 × .4 = 5.64 | **13.** 52.7 × .6 = 31.62 | **14.** 62.7 × .4 = 25.08 | **15.** 35.6 × .9 = 32.04 |

123

**goal** Introduction to multiplying two decimals, one decimal being greater than 1

**page 123** The first model showing .2 × .9 = .18 should be obvious to everyone, but be careful of the next ones. The models are used to introduce multiplication with a factor greater than 1 and are not easily understood by all children. First establish that 1 whole unit and .4 more of another unit are marked. Then .2 of that 1.4 is shown by the shaded part. In the third model, 2 units and .3 more of another unit are marked, and then .2 of that 2.3 is shown by the shaded part.

Consider having your pupils jot down answers for the columns at the top. Then correct answers and clear up questions before making a further assignment. Problems 1 through 15 increase in level of difficulty. Watch for two types of errors:

- Multiplication—especially renaming
- Placement of the decimal point

Reemphasize that tenths times tenths are hundredths. Examine the size of the products. Why are the products less than a whole region?

**lesson** Pages 124, 125

**goal** Applications involving multiplying, subtracting, and adding decimals

**page 124** There's lots of computation involved to complete the charts. Each line requires the pupil first to multiply, then to subtract. Finally the total cost must be added.

Since the youngsters have been computing decimals without a dollar sign, there may be a tendency to forget this symbol when recording answers for this page. A reminder may be in order.

1 You know how to compute with money. Go shopping.

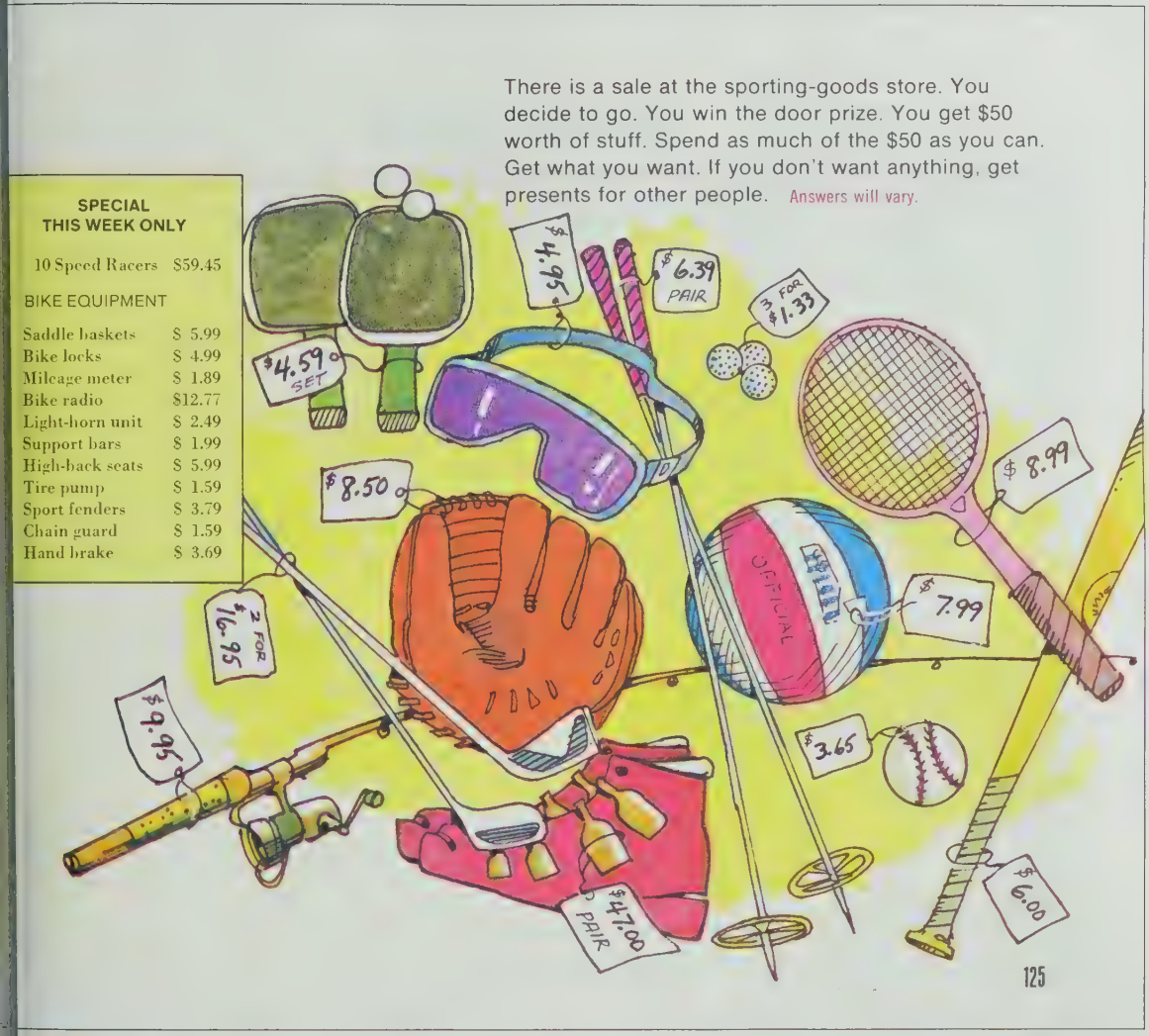
| | You want | Each costs | You have | Can you buy them? | How much money is left? |
|---|---|---|---|---|---|
| a | 3 rolls of film | $1.25 | $5.00 | ? Yes | ? $1.25 |
| b | 2 packages of flash bulbs | $ .79 | $2.00 | ? Yes | ? $.42 |
| c | 3 pounds of candy | $ .65 | $3.25 | ? Yes | ? $1.30 |
| d | 2 hamburgers | $ .55 | $1.25 | ? Yes | ? $.15 |
| e | 12 cookies | $ .08 | $1.00 | ? Yes | ? $.04 |

2 You have to buy these things. You are in charge of a field day.

| | You need | Each costs | You have | How much more do you need? |
|---|---|---|---|---|
| a | 24 train tickets | $1.50 | $24.00 | ? $12.00 |
| b | 24 tickets to the game | $ .75 | $15.00 | ? $3.00 |
| c | 19 lunch tickets (5 people will bring their own) | $1.10 | $20.00 | ? $.90 |
| d | How much money will be spent in all for this field day? $74.90 | | | |

124

There is a sale at the sporting-goods store. You decide to go. You win the door prize. You get $50 worth of stuff. Spend as much of the $50 as you can. Get what you want. If you don't want anything, get presents for other people. Answers will vary.




**things** newspaper ads

Put your researchers to work. Have them find ads similar to the one shown on the page. Decide on a specific amount of money each person may spend. Challenge: try to spend that **exact** amount—no more, no less.

**goal** Applications involving decision making

**page 125** Every individual will make choices for personal reasons. What is valuable to one person may not be for another. The direction any discussion takes is up to you.

To extend the page, be sure to have a group figure out what they would buy in order to have the maximum number of different presents. Could they buy a present for everyone in the whole class? Investigate the costs of all bike equipment offered on sale. If you bought the bike and all the equipment, how much money would you need?

Ask pupils to make up some more problems based on the information on the page and have fun.

**lesson** Page 126

**goal** **Progress Check** – writing, ordering, and rounding decimals; adding, subtracting, and multiplying decimals

**page 126** The Progress Check includes all the skills developed with decimals thus far. Use the chart below to help you plan appropriate additional help.

| Problems incorrect | Try |
|---|---|
| 1–6 | Further work with a place-value chart. |
| 7 | Practice locating points on a number line, then record in order. |
| 8–12 | Further tutoring under teacher or peer direction. |
| 13–22 | Check for computational errors first, then for confusion related to the decimal point. |

# PROGRESS CHECK

Here's what you were supposed to have learned so far.
Did you? Prove it.

Skill: Writing fractions as decimals

Write each as a decimal.

(1.) $\frac{2}{10}$ .2 (2.) $\frac{2}{100}$ .02 (3.) $\frac{2}{1000}$ .002 (4.) $\frac{20}{10}$ 2.0 (5.) $\frac{20}{100}$ .20 (6.) $\frac{20}{1000}$ .020

Skill: Ordering decimals

(7.) Arrange these decimals in order from smallest to greatest. 1.15, 10.01, 10.1, 11.1, 11.12, 100.0

10.1 10.01 1.15 11.1 11.12 100.0

Skill: Rounding decimals to the nearest tenth

Round each decimal to the nearest tenth.

(8.) 2.567 2.6 (9.) 10.16 10.2 (10.) 1.782 1.8 (11.) 1.021 1.0 (12.) 2.135 2.1

Skills: Adding, subtracting, and multiplying decimals

Compute. Watch the operation signs.

(13.) $3.7 + .9 = 4.6$ (14.) $10.3 - 1.2 = 9.1$ (15.) $8.3 - .4 = 7.9$ (16.) $3.526 + .758 = 4.284$ (17.) $.6 - .37 = .23$

(18.) $.2 \times .6 = .12$ (19.) $1.5 + .78 = 2.28$ (20.) $62.7 \times 1.8 = 112.86$ (21.) $15.4 \times .9 = 13.86$ (22.) $2.5 \times .1 = .25$

Keep your eyes open for decimals. Start a decimal collection from newspaper clippings, facts you hear on TV, stories from magazines. Copy information you might see in a store. Save labels from grocery items that show decimals being used. Keep your eyes and ears open.

126

See activity 5, page 144b.

See activity 6, page 144b.

The four members of the Junior Business Club got their first job on Saturday. They helped Ms. Wilton with her garden clean-up. Ms. Wilton paid them $6. The $6 was to be split among the four of them.

The math sentence is $\$6 \div 4 = \underline{?}$

To compute, you would write $4\overline{)\$6}$.

First try: $4\overline{)\$6}$ $\$1$ R2

$-4$

$2$

Answer: Each pays $1, but $2 remains to be divided. This answer isn't good enough.

Second try: Each member was given $1. That left $2 still to be paid out. Then Tricia, the president, suggested they each get another $0.25.

How much more was paid out? $1

How much was still left to be paid out? $1

How much did they each get in the end? $1.50

$\$1.50$

Is $4\overline{)\$6.00}$ true? Yes

Is $6 the same amount of money as $6.00? Yes

Ricky, the treasurer, wanted to make sure $1.50 was correct. He wrote

$\$1.50$

$\times \quad 4$

$\$6.00$ It checks out.

Ricky thought there had to be a way to do problems like this without so much trouble. Do you agree?

127

**lesson** Pages 127, 128

**goal** Introduction to division of decimals

**memo** If everyone, including you, has had enough of decimals for a while, stop now and go to chapters 6, 8, or 11. Any one of these will provide a change of pace. And you can come back here later.

On the other hand, your pupils' skills may be right where you want them for the introduction of division with decimals. It is a light, easy-to-handle entry. It should be fun for everyone. Please be sure to use pages 127 and 128 on the same day.

**page 127** When a $6 bill needs to be paid, a $2 unpaid remainder certainly is not a good enough answer. An exact answer is needed. Pages 127 and 128 lead the pupil into dividing with decimals by utilizing a real-world situation involving money. The problem is approached through an estimate that is verified by multiplication.

**goal** Introduction to division of decimals (*continued*)

**page 128** The algorithm for dividing a decimal is developed with the pupil on the page. For the present he should feel that these problems are no different from regular long division, except for the dollar sign and decimal in the quotient. Quotients should be verified by multiplication. You decide whether your pupils can handle the practice problems independently.

Ricky decided to write it as a division problem again. He just worked it like a whole-number division problem.

```
 1 50
   50
 1 00
4)6.00
- 4 00
  2 00
- 2 00
    0
```

Then he went back and put in the decimal points, so that it looked like this:

```
$1.50
 0.50
 1.00
4)6.00
- 4 00
  2 00
- 2 00
    0
```

When the Junior Business Club got their next job, the club had grown to 8 members. (Everybody wanted to get in on the act.) They worked all afternoon for Mr. Rees and were paid $10. How much was each person's share?

Ricky did the division this time.

He divided without paying attention to the decimal point.

Then he put the decimal points where he thought they belonged.

```
 1.25
    5
 0.20
 1.00
8)10.00
- 8 00
  2 00
- 1 60
   40
- 40
    0
```

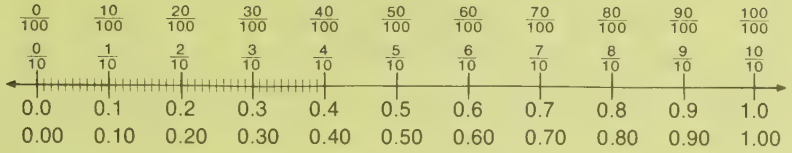
Check his work. Was he correct? Yes

```
 1.25
×   8
10.00
```

It's your turn to do division.

1. $5\overline{)8.00}$ = 1.60
2. $6\overline{)15.00}$ = 2.50
3. $4\overline{)4.80}$ = 1.20
4. $7\overline{)28.00}$ = 4.00
5. $8\overline{)8.80}$ = 1.10
6. $3\overline{)9.30}$ = 3.10

128




1. Do $\frac{10}{100}$, $\frac{1}{10}$, 0.1, and 0.10 mark the same distance on the number line? Yes
   Are they names for the same number? Yes

2. Do $\frac{30}{100}$, $\frac{3}{10}$, 0.3, and 0.30 mark the same distance on the number line? Yes
   Are they names for the same number? Yes

3. The decimal .5 tells that the distance between 0 and 1 has been divided into 10 equal parts.
   - **a** The decimal .50 tells that the distance between 0 and 1 has been divided into ? equal parts. 100
   - **b** Do .5 and .50 mark the same distance on the number line? Yes
   - **c** Would .500 also mark the same distance? Yes
   - **d** Would .5000 also mark the same distance? Yes

4. Do 1, 1.0, 1.00, and 1.000 mark the same distance on the number line? Yes

5. Do these quotients name the same number? Yes $2\overline{)4}$ $2\overline{)4.0}$ $2\overline{)4.00}$

129

**things** adding-machine tape

Develop a clearer understanding of decimal units by having the pupils take a strip of adding-machine tape and mark off a 1-metre length. Next, have them mark off and label the 100-centimetre lengths contained in the metre. Finally have them show all 1000 millimetres.

**lesson** Pages 129, 130, 131

**goal** Review of renaming decimals

**page 129** This page is more than review. It is essential for every pupil to understand that zeros can be annexed to the right of a decimal point as well as to the right of the last digit of a decimal. The zero(s) will not change the value of the number itself. Let the number-line diagram show how this renaming is possible.

You may want to make a much larger version of the number line on the chalkboard. Pupils can mark off 10 same-length segments, then label the tenths. Next have each segment divided into 10 same-length segments to form hundredths. Now have the hundredths labeled. Getting involved in such a project will yield more understanding of equivalent names for the same distance than merely looking at the finished product.

Renaming .5 as .50 is not new, but renaming 1 as 1.0 or 1.00 is. Focus on the names for 0 and 1 shown on the number line. This idea is essential for division with decimals.

**goal** Introduction to division resulting in a decimal quotient (tenths)

**page 130** There's no money involved this time. The idea is to divide up all that remains. The page is self-explanatory and leads the pupil right into the algorithm. Nice and easy—no need to rush. The division computations are quite simple. Anyone who has trouble with division of whole numbers should focus on that skill and forget about decimals for the time being.

Look at this division situation:

$$10\overline{)12} \quad 1\text{ R}2$$

Do you think we could annex a decimal point and a zero to the right of 12 and not change the value of 12? Yes

What would happen to the quotient and the remainder if we did that?

$$\begin{array}{r|l} & 1.2 \\ & .2 \\ & 1.0 \\ 10\overline{)12.0} & \\ -10.0 & 1.0 \times 10 \\ \hline & 2.0 \\ -\ 2.0 & .2 \times 10 \\ \hline & 0 \end{array}$$

Notice—the tenths are all in the same column. The ones also are in a single column.

**Hey!** The remainder is 0.

Can this be right? Check the problem.

$$\begin{array}{r} 1.2 \\ \times\ 10 \\ \hline 12.0 \end{array}$$

You know 5 ÷ 2 should have a remainder. Try annexing a decimal point and a zero (or zeros) and see what happens.

$$\begin{array}{r|l} & 2.5 \\ & .5 \\ & 2.0 \\ 2\overline{)5.0} & \\ -4.0 & 2.0 \times 2 \\ \hline & 1.0 \\ -1.0 & .5 \times 2 \\ \hline & 0 \end{array}$$

Check the problem:

$$\begin{array}{r} 2.5 \\ \times\ 2 \\ \hline 5.0 \end{array}$$

Annex a decimal point and zero if this is necessary to avoid a remainder. Divide and check.

**1.** $4\overline{)10.0}$ → 2.5; −8; 20; −20; 0

**2.** $5\overline{)18.0}$ → 3.6; −15; 30; −30; 0

**3.** $2\overline{)11.0}$ → 5.5; −10; 10; −10; 0

**4.** $6\overline{)27.0}$ → 4.5; −24; 30; −30; 0

**5.** $8\overline{)12.0}$ → 1.5; −8; 40; −40; 0

**6.** $5\overline{)32.0}$ → 6.4; −30; 20; −20; 0

130

```
   .2
  1.0
4)5.0
-4.0   1.0 × 4
 1.0
-  .8   .2 × 4
  .2
```

Now what? A zero has been annexed, but there is still a remainder.

Can you annex another zero and not change the value of 5.0? Try it.

Yes

```
   1.25
      ?
   .05
   .20
  1.00
4)5.00
-4.00   1.00 × 4
 1.00
-  .80   .20 × 4
  .20
-  .20   .05 × 4
     0
```

You can avoid remainders in many division problems. Sometimes you have to annex only one zero, sometimes more.

Your practice problems are arranged so that you won't have to annex more than three zeros. But please remember that there are some division problems that would need more.

In fact, there are some division problems that will have a remainder no matter how many zeros are annexed. Maybe you'll find one someday!

Practice on these.

**1.** $8\overline{)26.00}$ 3.25 **2.** $4\overline{)6.0}$ 1.5 **3.** $6\overline{)39.0}$ 6.5 **4.** $8\overline{)11.000}$ 1.375 **5.** $2\overline{)1.0}$ .5 **6.** $5\overline{)2.0}$ .4

**7.** $8\overline{)1.000}$ .125 **8.** $4\overline{)1.00}$ .25 **9.** $4\overline{)3.00}$ .75 **10.** $5\overline{)4.0}$ .8 **11.** $8\overline{)5.000}$ .625 **12.** $10\overline{)1.0}$ .1



**goal** Extension of division resulting in a decimal quotient (hundredths)

**page 131** The algorithm is expanded on this page. Annexing one zero just doesn't do the trick for these problems. Pick someone to read aloud that paragraph on the page. The information is too important to be missed by those youngsters who have a reading problem.

The computation remains quite simple. Children who can divide whole numbers should have no difficulty on this page.

**lesson** Pages 132, 133, 134

**goal** Exploration of renaming numbers in division

**memo** Understanding this page is essential for success with division of decimals.

**page 132** For anyone who still feels .3, .03, and .003 all name the same number, simply rewrite these decimal fractions as common fractions — $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$.

Two technical words will prove helpful for communicating when talking about problem 2 —

Divisor → $2\overline{)4}$ ← Dividend

Stress that both the **divisor** and the **dividend** of the original problem have been multiplied by the same number. Praise the sharp one who realizes that this is similar to multiplying the numerator and denominator of a fraction when renaming.

It is very important that each learner convince himself that multiplying the divisor and dividend by the same number will not change the answer. You may want to try still more simple examples before tackling problems 3, 4, and 5. Consider $2\overline{)6}$. Multiply the divisor and dividend by 2, then 3, then 4, then 5, and finally 10. What is the quotient for each?

After the pupils complete problem 3, call time-out to ask a question. By what number have the divisor and dividend each been multiplied?

A game must be played by the rules. Change one rule, and look what happens! Examine problem 5.

**1** Annexing zeros without paying attention to the decimal point is unwise.

**a** Do 3, 3.0, 3.00, 3.000 name the same number? Yes

**b** Do 3, .3, .03, .003 name the same number? No

**2** **Look at these division problems.**

$2\overline{)4}$ → Take both × 2. → $4\overline{)8}$ Is the quotient the same? Yes

Take both × 3. → $6\overline{)12}$ Is the quotient the same? Yes

Take both × 10. → $20\overline{)40}$ Same quotient? Yes

Take both × 100. → $200\overline{)400}$ Same quotient? Yes

If you multiply both numbers by the same number, it seems that the quotient doesn't change. Test the idea on these problems.

**3** $4\overset{2}{\overline{)8}}$ $40\overset{2}{\overline{)80}}$ $400\overset{2}{\overline{)800}}$ $4000\overset{2}{\overline{)8000}}$ Are the quotients the same? Yes

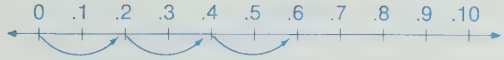
**4** $5\overset{2}{\overline{)10}}$ $50\overset{2}{\overline{)100}}$ $500\overset{2}{\overline{)1000}}$ $5000\overset{2}{\overline{)10,000}}$ Are the quotients the same? Yes

**5** What happens to the quotient if only one number is multiplied by another number? Does it stay the same? No

$5\overset{2}{\overline{)10}}$ $5\overset{20}{\overline{)100}}$ $5\overset{200}{\overline{)1000}}$ $5\overset{2000}{\overline{)10,000}}$

132

## HOW MANY .2s IN .6?




It just doesn't make sense to draw a number line every time you want to do a simple division problem. There is a way to compute the quotient.

You know how to divide whole numbers. You can use that skill. The only thing you must do is figure out a way to make .2 a whole number.

Is there a number you can multiply times .2 to make it a whole number?

$1 \times .2 = .2$ That won't work.

O.K., try again. $10 \times .2 = 2.0$ THAT'S IT!

$.2\overline{).6}$ ← What about that part of the problem?
↑ If .2 is multiplied by 10, .6 must also be multiplied by 10.

$.2\overline{).6}$ **Think** $10 \times .2 = 2$
$10 \times .6 = 6$

Both numbers have been multiplied by 10. The quotient of $.2\overline{).6}$ and $2\overline{)6}$ will be the same. Now the problem is $2\overline{)6}$.

Rename so that you can divide by a whole number. Write the new problem. Then compute.

**1.** **a** $.4\overline{).8}$ $4\overline{)8}^{\,2}$ **b** $.4\overline{)8}$ $4\overline{)80}^{\,20}$ **c** $.3\overline{).6}$ $3\overline{)6}^{\,2}$ **d** $.3\overline{).06}$ $3\overline{).6}^{\,.2}$

**2.** **a** $.3\overline{)60}$ $3\overline{)600}^{\,200}$ **b** $.3\overline{)6}$ $3\overline{)60}^{\,20}$ **c** $.4\overline{)80}$ $4\overline{)800}^{\,200}$ **d** $.4\overline{)800}$ $4\overline{)8000}^{\,2000}$



**goal** Introduction to dividing by a decimal

**page 133** When a zero is annexed to a whole number, the number has been multiplied by 10. Annexing two zeros multiplies the number by 100. When a decimal is multiplied by 10, the decimal point moves one place to the right. Multiplying by 100 moves the decimal point two places to the right. These understandings are prerequisite for dividing decimals. Although these concepts are not new, you may do well to check whether they are generally understood before applying them in the division work.

After multiplying so that the decimal divisor becomes a whole number, have the youngsters actually rewrite the division problem in order to see the results of the complete multiplication step.

**goal** Extension of dividing by a decimal

**page 134** The page is self-explanatory. The emphasis is on getting a whole-number divisor. Continue having the youngsters rewrite the problem after the multiplication step on this page. A shorter method will be shown on the next page.

What can you do here? $.14\overline{).28}$

What number times .14 is equal to 14?

$10 \times .14 = 1.40$ Nope!

What about $100 \times .14 = 14.00$? That's it! 14.00 names 14.

$.14\overline{).28}$ **THINK** $100 \times .14 = 14$

$100 \times .28 = 28$

Now the problem is $14\overline{)28}$

What is the quotient? 2

Rename so that you can divide by a whole number. Write the problem. Then compute.

| | a | b | c | d |
|---|---|---|---|---|
| 1. | $.25\overline{).75}$ $\quad 25\overline{)75}$ $\;3$ | $.25\overline{)7.5}$ $\quad 25\overline{)750}$ $\;30$ | $.25\overline{)75}$ $\quad 25\overline{)7500}$ $\;300$ | $.25\overline{).075}$ $\quad 25\overline{)7.5}$ $\;.3$ |

No more rigged problems. You are on your own. You know the model for whole-number division works. All you have to do is make sure you divide by a whole number.

| | a | b | c | d |
|---|---|---|---|---|
| 2. | $.06\overline{).18}$ $\;3$ | $.3\overline{).9}$ $\;3$ | $.27\overline{).81}$ $\;3$ | $.5\overline{).85}$ $\;1.7$ |
| 3. | $.2\overline{)1.4}$ $\;7$ | $.5\overline{)2.5}$ $\;5$ | $.7\overline{)4.2}$ $\;6$ | $.3\overline{)1.2}$ $\;4$ |
| 4. | $.70\overline{)2.8}$ $\;4$ | $.15\overline{)3.00}$ $\;20$ | $.10\overline{)10}$ $\;100$ | $.1\overline{)1}$ $\;10$ |

5. Don't let this one fool you. Operate in the same way. $2.5\overline{)5}$ $\;2$

134

Someone discovered a way to write the action of multiplying by 10 or 100 or 1000. Rather than rewrite the problem, he used an arrow to show the placement of the decimal. It can help remind you about the decimal placement in the quotient, too.

$2.3\overline{)3.22}$ Usually you rewrite the problem. Multiply both times 10. Use an arrow instead of rewriting. $2.3\overline{)3.22}$

$.12\overline{).516}$ (quotient $4.3$) By what number were both these parts multiplied? Find the quotient. 100

## PROGRESS CHECK

Skill: Dividing with decimals

Compute the quotients. Check each quotient to see that your placement of the decimal point makes the answer reasonable.

1. $.5\overline{)2.5}$ (5)
2. $.7\overline{).49}$ (.7)
3. $.4\overline{).072}$ (.18)
4. $.6\overline{)13.8}$ (23)
5. $1.3\overline{)6.5}$ (5)
6. $3.2\overline{)86.4}$ (27)
7. $.23\overline{)3.68}$ (16)
8. $.35\overline{).0105}$ (.03)
9. $.07\overline{).0063}$ (.09)
10. $1.35\overline{)3.510}$ (2.6)
11. $2.65\overline{)9.805}$ (3.7)
12. $.03\overline{).0051}$ (.17)

135

See activity 7, page 144c.

See activity 8, page 144c.

**lesson** Page 135

**goal** Introduction to using an arrow to indicate placement of the decimal after multiplication; **Progress Check** – dividing by a decimal

**page 135** Some learners will want to continue rewriting the division problem after the multiplication step. That's fine! Others will feel that this is an unnecessary step. A shortcut is helpful only if it makes sense to the person using it.

Anyone who can compute through problem 6 is progressing satisfactorily. Beginning with problem 7, each problem presents something new and different. You may wish to work some of these as a group since they require unusual manipulations. Or you may choose to use them for additional practice in relocating the decimal only – no computation necessary. Reserve problems 7 through 12 as independent work for your really capable students.

Look for two types of errors:

- Decimal point errors
  $.6\overline{)13.8}$ changed to $.6\overline{)13.8}$
  $.7\overline{).49}$ changed to $.7\overline{).49}$
- Division errors

$$1.3\overline{)6.5}\quad \text{(quotient } 6\text{)}$$
$$-6\,8 \rightarrow \text{multiplication error } (6 \times 13 = 78)$$
$$3 \rightarrow \text{subtraction error}$$

**lesson** Page 136

**goal** Practice in estimating quotients and checking division

**page 136** Some pupils may need to rewrite the problems into computation form in order to determine where the decimal should be placed for each quotient in problem 1. The emphasis should be on **estimation** rather than on operating mechanically. If rewriting is necessary, have them write only this much of the problem, then use estimation:

$6\overline{)26.4}$ $3\overline{)38.4}$

When checking the completed problems, make sure the youngsters don't get mixed up placing the decimals in the product. Either of these two ways of checking is O.K.

$$\begin{array}{r} 7.5 \\ \times\ .3 \leftarrow \text{decimal in original placement} \\ \hline 2.25 \leftarrow \text{decimal in original placement} \end{array}$$

$$\begin{array}{r} 7.5 \\ \times\ 3 \leftarrow \text{decimal removed by multiplication} \\ \hline 22.5 \leftarrow \text{decimal moved by multiplication} \end{array}$$

Stress the consistencies:

- Multiply by the original divisor, get the original dividend.
- Multiply by the renamed divisor, get the renamed dividend.

## 1

The digits for the quotients are given for each problem below, BUT there is no hint about the value of the digits. Estimate the quotients. Put the decimal in the correct place in the quotients.

**a** $26.4 \div 6 = 44$ 4.4 **b** $38.4 \div 3 = 128$ 12.8 **c** $79.48 \div 4 = 1987$ 19.87

**d** $19.2 \div 8 = 24$ 2.4 **e** $1.33 \div 7 = 19$ .19 **f** $40.08 \div 12 = 334$ 3.34

**g** $3.164 \div 14 = 226$ .226 **h** $485.1 \div 21 = 231$ 23.1 **i** $74.88 \div 32 = 234$ 2.34

## 2

In each example below, the decimal point is put in the quotient directly above the decimal point in the dividend. Are the quotients reasonable answers? Check to make sure the answers are correct.

**a** $3\overline{)2.25}$

$$\begin{array}{r} 7.5 \\ -21 \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 7.5 \\ \times\ .3 \\ \hline 2.25 \end{array}$$

**b** $.07\overline{)2.394}$

$$\begin{array}{r} 34.2 \\ -21 \\ \hline 29 \\ -28 \\ \hline 14 \\ -14 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 34.2 \\ \times\ .07 \\ \hline 2.394 \end{array}$$

**c** $1.2\overline{)1.68}$

$$\begin{array}{r} 1.4 \\ -12 \\ \hline 48 \\ -48 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1.4 \\ \times\ 1.2 \\ \hline 1.68 \end{array}$$

You align decimals when you subtract. Does it make sense to align decimals in your division computation too?

Yes—you really need to, since you subtract in the division computation, too.

## 3

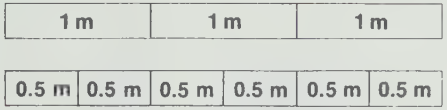
You are on your own. Try these.
Make sure the quotient is reasonable.

**a** $2\overline{)5.32}$ 26.6 **b** $.05\overline{)2.55}$ 51. **c** $.03\overline{).243}$ 8.1

136

Any measurement you make is as accurate as your measuring device. Pretend you have a measuring stick 3 metres long. It has no markings except metres.

You could say the mark is more than 1 metre and less than 2 metres. Which is it closer to?




If .5 metres were marked, you could say the mark is more than 1.0 metre and less than 1.5 metres. Which is it closer to?

If someone came along with a metrestick marked in centimetres, the mark could be measured to the nearest centimetre.

Someone else could come along with a metrestick marked in millimetres, and so on, and so on, and so on.

If you are computing measurements, you operate in a very special way.

1.530 m ←——This is measured to the nearest millimetre.
+ 1.4 m ←——This wasn't. If it had been measured to the nearest millimetre, it would be written 1.400 m. Does that mean you can't add?

You can add. But both measurements should be the same. You can't improve the 1.4 m measure. But you can round the 1.530 to tenths.

1.530 rounds to 1.5 m
+ 1.4 m
O.K. now!
The sum is 2.9 m.

This is the way you should work with measurements if you want to be accurate. When you are practicing computation, you can annex zeros if you need to.

137

**lesson** Pages 137, 138

**goal** Rounding measurements written in decimal notation for computation

**memo** Pages 137 and 138 are extension pages. You may not want to use them with everyone.

**page 137** This idea can be hard for adults as well as for children. A lot of rules change when a decimal numeral becomes a measurement. The whole thing has to do with the accuracy of a measuring device. For example, measuring with a stick that is 1 metre long, a metrestick marked with decimetres, another marked with centimetres, and still another marked with millimetres will obviously allow measurements of varying accuracy. The pupil page is self-explanatory, but there are lots of words. Why not use this page as a basis for discussion and learning together?

**goal** Computing measurements in decimal notation

**page 138** A clear distinction is made between computing numbers and computing measurements. Check to make sure this distinction is understood. Pupils who annex zeros when they should be rounding do not understand.

If you are doing computation practice, you are free to annex zeros to the right of any digit.

$$\begin{array}{r} 1.7 \longrightarrow 1.70 \\ +\,3.85 \longrightarrow +\,3.85 \\ \hline \end{array}$$ O.K.? Yes

$$\begin{array}{r} 1.7\ \text{m} \longrightarrow 1.7\ \text{m} \\ +\,3.85\ \text{m} \longrightarrow +\,3.9\ \text{m} \\ \hline \end{array}$$ O.K.? Yes

$$\begin{array}{r} 9.8 \longrightarrow 9.800 \\ -\,5.736 \longrightarrow -\,5.736 \\ \hline \end{array}$$ O.K.? Yes

$$\begin{array}{r} 9.8\ \text{m} \longrightarrow 9.8\ \text{m} \\ -\,5.736\ \text{m} \longrightarrow -\,5.7\ \text{m} \\ \hline \end{array}$$ O.K.? Yes

Decide if you should round before you compute. Then compute. Look out! Some of them are tricky.

1. He was to find out how much gold wire the goldsmith had in stock. One box was marked 10.55 cm . Another was marked 3.5 cm . The third box was marked 6.4 cm . About how much gold wire was in the three boxes?
   $10.6 + 3.5 + 6.4 = 20.5$ cm
2. The car odometer read 1782.5 km when she started her drive. It read 1925.0 km when she arrived. How far had she driven?
   $1925.0 - 1782.5 = 142.5$ km
3. One label on a package of trim said 100 cm . Another package label said 1 m . She bought both of them. How much trim did she buy?
   $100\ \text{cm} = 1\ \text{m} \quad 1\ \text{m} + 1\ \text{m} = 2\ \text{m}$

*4. Read this one carefully. A carton contains forty 25 g packages of spice. Another carton contains two 25 g packages of spice. How many kilograms of spice in all? 1.050 kg



**International travel**

Travel between Canada and other countries grew at a slower rate in 1971. Both receipts and payments from international travel increased by less than 5%, compared with gains of close to 15% a year earlier, and receipts rose at a slightly higher rate than payments, reducing the deficit on travel account. The growth in travel in 1971 was dampened to a greater degree in North America than in Europe, where rapid growth rates were experienced for the second consecutive year. Travel earnings for both Canada and the United States advanced by less than 7%, while European countries registered substantial gains of about 20%.

**50% Off**

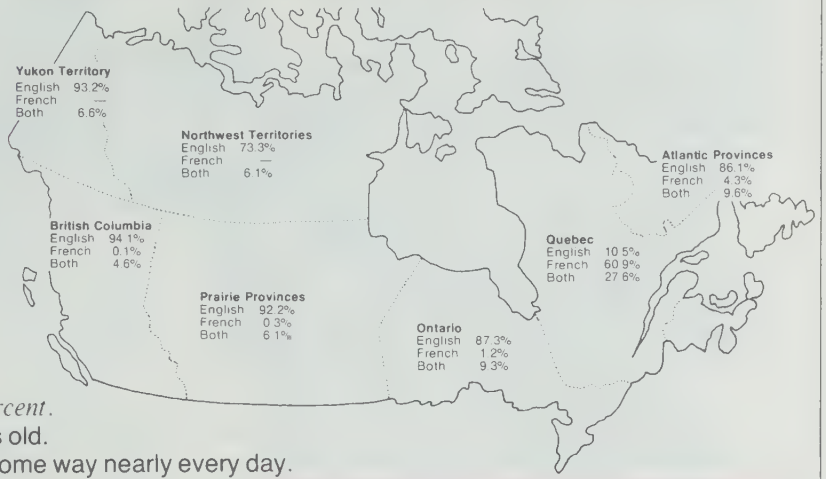
Age d

| Age group |
|---|
| 0-4 years |
| 5-9 years |
| 10-14 years |
| 15-19 years |
| 20-24 years |
| 25-29 years |
| 30-34 years |
| 35-39 years |
| 40-44 years |
| 45-49 years |
| 50-54 years |
| 55-59 years |
| 60-64 years |
| 65-69 years |

**Employment**

Over the 1966-71 period, the industrial composite index of employment for Canada rose by 5.9%. Among the industry divisions showing gains over this period, service led with a 34.1% advance, followed by finance, insurance, and real estate (21.3%), trade (15.0%), mining (7.4%), and transportation, communication, and other utilities (6.5%). Declines occurred in forestry (25.1%), construction (10.4%) and manufacturing (1.5%) during the same period. Compared with 1970, the industrial composite index for 1971 was up 0.6%.

**Percentage distribution of the population speaking one or both of the official languages (preliminary figures for 1971).**




% is a symbol for the word *percent*.
The idea is nearly 2000 years old.
We still see percent used in some way nearly every day.
Look at the pictures on this page.
List the different ways people use percent.

To show increase or decrease, to describe interest rates, to describe part of a whole, to describe discounts

1971.

| 1966 | 1971 |
|---|---|
| % | % |
| 11.0 | 8.4 |
| 11.5 | 10.4 |
| 10.5 | 10.7 |
| 9.2 | 9.8 |
| 7.3 | 8.8 |
| 6.2 | 7.3 |
| 6.2 | 6.1 |
| 6.4 | 5.9 |
| 6.3 | 5.9 |
| 5.4 | 5.7 |
| 4.9 | 4.9 |
| 4.1 | 4.4 |
| 3.3 | 3.6 |
| 2.7 | 2.9 |

**Certificates and Savings Accounts**

**9.00%**
new effective annual yield on
**7.75%** a year

their spaceships. They have reason: last year, Soviet output of goods and services rose less than 2%, the smallest gain in a decade. That contrasts with a 1972 rise of 9.7%, or 6.5% after subtracting price increases, in the U.S. gross nation- predictably, the So-

**15% to 50% OFF**
**on a Select Group of men's Suits, Sport Coats and Slacks**
**20% OFF**
on a Sele

139

**lesson** Pages 139, 140

**goal** Examining uses for the word PERCENT and the symbol %

**memo** Pages 139 through 143 are exploratory pages. Their purpose is to introduce the concept of percent as an extension of working with decimals. Percent is another way of writing a decimal. A percent must be rewritten as a decimal before computation can be done. Mastery of percent is not expected in this level.

**page 139** Strictly for discussion. Some pupils may already have a good grasp of percent from their own experiences, but others will find the word and symbol a mystery. The purpose of the page is to point out where the word or symbol can be seen and how it is used. Continue on to page 140 to find out what percent means.

**goal** Introduction to the meaning of percent

**page 140** Everything you need to introduce the meaning of percent is right on the page. If a 1-digit percent, such as 9%, is written as tenths on anyone's paper, take time to examine problem **2c** together. Begin with examining the error of 9% written as 9 tenths.

$$.9 = \tfrac{9}{10}$$

$$.9 = .90$$

$$\tfrac{9}{10} \times \tfrac{10}{10} = \tfrac{90}{100}$$

$$.90 = \tfrac{90}{100} = 90\%$$

Now 90% and 9% surely aren't the same! You can use the same approach if 9% was written as .90 rather than as .09.

The sign says you pay 10% less than the selling price.

But what does 10% mean?

Percent comes from Latin. It means "in each hundred." Percent is really another way to write a decimal for hundredths.

10% is the same as .10 $\left(\frac{10}{100}\right)$.

Suppose you went to the sale.
You bought a book marked $1.00.
10% of $1 is $.10.
You would get the book for $1.00 – $.10, or $.90

No matter what the price was, you would get 10 cents taken off every dollar you spend.

---

Time out to look at other percents. Remember percent means hundredths.

5% is the same as .05 $\left(\frac{5}{100}\right)$.

That's an important digit! Why?
It shows 5 hundredths rather than 5 tenths.

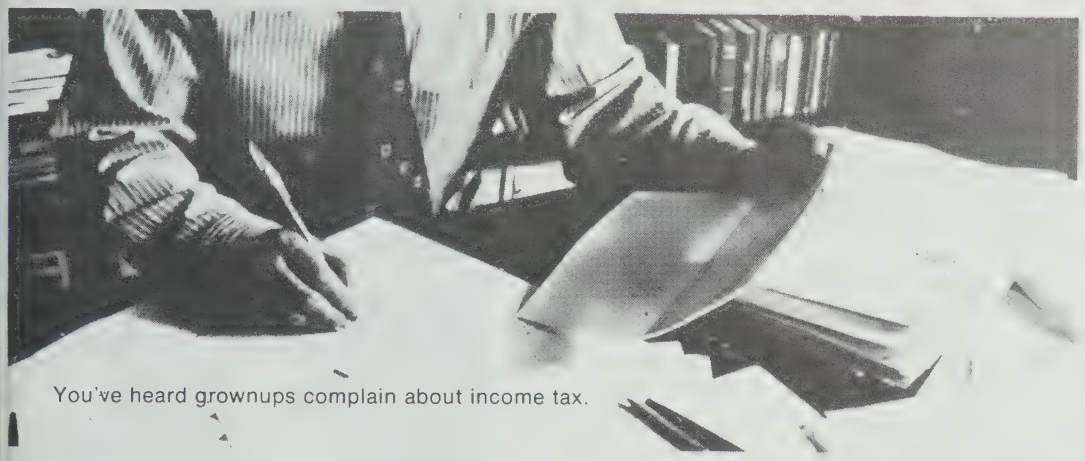
37% is the same as .37 $\left(\frac{37}{100}\right)$.

Write each of the following percents as a decimal.

| | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|
| 1. | 15% .15 | 80% .80 | 40% .40 | 35% .35 | 12% .12 | 98% .98 | 30% .30 | 4% .04 |
| 2. | 25% .25 | 10% .10 | 9% .09 | 1% .01 | 5% .05 | 3% .03 | 15% .15 | 99% .99 |

---

140



You've heard grownups complain about income tax.

1. What is an income tax? A government tax on total annual income
   - **a** Who has to pay income tax? Every wage earner with income over a certain amount
   - **b** What is the tax money used for? To finance government services
2. Some provinces have their own income tax. Does yours? Find out. Answers will vary
3. Most places have a sales tax. Do you pay any money in tax when you buy something for 10¢? for 25¢? for $1.00? If you pay a sales tax, find out how much. Answers will vary according to the particular sales tax.
4. If you sell something, do you charge tax? Usually not, unless you own a business
5. Lorna lives in a place where they have a 5% sales tax. She must pay $0.05 every time she buys something that costs $1. How much tax would she pay if she bought something for $2? $3? $5? $10? $.10; $.15; $.25; $.50
6. Do you think Lorna would pay a tax on something that costs less than $1? Probably Would she pay $0.05 or more? Why? Probably less Usually you pay less tax on amounts under $1.

Do some research. Interview an adult. Find out what kind of taxes that person pays.

141

**lesson** Pages 141, 142, 143

**goal** Examining income and sales taxes

**page 141** There's lots to talk about on this page. Most people are aware of income tax as well as sales taxes. However, some people are lucky enough to live where there is no sales tax. If so, this particular tax will need a bit of explanation.

**goal** Relating percent to test scores

**page 142** Test scores reported as a percent may be a new experience for your pupils. If so, go nice and easy. The method is not complicated when the total number of questions is 100. Some pupils may need a little help getting organized for problem 4. And problem 5 is strictly for discussion.

1. Suppose there was only one question on a test. Your score was 100%. How many did you get right? 1 How many wrong? 0

2. There were 10 questions on a test. Your score was 100%. How many right? 10 How many wrong? 0

3. There were 100 questions on a test. Your score was 100%. How many right? 100 How many wrong? 0

4. There were 100 questions on the test. Look at these scores. Tell how many questions each person got right and how many each got wrong.

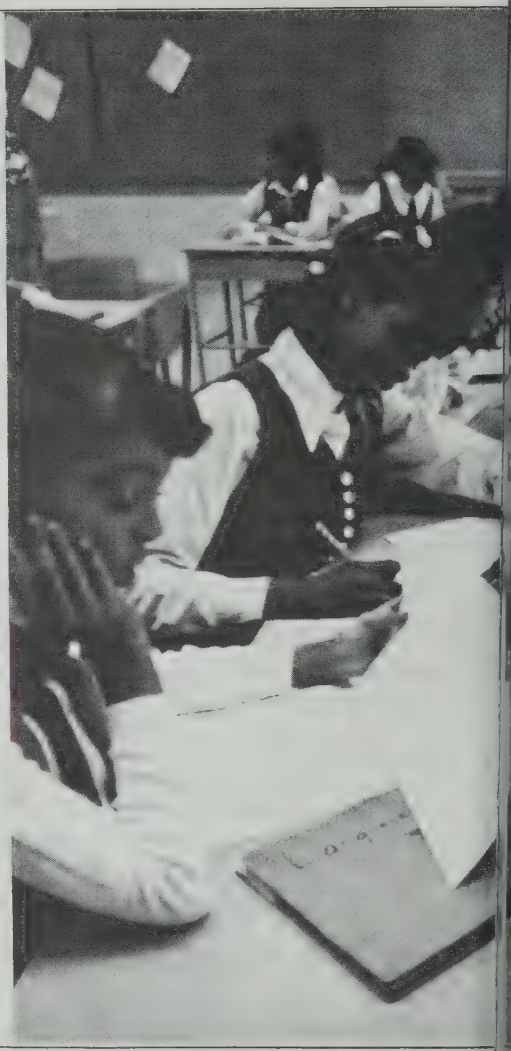
| | | R | W | | | R | W | | | R | W |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Allan | 78% | 78 | 22 | Betty | 95% | 95 | 5 | Carol | 77% | 77 | 23 |
| Darcey | 82% | 82 | 18 | Ellen | 88% | 88 | 12 | Frank | 90% | 90 | 10 |
| Gene | 98% | 98 | 2 | Hank | 80% | 80 | 20 | Irma | 100% | 100 | 0 |
| John | 92% | 92 | 8 | Karen | 85% | 85 | 15 | Lee | 99% | 99 | 1 |
| Mary | 75% | 75 | 25 | Nancy | 94% | 94 | 6 | Owen | 82% | 82 | 18 |

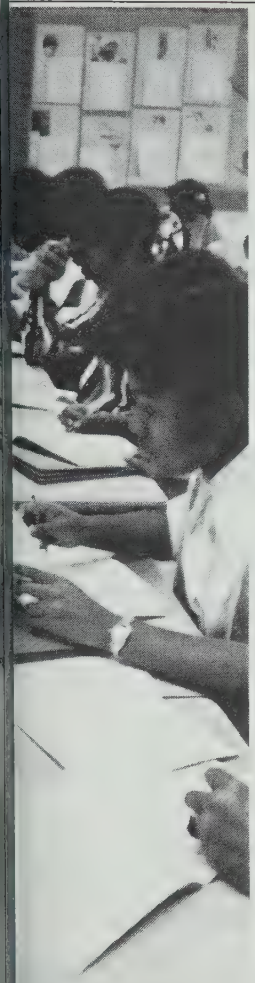
5. Percent is used in other ways. Think carefully. What information is missing?

   **a** Ron said he got 60% in his history test. How many questions did he get right? How many questions there were in all

   **b** Sally said she got a 10% increase in her salary. How much salary does she get now? What her salary was before the increase.

142





Percent is a good way to describe situations. But you can't compute with percent. You must rename the percent as a decimal. Then compute.

Sylvia said, "I got 30 words right in my test last week.
In this week's spelling test I got 20% more."

How many words did she get right this week?

**THINK** 30 last week. What's 20% more? Rename as a decimal and multiply.

$\times$ 0.20

6.00 That's 6 more words.
30 words plus 6 more.
She had 36 words right this week.

Elma said, "I only had 20 words right last week.
But I improved by 40% this week."

How many words did she get right this week?

**THINK** 20 last week. What's 40% more? Rename and multiply.

$\times$ 0.40

8.00 That's 8 more. You finish the problem.
20 + 8 = 28; She had 28 words right this week.

1. Sara got a 15% raise. She was earning $100 a week. How much will she earn after the raise? $115

2. Dave had 500 bricks for the wall. He needed 50% more. How many bricks did he need in all? 750

3. A year ago his mass was 70 kg . It has increased by 20%. What is his mass now? 84 kg

143

**goal** Applications involving percents

**page 143** Everything you need for a good discussion is on the page. Watch out for the decimal point in the product. You'll have to decide whether your pupils can compute problems 1 through 3 independently, or whether this should be a group project. Mastery is not expected at this level.

**lesson** Page 144

**goal** **Checkout**—operating with decimals

**page 144** There's lots of computation. You may wish to split the page and use part in each of two sessions.

Look first for computational errors, especially where renaming is involved. The pupils making these errors probably have trouble working with whole numbers also.

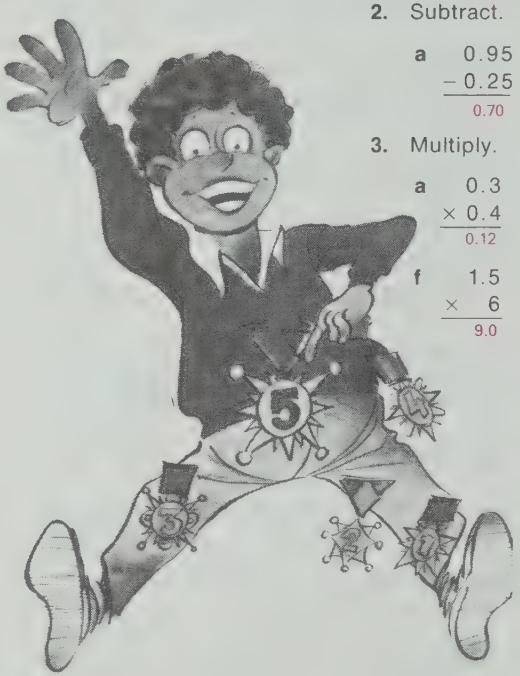
With problem sets 3 and 4, look for decimal-point placement problems. Use a place-value chart together with the following rules to help:

- ones × ones = ones
- ones × tenths = tenths
- ones × hundredths = hundredths
- ones × thousandths = thousandths
- tenths × tenths = hundredths
- tenths × hundredths = thousandths
- tenths × thousandths = ten-thousandths

It is best for the pupil to learn about the structure of place value rather than blindly learn to count the digits to the right of the decimal point. But we must accept that this rote system does work, and it may be the only way to let the person in real trouble enjoy any degree of success in computation.

It's been a long chapter. Everyone—including you—deserves a big pat on the back for fortitude and perseverance.

## CHECKOUT



1. Add.

| a | b | c | d | e |
|---|---|---|---|---|
| 0.2 + 0.7 = 0.9 | 0.26 + 0.56 = 0.82 | 0.75 + 0.25 = 1.00 | 1.2 + 0.8 = 2.0 | 0 + 0 = |

2. Subtract.

| a | b | c | d | e |
|---|---|---|---|---|
| 0.95 − 0.25 = 0.70 | 0.36 − 0.09 = 0.27 | 0.5 − 0.12 = 0.38 | 1.05 − 0.76 = 0.29 | 3 − 0 = |

3. Multiply.

| a | b | c | d | e |
|---|---|---|---|---|
| 0.3 × 0.4 = 0.12 | 0.2 × 0.4 = 0.08 | 0.04 × 5 = 0.20 | 0.03 × 3 = 0.09 | 0 × = |

| f | g | h | i | j |
|---|---|---|---|---|
| 1.5 × 6 = 9.0 | 2.8 × 0.2 = 0.56 | 0.23 × 0.7 = 0.161 | 0.23 × 0.3 = 0.069 | 0 × 0 = 0 |

4. Writing the correct digits does not mean you have the correct answer. Your answer must als have the correct place value. Multiply again. Th emphasis is on place value here.

| | |
|---|---|
| a 1 × 1 = ? 1 | b 0.1 × 0.1 = ? 0.01 |
| c 1 × 0.1 = ? 0.1 | d 0.1 × 0.01 = ? 0.001 |
| e 1 × 0.01 = ? 0.01 | f 0.1 × 0.001 = ? 0.000 |
| g 1 × 0.001 = ? 0.001 | Would you believe place value is important? |

5. Divide.

| a | b | c | d | e |
|---|---|---|---|---|
| $4\overline{)3.00}$ = 0.75 | $5\overline{)9.0}$ = 1.8 | $0.4\overline{)1.2}$ = 3.0 | $1.2\overline{)6.0}$ = 5. | $0.5\overline{)}$ |

144

See activity 9, page 144c.

See activity 10, page 144c.

# RESOURCES

## another form of evaluation

**for progress check**—page 106

Copy and order each set of decimals from smallest to largest.

1. 3.2, 5.1, 2.6, 4.2 2.6, 3.2, 4.2, 5.1
2. .6, .3, .8, .4, 1.1 .3, .4, .6, .8, 1.1
3. .376, .243, .359, .481 .243, .359, .376, .481
4. 2.601, 1.620, 1.602, 2.106 1.602, 1.620, 2.106, 2.601
5. .264, .35, 6.4, 5.78 .264, .35, 5.78, 6.4
6. 4.635, 3.45, 3.546, 4.356 3.45, 3.546, 4.356, 4.635

**for progress check**—page 115

Add.

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| 4.36 | 3.24 | 2.78 | 6.47 |
| +2.51 | +4.63 | +3.15 | +3.48 |
| 6.87 | 7.87 | 5.93 | 9.95 |
| **5.** | **6.** | **7.** | **8.** |
| 1.09 | 5.84 | 3.08 | 4.65 |
| +2.25 | +2.67 | +5.94 | +2.78 |
| 3.34 | 8.51 | 9.02 | 7.43 |
| **9.** | **10.** | | |
| 25.49 | 43.76 | | |
| + 7.63 | +28.98 | | |
| 33.12 | 72.74 | | |

Subtract.

| 11. | 12. | 13. | 14. |
|---|---|---|---|
| 6.78 | 8.94 | 5.84 | 6.72 |
| −2.53 | −6.81 | −2.58 | −4.36 |
| 4.25 | 2.13 | 3.26 | 2.36 |
| **15.** | **16.** | **17.** | **18.** |
| 3.50 | 8.53 | 9.46 | 12.38 |
| −1.34 | −4.94 | −5.78 | − 7.89 |
| 2.16 | 3.59 | 3.68 | 4.49 |

| 19. | 20. |
|---|---|
| 20.00 | 52.04 |
| − 6.59 | −37.86 |
| 13.41 | 14.18 |

*Compute. Watch the operation signs.

| 21. | 22. | 23. | 24. |
|---|---|---|---|
| 7.256 | 8.765 | 3.207 | 5.684 |
| +2.431 | −3.420 | +5.691 | +2.497 |
| 9.687 | 5.345 | 8.898 | 8.181 |
| **25.** | **26.** | **27.** | **28.** |
| 3.790 | 4.274 | 8.483 | 9.257 |
| +4.650 | +5.382 | −2.604 | +3.783 |
| 8.440 | 9.656 | 5.879 | 13.040 |
| **29.** | **30.** | | |
| 6.351 | 4.056 | | |
| −3.872 | −1.388 | | |
| 2.479 | 2.668 | | |

*31. Somebody made some errors. Find and correct them.

| | |
|---|---|
| 5.307 | 4.327 |
| +2.896 | −2.649 |
| ~~8.193~~ | ~~1.776~~ |
| 8.203 | 1.678 |

**for progress check**—page 126

Write each as a decimal.

1. $\frac{3}{10}$ .3
2. $\frac{3}{100}$ .03
3. $\frac{3}{1000}$ .003
4. $\frac{30}{10}$ 3.0 or 3
5. $\frac{30}{100}$ .30 or .3
6. $\frac{30}{1000}$ .030 or .03
7. Arrange these decimals in order from smallest to largest.
   3.40 3.04 .043 40.3 .345 5.04
   .043, .345, 3.04, 3.40, 5.04, 40.3

Round each decimal to the nearest tenth.

8. 15.24 15.2
9. 3.478 3.5
10. 5.639 5.6
11. 25.892 25.9
12. 13.055 13.1

Compute. Watch the operation signs.

| 13. | 14. | 15. | 16. |
|---|---|---|---|
| 2.5 | 17.5 | 9.6 | 4.562 |
| + .8 | − 9.3 | − .8 | +2.918 |
| 3.3 | 8.2 | 8.8 | 7.480 |
| **17.** | **18.** | **19.** | **20.** |
| .9 | .4 | 3.6 | 47.8 |
| −.43 | ×.3 | + .74 | × 1.3 |
| .47 | .12 | 4.34 | 62.14 |
| **21.** | **22.** | | |
| 25.6 | 4.7 | | |
| × .7 | × .1 | | |
| 17.92 | .47 | | |

**for progress check**—page 135

Compute the quotients. Check each quotient to see that your placement of the decimal point makes the answer reasonable.

| 1. | 2. | 3. |
|---|---|---|
| 7 | .9 | .13 |
| .6)4.2 | .4)3.6 | .5)0.65 |
| **4.** | **5.** | **6.** |
| 26 | 7 | 37 |
| .7)18.2 | 1.2)8.4 | 2.3)85.1 |
| **7.** | **8.** | **9.** |
| 18 | .04 | .07 |
| .31)5.58 | .46)0.0184 | .08)0.0056 |
| **10.** | **11.** | **12.** |
| 3.2 | 2.6 | .14 |
| 1.57)5.024 | 3.48)9.048 | .06)0.0084 |

**for checkout**—page 144

1.

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| .3 | .36 | .48 | 3.4 | .6 |
| +.4 | +.28 | +.57 | + .7 | +.7 |
| .7 | .64 | 1.05 | 4.1 | 1.3 |

2.

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| .76 | .64 | .8 | 3.08 | 5. |
| −.46 | −.28 | −.15 | − .19 | − .16 |
| .30 | .36 | .65 | 2.89 | 4.84 |

3.

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| .4 | .3 | .07 | .02 | .75 |
| ×.8 | ×.3 | × 4 | × 4 | × 7 |
| .32 | .09 | .28 | .08 | 5.25 |

| f) | g) | h) | i) | j) |
|---|---|---|---|---|
| 4.8 | 1.7 | .46 | .32 | .06 |
| × 6 | × .3 | × .8 | × .2 | × .7 |
| 28.8 | .51 | .368 | .064 | .042 |

4. Writing the correct digits is not enough. Your answer must also have the correct place value. Multiply again.
   a) $2 \times 1 = ?$ 2
   b) $.2 \times .1 = ?$ .02
   c) $2 \times .1 = ?$ .2
   d) $.2 \times .01 = ?$ .002
   e) $2 \times .01 = ?$ .02
   f) $.2 \times .001 = ?$ .0002
   g) $2 \times .001 = ?$ .002

5.

| a) | b) | c) | d) | e) |
|---|---|---|---|---|
| 1.5 | .75 | 3 | 6 | 145 |
| 4)6.0 | 8)6.00 | .5)1.5 | 1.5)90. | .4)580 |

# activities

**1. things** for each pair of pupils: 4 dice; 2 place-value charts (as shown on page 104); 5 markers each of 2 colors

Each player needs 2 dice, a place-value chart, and 5 markers of the same color. The players throw their dice at the same time and each player adds the dots that land faceup on his pair of dice. This sum is recorded in the thousandths place on the player's chart. If the number of dots is 10 or more, a zero is recorded. If both players have the same sum for this first round, they begin again.

For the second round, the number of dots is recorded in the hundredths place; the third round, the tenths place; and so on, proceeding from right to left on the chart. Play stops when both players throw the same sum or when the thousands position is filled—whichever happens first. Each player reads aloud the number shown on his chart. The player with the smaller number gives one of his markers to his opponent. Play is resumed, using a new row on the place-value chart. The game ends when one player wins all his opponent's markers.

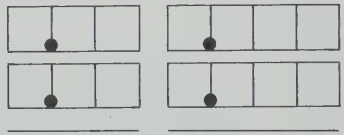
You may want to decrease the number of markers to 3 to speed up the game.

**2.** Complete a portion of a grid as shown. Challenge the youngsters to find the pattern and complete the grid. Note that the grid shown contains a vertical as well as a horizontal pattern. These two patterns need not be the same.

| | | | |
|---|---|---|---|
| .8 | .9 | 1.0 | |
| .6 | .7 | .8 | |
| .4 | .5 | | |
| | | | |

**3. things** spirit master of frames shown; small cards

Prepare at least 2 cards for each digit 0 through 9. The cards are shuffled and placed facedown in a stack. Each player will need a page of frames as shown.

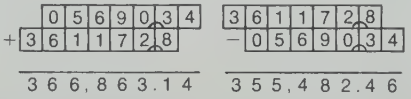


To begin a round, the players determine which kind of frame to use and whether to add or to subtract. The appropriate operation sign is then recorded. The top card is turned faceup and displayed for all players to see. Each player is free to record the digit in any position of the frame that he chooses. This procedure continues until the appropriate number of digits has been turned faceup. Then the operation is performed. When addition is chosen, the player with the largest sum is the winner. When subtraction is chosen, the player with the smallest difference wins. Shuffle the cards before beginning a new round.

**4. things** oaktag strips; paper clips

Mark off the strips to form squares, and randomly write one digit in each square. Take 2 strips and indicate a decimal point on each with a paper clip. Place the decimal to show no more than 3 places to the right.

Hand 2 strips to a pupil and ask him to add or to subtract the two numbers. For example:




Shift the decimal point by moving the paper clip for additional challenges.

**5. things** spirit master of place-value chart

Provide the pupil with a place-value chart on which some positions have been crossed out, and a list of numbers. He is to round each number so that it can be written in the blank spaces of the chart.

1. 17.08 (17.1)
2. 6.345 (6)
3. 25.427 (25.43)

| | thousands | hundreds | tens | ones | tenths | hundredths | thousandths |
|---|---|---|---|---|---|---|---|
| 1. | | | | | | X | X |
| 2. | | | | | X | X | X |
| 3. | | | | | | | X |

**6.** Your job—find the missing numbers.

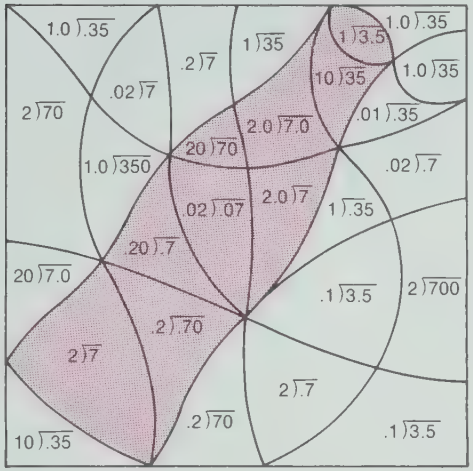
**a** 0.08, (0.10), (0.12), 0.14, 0.16, (0.18), 0.20
**b** 0.06, (0.12), 0.18, 0.24, (0.30), 0.36, 0.42
**c** 0.52, 0.48, 0.44, (0.40), (0.36), 0.32, (0.28)
**d** 0.12, 0.14, 0.18, (0.26), 0.42, (0.74), (1.38)
**e** 0.2, 0.4, 0.8, (1.6), 3.2, (6.4), (12.8)
**f** 7.29, 2.43, (0.81), 0.27, 0.09, (0.03), (0.01)
**g** 0.2, 0.4, 0.8, (1.4), 2.2, (3.2), (4.4)

Now tell how you found those numbers.

**a** (Add 0.02)
**b** (Add 0.06)
**c** (Subtract 0.04)
**d** (Subtract, multiply the difference by 2, add)
**e** (Multiply the preceding number by 2)
**f** (Divide by 3)
**g** (Subtract, add 0.2 to the difference)

**7. things** spirit master as shown

Examine the problems carefully. You need not compute every one. Your goal—to shade only the regions where the quotient is 3.5.




**8.** All distances for the Olympic games and international track competition are measured in metric units. The individual running events in the Olympic games include races of the following distances: 100 metres; 200 metres; 400 metres; 800 metres; 1500 metres; 5000 metres; and 10,000 metres.

1. How many lengths of 100 m in 200 m? (2) in 400 m? (4) in 800 m? (8) in 1500 m? (15) in 5000 m? (50) in 10,000 m? (100)
2. The winning time for the men's 100-m dash in the 1972 Olympic games held in Munich, Germany, was 10.1 seconds (won by Valery Borzov of Russia). If all races were run at this rate of speed, what would have been the winning times for the other running events? (Answers follow.)
3. For each running event other than the 100-m dash estimate a reasonable winning time (in seconds).
4. What events usually run in United States meets are comparable to the Olympic running events?
5. Compare your results for step 3 with the actual winning times. How close were your guesses? (See answers below.)

Answers:

2. 200 m – 20.2 sec.
   400 m – 40.4 sec.
   800 m – 80.8 sec. (1 min. 20.8 sec.)
   1500 m – 151.2 sec. (2 min. 31.2 sec.)
   5000 m – 505 sec. (8 min. 25 sec.)
   10,000 m – 1010 sec. (16 min. 50 sec.)

4. 100 m – 100 yd.
   200 m – 220 yd.
   400 m – 440 yd. (quarter mile)
   800 m – 880 yd. (half-mile)
   1500 m – mile run
   5000 m and
   10,000 m – none comparable in United States track events

5.

| *Event* | *Winning Time* | *Winners* |
|---|---|---|
| 200 m | 20.0 sec. | Valery Borzov, |
| 400 m | 44.7 sec. | Vince Matthews |
| 800 m | 105.7 sec. (1:45.7) | Dave Wottle |
| 1500 m | 216.3 sec. (3:36.3) | Pekka Vasala |
| 5000 m | 806.4 sec. (13:26.4) | Lasse Viren |
| 10,000 m | 1658.4 sec. (27:38.4) | Lasse Viren |

It is interesting that the time for 200 m is less than twice the time for 100 m. This is an unusual situation, although the Olympic record for 200 m (19.8 sec.) is exactly twice the record for 100 m (9.9 sec.).

**9.** Engineers use a slide rule to multiply decimals—BUT a slide rule usually gives a rounded product, not all the digits you get by computing with paper and pencil or a calculator. For example, the slide-rule reading for $2 \times 30$ would be 6. Now you know 6 is not the product! One zero must be annexed to the right of the 6 to write the correct product 60. Imagine you are an engineer. The problems you have to solve are in the first column. The slide-rule reading is given in the second column. Your job—write the correct product in the third column.

| *Problem* | *Slide-Rule Reading* | *Product* |
|---|---|---|
| $25 \times 4$ | 1 | (100) |
| $4 \times 20$ | 8 | (80) |
| $20.4 \times 410$ | 836 | (8360) |
| $40 \times 2.5$ | 1 | (100) |
| $2.4 \times 25$ | 6 | (60) |
| $400 \times 1.5$ | 6 | (600) |
| $45 \times 6$ | 27 | (270) |
| $250 \times 0.6$ | 15 | (150) |

**10.** Provide the pupil with these directions: Make a 4-by-4 array of squares. Now arrange the numbers 0, .1, .2, .3, .4, .5, .6, .7, .8, .9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5 so that the sum of each row, each column, and the two diagonals is 3. **And** to make the puzzle even more interesting, you must arrange the decimals so that the sum of the four corners is also 3.

Answer:

| | | | |
|---|---|---|---|
| 0 | 1.3 | 1.0 | .7 |
| 1.1 | .6 | .1 | 1.2 |
| .5 | .8 | 1.5 | .2 |
| 1.4 | .3 | .4 | .9 |

## additional learning aids

**concept**—chapter objectives 3, 8

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: F-20, 21
*Computapes*, SRA (1972)
Module 6, Lesson: DP 3

**notation**—chapter objectives 1, 2

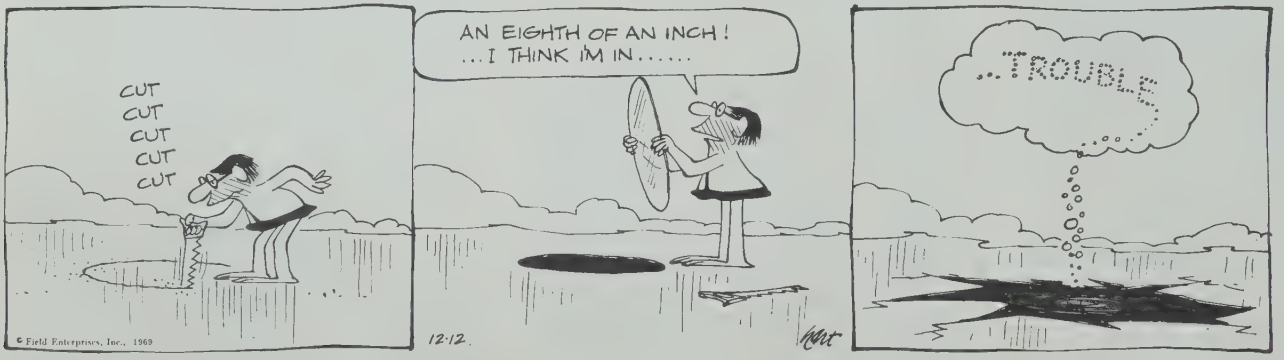
**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit master: F-22
*Computapes*, SRA (1972)
Module 6, Lesson: DP 11

**operation**—chapter objectives 4, 5, 6, 7

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: F-13, 19, 24, 25, 26, 27
*Computapes*, SRA (1972)
Module 6, Lessons: DP 4–10
*Computational Skills Development Kit*, SRA (1965)
Addition with Decimals cards: 3–5
Subtraction with Decimals cards: 1–5
Multiplication with Decimals cards: 1, 2
Division with Decimals card: 4
*Cross-Number Puzzles* (*Decimals and Percent*), SRA (1966)
Addition cards: 1–3
Subtraction cards: 1–3
Multiplication cards: 1–5
Division cards: 1–4, 7–10
*Cross-Number Puzzles* (*Story Problems*), SRA (1966)
Decimal cards: 1–5
*Math Applications Kit*, SRA (1971)
Appeteasers card: 10
Sports and Games cards: 10, 34
Everyday Things cards: 20, 39, 45
*Mathematics Involvement Program*, SRA (1971)
Cards: 316, 157, 327, 337
*Skill Modes in Mathematics*, SRA (1974)
Level III, Molecules: S, T, W, X
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 41, 53, 65, 66, 67, 68, 69, 70
*Visual Approach to Mathematics, Rational Numbers*, SRA (1967)
Visuals: 15, 16

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MILLIONS
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hundred
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ten
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hundreds
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BILLIONS
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MILLIONS
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ONES
hundred
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ten
one
hundreds
tens
ones
tenths
hundredths
thousandths
ten-thousandths
hundred-thousandths

# People and jobs

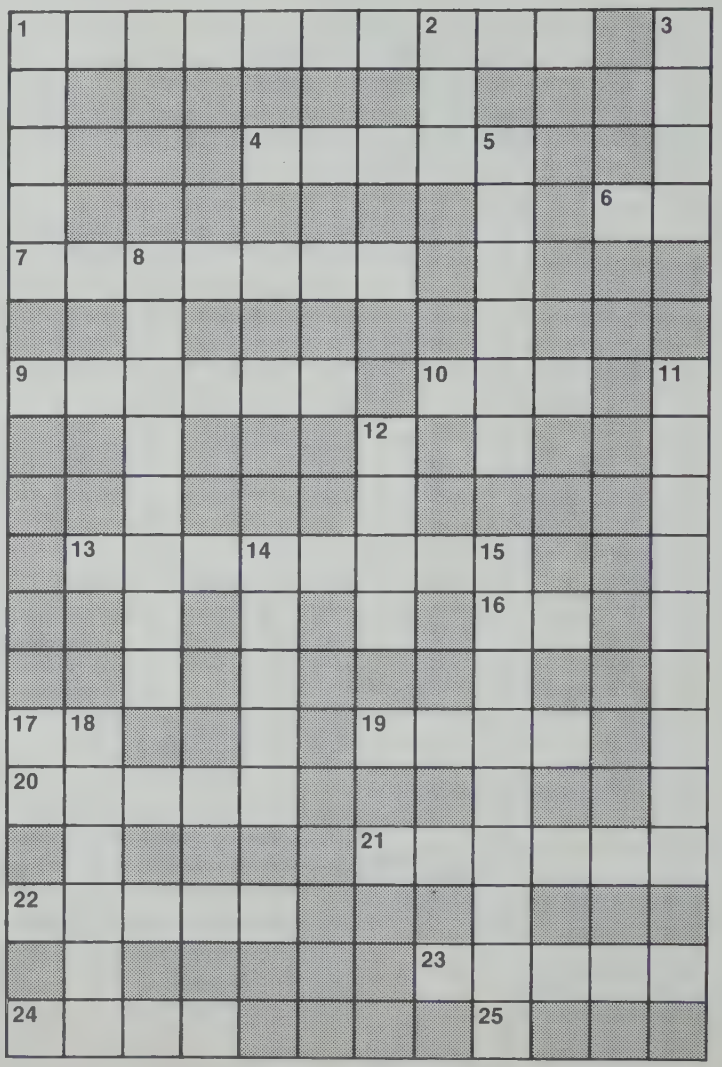
**What unit of measure might be used?**

## ACROSS

1 European shirtmaker—how wide is the button
4 butcher—how much hamburger for supper
6 cook—how much cream to whip
7 race driver—how much time to refuel
9 milkman—how much milk to deliver
10 biologist—how long does an insect live
13 French grocer—how much coffee in the large can
16 cabinetmaker—how wide is the drawer (abbreviation)
17 photographer—what size film (symbol)
19 long-distancer runner—how long is the race
20 German service station worker—how much gas
21 nurse—how much temperature the baby has
22 mechanic—how much oil goes in the car
23 tailor—how much fabric to buy
24 laundryman—how often to stop at the Smith's
25 swimmer—how long is the Olympic pool (symbol)

## DOWN

1 mailman—how much postage is due on the letter
2 miner—how much coal taken out of the mine
3 furniture salesman—how long is the sofa
5 banker—how much money
8 dietitian—how many to lose weight
11 salesman—how far from Rome to Venice
12 farmer—how much land in one field
14 jewelry maker—how much gold to buy
15 doctor—how much medicine to give
17 pharmacist—how much liquid to use
18 anybody—how long you're told to wait





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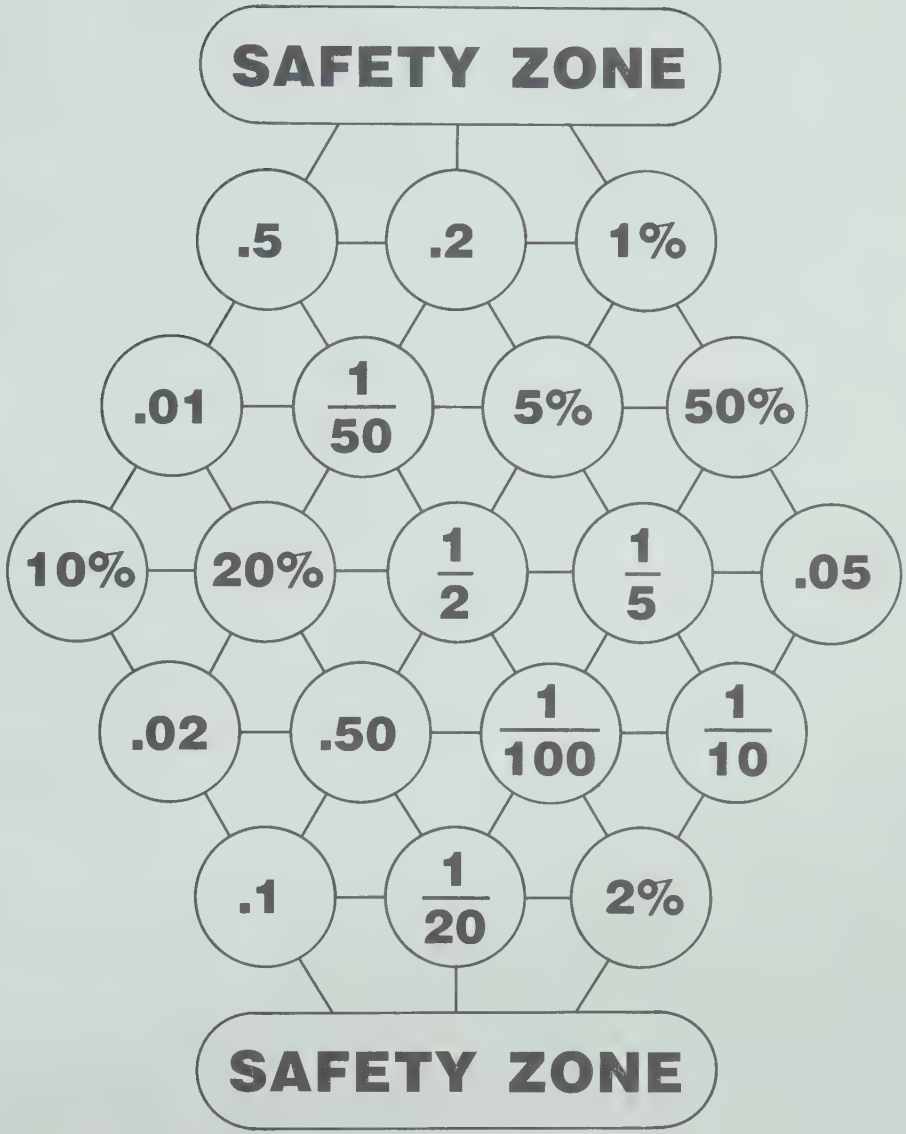
the
## Notation Network
game

Pick a partner. Each of you gets 3 tokens and a playing board. Put your own tokens in one of your own safety zones. Take turns. Try to get your tokens across the board to the opposite safety zone. You can move forward or sideward along the lines but never backward! Move one space at a time. You can't move into a circle that already has a token in it, but the safety zone can have more than one token in it.

When it is your turn to play, you must move one token. You can capture your partner's token if you move to a circle whose number is equivalent to the number your partner's token is on. If your partner has two tokens sitting on numbers that are equivalent, you can capture both of them!

The game is over when you both have reached the opposite safety zone with all your uncaptured tokens. The winner is the one with the most tokens in a safety zone.

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Here's another idea for the Super Sleuth Box.

## IXOHOXI

Here is a very famous and unusual magic square. It even has its own name. The name is pronounced Ixo-hocksie. You can look at the name in a mirror and the name is the same. You can turn the name upside down and it is still the same. No matter how you look at it or how you hold it, it is always the same. Now look at the puzzle.

| | | | |
|---|---|---|---|
| 8818 | 1111 | 8188 | 1881 |
| 8181 | 1888 | 8811 | 1118 |
| 1811 | 8118 | 1181 | 8888 |
| 1188 | 8881 | 1818 | 8111 |

Add the rows. What is the sum?
Add the columns. What is the sum?
Add the diagonals. What is the sum?
Add the numbers in each of the four corners. What is the sum?

You will find many more combinations of four cells within the puzzle that also add to the same sum. How many more can you find?

Detach this BEFORE you duplicate the magic square.

There are 25 different combinations of numbers that have the same sum when the magic square is in this position. There are 25 more if the square is turned upside down.

Letters have been placed in the square to replace the numbers so you can see just some of the answers.

| | | | |
|---|---|---|---|
| A | B | C | D |
| E | F | G | H |
| I | J | K | L |
| M | N | O | P |

EIHL CHNI

BCNO ABEF

CDOP BCFG

EBLO CDGH

# 6 AREA AND PERIMETER

**before this chapter, the learner has –**

1. Estimated and found the sum of any two 4-digit numbers
2. Estimated and found the product of any 2- and 3-digit factors
3. Found the distance around a figure on graph paper by counting
4. Found the surface area of a figure on graph paper by counting

**in chapter 6, the learner is –**

1. Constructing different regions having a given area or given perimeter
2. Finding the perimeter of a rectangle by counting or by computation
3. Finding the area of a rectangle by computation
4. Diagnosing and reinforcing addition and multiplication skills

**in later chapters, the learner will –**

1. Estimate the area of a circular region on graph paper
2. Find the area and perimeter of polygons by computation

# Notes & Things

This chapter will explore the concept of area and perimeter. It is full of activities and very real problem-solving situations. There are some challenges for good thinkers and many opportunities to share ideas. For example, fashioning a rectangle out of a folding ruler clearly reveals the perimeter. Laying that rectangular shape on inch-squared paper will dramatically show the rectangle's area. No problem. If the folding ruler is "squished" to assume the shape of a parallelogram, the same perimeter is still visible. But most children will stare in wonder as the shape is laid down on inch-squared paper, and they find that some of the area has disappeared. That must be magic.

There will be very few new words, no emphasis on square measure, none of the expected computation. The purpose will be to find that there is no relationship between the area and the perimeter of any common polygon. The pupils will find perimeters and areas, but they will be free to compute the measurements in any way they can.

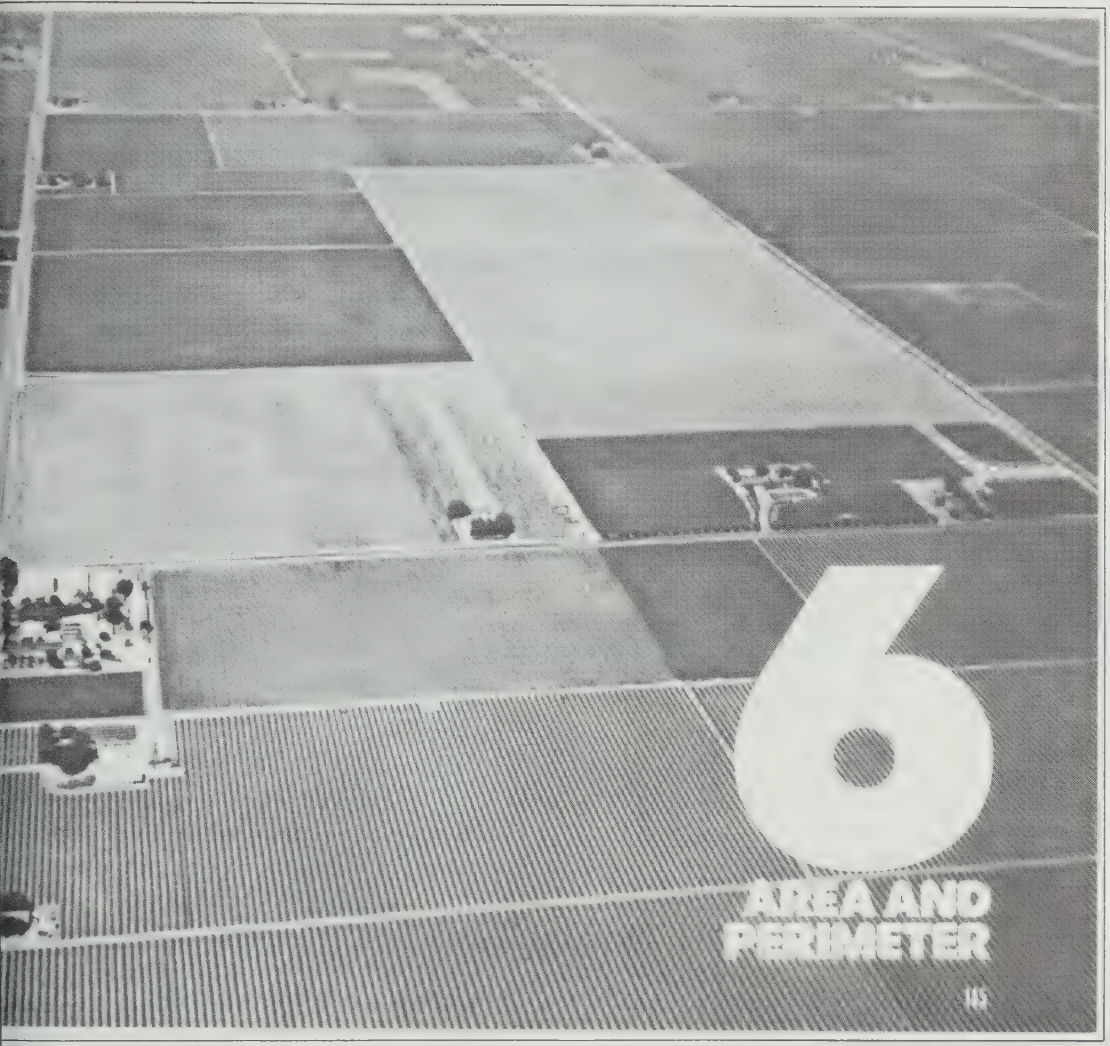
A maintenance program of related addition and multiplication skills is also a feature of the chapter. These are the skills that must be mastered before any sophisticated computational work with perimeter and area can be done.

You will all enjoy this chapter!

## things

graph paper
scissors
empty boxes
transparent tape
wood cubes (optional)

For the extra activities you will want to have available newspaper real estate ads.




**goal** Think about and explore ideas through a picture clue

**page 145** The concept of **area** seems so simple. Anyone can look at a 10 cm square of paper and accept the idea that the surface covered can be called area. But area is used in much more informal situations also.

- Her area of study is agronomy.
- I can't remember my area code.
- He has travelled in that area.
- This area is not polluted.

Take time to discuss these uses of the word **area**. This discussion can be related to the picture because each of the above statements could be made by a family living in one of the houses in the aerial photograph. Then you can also discuss the area of the land.

The fields are probably rectangular shaped but they certainly don't look like it. Why not? What area of land do the pupils think is pictured?

If your pupil's experience is limited, use the following approach. *How many buildings are there usually on a city block?* Find a building in the picture. Take time to think. *Is more than 1 city block shown? Are more than 10? more than 15? Are 10 blocks more than 1 km? Is there more than 1 $km^2$ in the photograph? Are there more than 2 $km^2$?*

To make the concepts of length and a square unit real for the children, it would be worth while to take time for an outside project of walking a kilometre, then determine what area in the vicinity of the school is a square kilometre. Then everyone should be ready to go with this chapter.

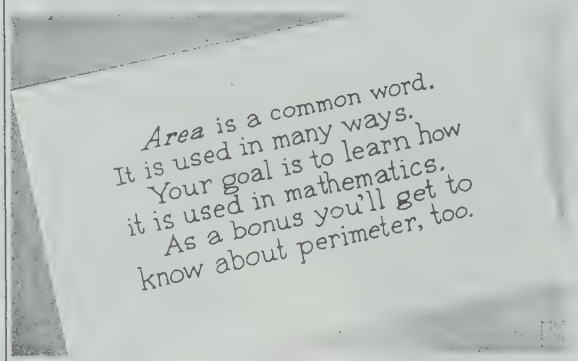
**lesson** Pages 146, 147, 148

**goal** Exploring the concepts of area and perimeter

**things** graph paper

**page 146** **Explore** is the key word for this chapter. Get the notion of wide open problem solving started right on this first page. Keep challenging until someone comes up with the rectangular dimensions $1 \times 24$, $2 \times 12$, $3 \times 8$, and $4 \times 6$. You may get an interesting argument as to whether a $2 \times 12$ community is the same as a $12 \times 2$ one. No doubt about it, the dimensions are the same. But if one edge of a real community were on a riverbank or next to a railroad track, it might make a big difference. The sky's the limit once your pupils know the community need not be rectangular.

The idea of the distance around a shape was explored only slightly in chapter 3 in connection with congruence. For the most part, this will be the pupil's first real encounter with the concept.

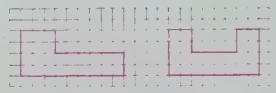

*Area* is a common word.
It is used in many ways.
Your goal is to learn how
it is used in mathematics.
As a bonus you'll get to
know about perimeter, too.

Bill explained that his community was 24 square blocks. Was this a complete description of the size of his community? No

Assume that it had the shape of a rectangle. Use graph paper and show the different rectangular shapes it might have. (There can be more than one.) 4 x 6, 3 x 8

Suppose that you are going to walk the outside boundaries of each of the shapes. How many blocks would you walk on the first rectangular shape? How many blocks o the second? How many blocks on each of the other rectangular shapes that you have drawn? 20 blocks around 4 x 6; 22 blocks ar 28 blocks around 2 x 12; 50 blocks (Answe if o

Bill didn't say that his community was shaped like a rectangle, did he? No
What are the other shapes it could have? See
It must have 24 square blocks, remember.
(There are a lot more shapes than one now.)



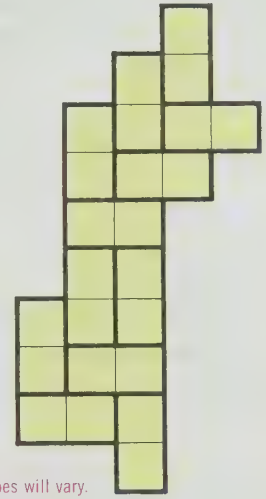
and many others.

146

1. Did you remember something about factors and multiples when you tackled the problem on other shapes for Bill's community?

2. If you didn't, take another look at your work. Did you put Bill's community together with square shapes that are multiples of 2? multiples of 3? of 4? of 6? of 8? of 12? You probably used multiples of at least two numbers.

3. For example, one shape made up of two square-block units might look like this:



4. Are there other shapes that contain 24 square blocks? Yes
Keep on building, using multiples of two square blocks. Shapes will vary.

5. Walk the outside boundaries of this shape. How many blocks did you walk? How many square blocks are there inside the boundary? 30 24

6. Now it's your turn. Prove that you can build another shape made up of two square-block units. Remember—you need a total of 24 square blocks. Shapes will vary.

7. Walk around each of the shapes you have constructed. How far is it around the outside boundary of each shape? Answers depend on shapes drawn for problem 6.

The amount of surface is called *area*.
The distance around the boundary is called *perimeter*.

147

**goal** Finding that two shapes can have the same AREA but different PERIMETERS

**things** graph paper

**page 147** Pupils are not limited to two square-block units in problem 4. Encourage them to experiment. What other type units are possible? (Any factor of 24—3, 4, 6, 8, 12) They will have to remember that the total size of the community remains 24 square units.

Your community designers will naturally want to show their work to the public. One thing is known for sure—the **area** of each shape is the same (unless someone didn't count correctly). But, what about the **perimeters**?

**goal** Finding that two shapes can have the same perimeter but different areas

**things** graph paper
scissors

**page 148** Graph paper and scissors for everyone—this is an activity page. The focus is on keeping the **perimeter** the same, then checking to find out what happened to the area. The pieces cut away need not be identical to those shown on the page. Encourage experimentation before making a generalization.

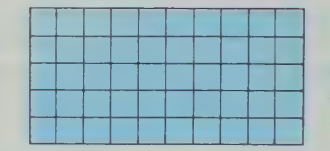
Let the last problem on the page be the real challenge. The results of some determined efforts could be surprising to everyone.

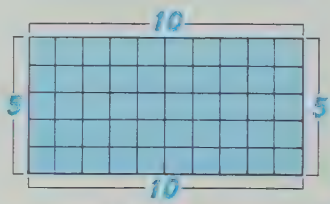
1. It appears that two shapes can have the same area but different perimeters. Can you have two shapes with the same perimeter but different areas? Yes

2. You'll need graph paper and scissors for this one. Start with a rectangular piece of paper 5 squares wide and 10 squares long.

   **a** What is the number of squares contained in this shape?
   5 rows of 10, or 50

   **b** What is the number of squares around?
   10 + 5 + 10 + 5, or 30

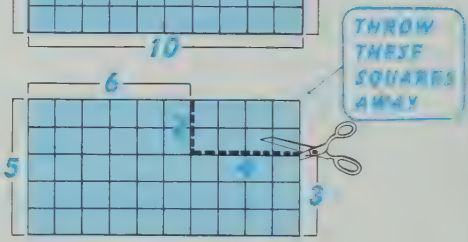



3. Now cut away part of the rectangle.

   **a** Now how many squares for the area? 42

   **b** What is the number of squares around? 30

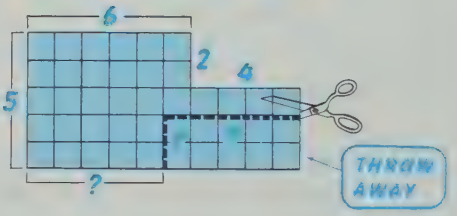



4. Cut some more squares away.

   **a** Now how many squares for the area? 32

   **b** What is the perimeter? 30

5. How small can you make the area and keep the perimeter at 30 units?
   14 is the best answer




148

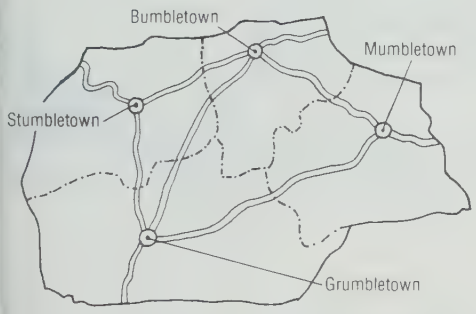
**things** graph paper

Individual activity. (Provide the pupil with graph paper and the following directions.)
1. Lay your hand on the graph paper and spread your fingers apart.
2. Trace your hand.
3. What is the area of your hand? Estimate as best you can.
4. Lay your hand on another piece of graph paper. This time keep your fingers together.
5. Trace your hand again.
6. What is the area of your hand this time?
7. Are your answers to step 3 and step 6 the same?

The idea of a perimeter is used in many situations. You don't have to build a fence before you have a perimeter. You don't have to walk around some city blocks. You might plan a trip and have the path of that trip be "the distance around."

Mr. Trumble lives in Bumbletown. He wants to visit Mumbletown, Grumbletown, and Stumbletown on his next trip. His time is short. He does not want to travel more than 1000 km round trip. He may not get to all the cities he wants if he holds to this rule.

Mr. Trumble checks his road map to find the distances.




| | Bumbletown | Mumbletown | Grumbletown | Stumbletown |
|---|---|---|---|---|
| **Bumbletown** | 0 | 306 | 459 | 263 |
| **Mumbletown** | 306 | 0 | 569 | 494 |
| **Grumbletown** | 459 | 569 | 0 | 251 |
| **Stumbletown** | 263 | 494 | 251 | 0 |

Trace the map. Plan Mr. Trumble's trip for him. How many cities can he visit? Does he have a choice? 2 No
Which cities can he visit? What path should he take? See below
How many kilometres does he travel in all? 973 km
How many kilometres less than 1000 is his journey? 27 km
Bumbletown → Stumbletown → Grumbletown or vice versa = 973 km



**things** graph paper

Just about everyone has more than one way he can walk or ride to school. Distribute graph paper and have each person make a map showing his usual route to school and then an unusual, but possible, route home.

If all your pupils live near the school, each square on the paper can represent a city block. If some children come by car or bus, they will have to decide what they want a square to represent and write that as a key at the bottom of the page.

**lesson** Page 149

**goal** Applying the concept of perimeter

**page 149** This looks like an easy activity, but it isn't. First of all, some pupils may have no experience in reading this type of table. If this is true, you'll want to make a sketch of the table on the board, leaving all the numbers out. Then, by using arrows, demonstrate how such a table is read. Confirm understanding, using the table on the page. How far from Bumbletown to Mumbletown? How far from Bumbletown to Stumbletown? How far from Stumbletown to Grumbletown? How far from Bumbletown to Bumbletown? Let everyone confirm that the table really does say 0 km.

Each person can plan Mr. Trumble's trip independently. Encourage answers such as, "He decided to go to Grumbletown only, and then drive home." It's a decision Mr. Trumble could make. He could decide to stay home too, but that really is not playing the game. Your travel agents will want to share their possible trips.

**lesson** Page 150

**goal** Diagnosis and reinforcement of addition skills

**page 150** Talk about problem 1 once the answers have been computed. How was the shortest route determined? Did anyone use estimation? What can the dirt roads and the fence around the farm be called? (Perimeter)

Addition is a prerequisite skill for computing perimeter. Use sets 1 and 2 to help you identify pupils who are slipping in their addition skills and need some type of regular additional practice for maintenance. When the problems are completed, consider having pupils exchange papers to correct. Before papers are returned to the owners, have the checker try to find out the source of the error, if there was one, and then discuss his findings with the owner of the paper.

**1.** George Greenbean owns a large farm. One day a farmhand tells George that his fence in the northwest corner (point $B$) needs fixing. George and the farmhand drive from the farmhouse to the broken fence and fix it. They drive on the dirt roads by the fence line. They take the shortest route.

2343 → 1725 → 78 → 3125 (north, then west)

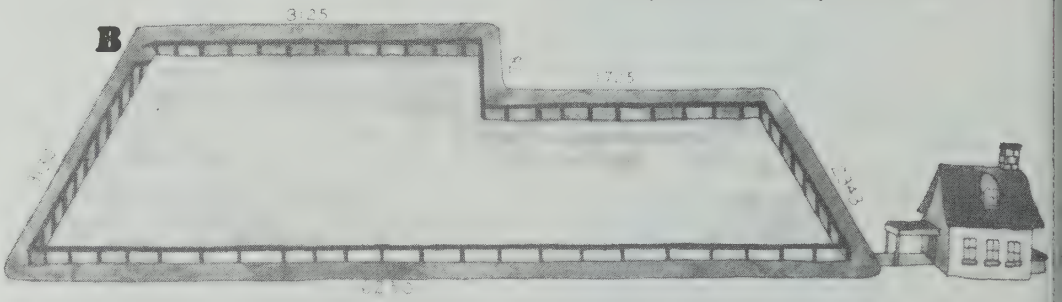
**a** Which route appears to be shorter?

**b** How many metres did they drive to point $B$? 7271

**c** How many metres shorter is this path than the other path to point $B$? 2109

**d** If they drive back the way they came, how many metres will they drive in all? 145

**e** If they drive back the other way, how many metres will they drive in all? 16 651

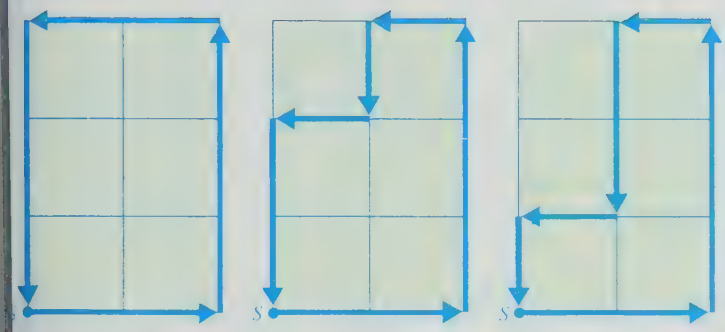



**2.** Finding the answers to the questions above meant that you had to know how to add. Check your addition skills with these problems. Work for accuracy and speed.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| SET 1 | 2746<br>+3251<br>5997 | 3104<br>+5087<br>8191 | 5658<br>+3274<br>8932 | 2546<br>+7454<br>10 000 | 2556<br>+1765<br>4321 |
| SET 2 | 121<br>32<br>+ 43<br>196 | 256<br>310<br>+ 25<br>591 | 62<br>158<br>+5213<br>5433 | 225<br>132<br>371<br>+ 50<br>778 | 267<br>113<br>1506<br>+ 251<br>2137 |

150

Tom, Dick, and Harry decided to have a footrace around six square blocks. They started at point *S*. Tom got ahead right away. Dick wanted to win, so he decided to cheat. Poor Harry was way behind. He saw Dick cheat, so he decided to cheat more. Tom still won.




Tom's path Dick's path Harry's path

Each of the paths is different. Each path is a perimeter. Each path goes around a different number of squares. A polygon is formed by each perimeter. The amount of surface inside the polygon is an area.

. Each side of each square in the picture represents 100 metres.

**a** How far did Tom run? 1000 metres

**b** How far did Dick run? 1000 metres

**c** How far did Harry run? 1000 metres

**d** Did Tom, Dick, or Harry run around the greatest number of 100-metre squares?
Tom ran around the greatest number of squares, even though they each ran the same distance.

151

Extend the activity on the page. Try a 3-by-3-block race. How many different paths could be taken now? Let the ambitious try a 4-by-4-block, 4-by-5-block, or 5-by-5-block race as a special project.

**lesson** Page 151

**goal** Relating perimeter to the sides of a polygon; relating the surface inside a polygon to area

**page 151** Have fun with this page. The purpose of the race is to reinforce the concept that shapes can have the same perimeter—in this case the path walked by each boy, but different areas—the number of blocks around which each boy walked.

Someone may have forgotten what a polygon is—better check. (Closed figure having straight sides)

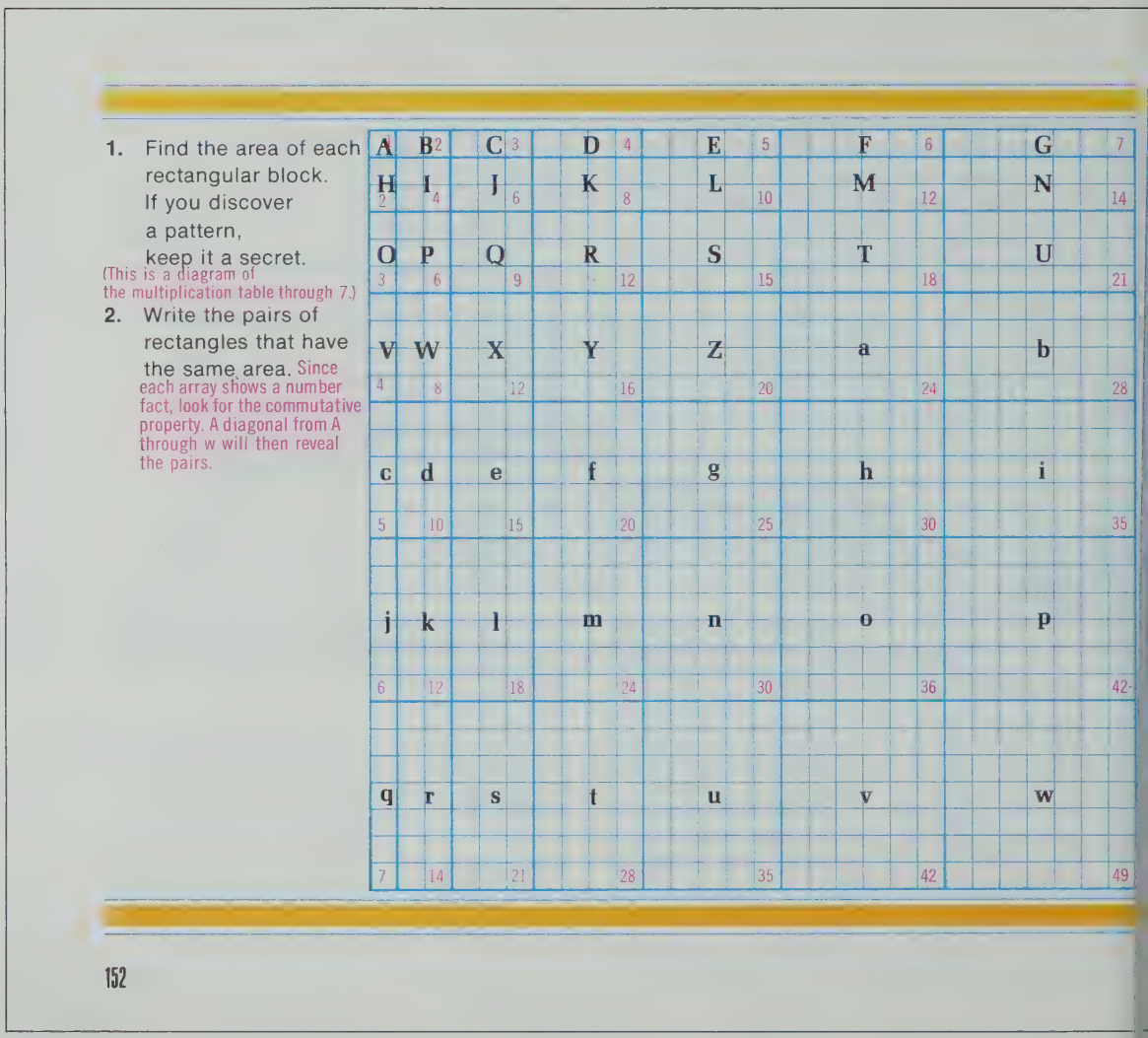
**lesson** Pages 152, 153

**goal** Finding the area of rectangles

**memo** Pages 152 and 153 are related and should be used together.

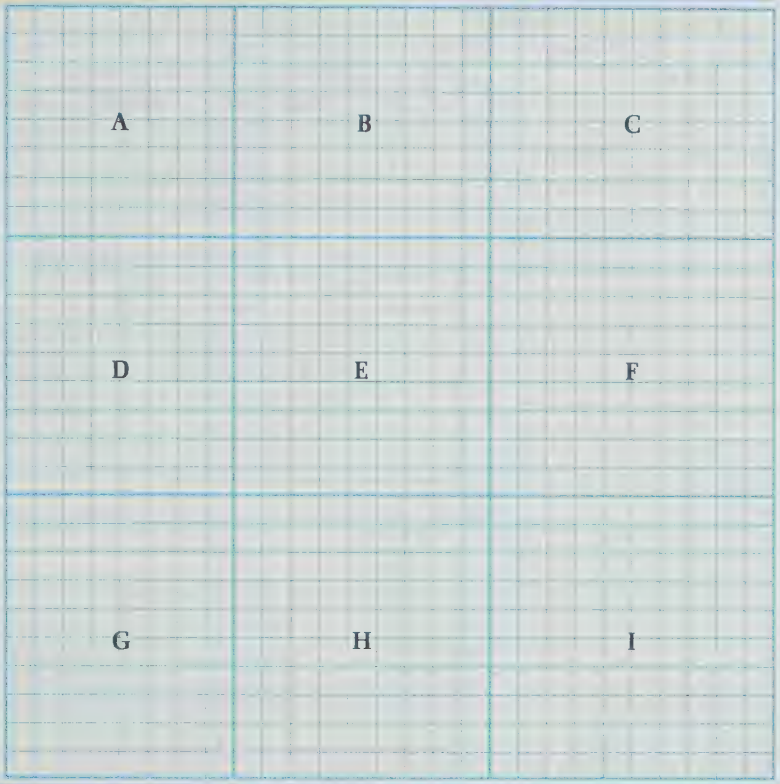
**page 152** The joy of this page is in the disguise. Some of your sharp pupils will discover that the rectangular blocks are a graphic representation of the good old-fashioned multiplication table, showing the facts through 7. These pupils will complete the page in a flash. Others may find the area by counting. Either method – multiplication facts or counting – is O.K. Please don't teach the formula for finding area now. This is a time to explore.

When everyone is finished with the page, explore the notion of commutativity by examining rectangles that are congruent. The pupils will find that matching rectangles have been turned.



. Look at the nine rectangular regions on this page. Are any the same size? Yes
Which ones?
B and D, C and G, F and H
(This shows 8-, 9-, and 10-facts.)

. What is the area of the total set of nine rectangles?
729 square units




153

**goal** Finding rectangles that have the same area

**page 153** These rectangles complete the remaining multiplication facts and include $10 \times 10$.

After the pupils complete the page, ask which rectangles do not have a match. Anything special about these? (Square) Go back to page 152. Are there any square rectangles? (Yes) Where do they fall? (On the diagonal)

**lesson** Page 154

**goal** Computing the area and the perimeter for a rectangle

**page 154** Note that the pupil is directed to select a unit of measure for each figure. Encourage freedom of choice. Two what? 2 feet by 2 feet? 2 metres by 2 metres? 2 miles by 2 miles? 2 centimetres by 2 centimetres?

Emphasize that if centimetres are selected, for example, the perimeter is computed in centimetres, while the area is computed in square centimetres. When correcting the work, discuss what is the same about everyone's answer, what is different. (The number of units will be the same, the unit names may be different.)

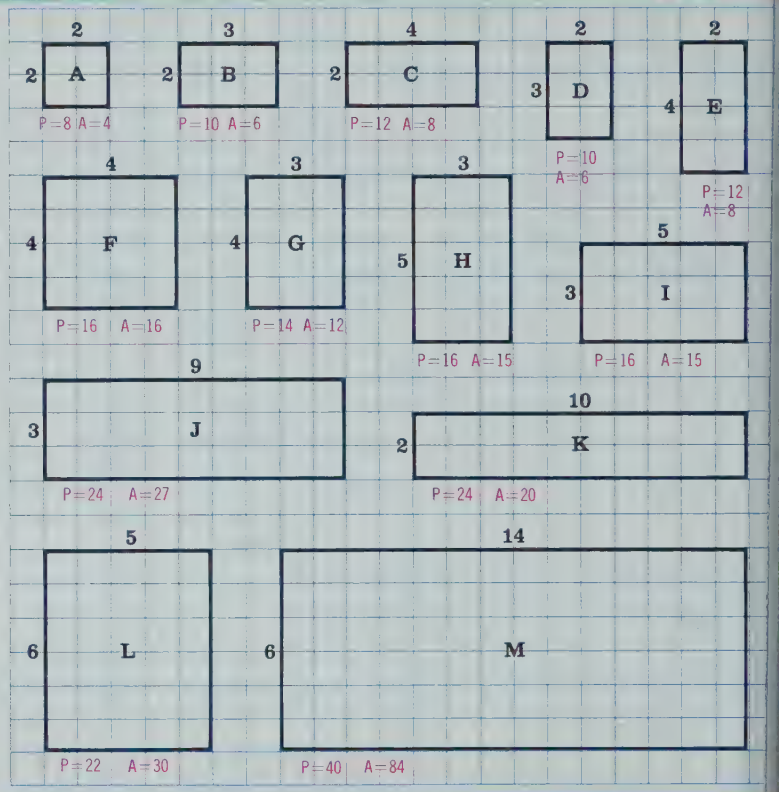
Did anyone discover a better way to find the area than by counting? Now is the time to move toward the formula for area.

area = length × width

To find the perimeter of any figure, you add the lengths of the sides.

Area is always in square units of measure. To find the area, you can count the number of squares a figure covers. OR maybe you have found a better way to find the area than counting.

Find the perimeter and the area of each figure. You name the unit of measure.




154

1. You must know how to multiply if you compute area. How are your multiplication skills? Do these problems. Work for accuracy and speed.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| SET 1 | 57 × 8 = 456 | 79 × 6 = 474 | 86 × 7 = 602 | 78 × 9 = 702 | 96 × 4 = 384 |
| SET 2 | 40 × 75 = 3000 | 19 × 18 = 342 | 36 × 36 = 1296 | 39 × 95 = 3705 | 62 × 58 = 3596 |
| SET 3 | 647 × 481 = 311,207 | 518 × 926 = 479,668 | 3967 × 207 = 821,169 | 2740 × 509 = 1,394,660 | 3958 × 471 = 1,864,218 |

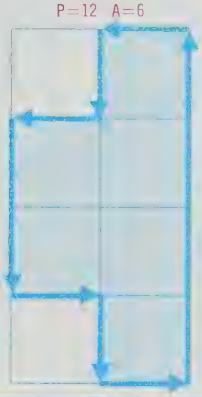
2. Mr. Jones bought 60 feet of fence. Use graph paper and show at least five different rectangular pieces of land that he could enclose with that length of fence.
Possible dimensions of some rectangles:
5 × 25; 10 × 20; 15 × 15; 18 × 12; 23 × 7; 27 × 3;
many more are possible.

3. Name the perimeter and the area of each of the figures.

a

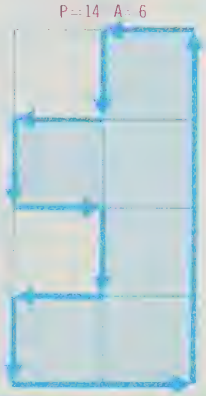
P=14 A=6

b



P=12 A=6

c



P=14 A=6



**lesson** Page 155

**goal** Diagnosis and reinforcement of multiplication skills

**things** graph paper

**page 155** The first sentence isn't completely true. The pupil could continue to count squares to find area, but it would take lots of graph paper and lots of counting to find the area of a lot 150 feet by 75 feet, for example.

Assign sets 1, 2, and 3 as independent work. These sets are designed to maintain computational skills and to help identify pupils whose multiplication skills have slipped. Observe pupils' speed and apparent confidence in completing the computation.

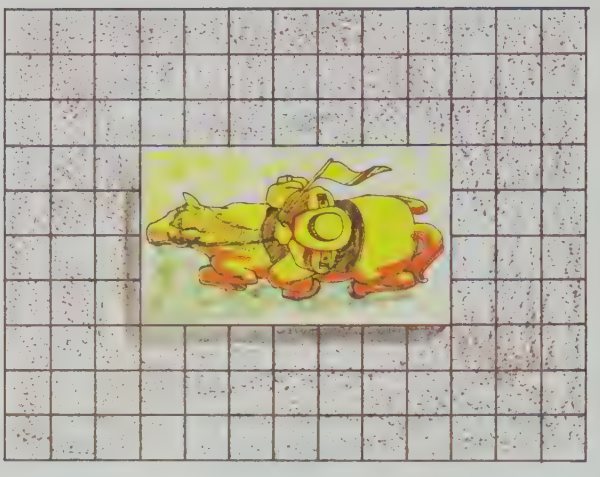
For those pupils who made no errors on any of the problems, establish the ground rule that they will be resource people to help those who made errors.

Enjoy the remainder of the page. Graph paper and exploration are the techniques needed for problem 2.

**lesson** Pages 156, 157

**goal** Application of area and perimeter

**page 156** Treat this real-world problem as an independent problem-solving activity. Then talk about the variety of ways to find the answers—other than by counting.



There is a long way and several short ways to do the next problem. Counting is the long way. That way will get an answer. It's O.K. But try to find one of the short ways.

The people of the town had wanted a statue for many years. They finally got one. It was beautiful from all sides. They wanted a sidewalk 3 feet wide around the statue. The base of the statue measured 4 feet by 7 feet.

This picture shows the shape of the sidewalk.

1

How many square feet of sidewalk does the town need? 102 square feet

*2

If the sidewalk costs $2.50 a square foot, how much will it cost in all? $255

156

See activity 1, page 165a.

Today you are a problem solver. Help Mr. Smith.

Mr. Smith didn't have a very big yard. But he had a dog. The dog needed an outdoor pen. Mr. Smith thought he needed at least 36 square feet.

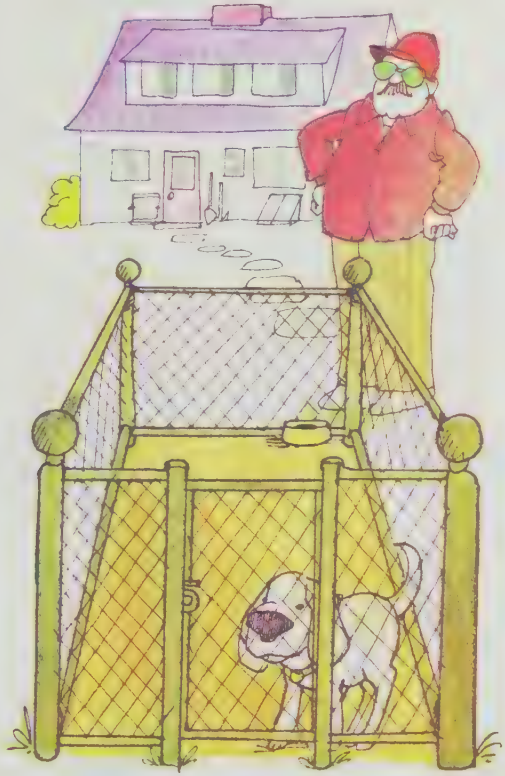
Use graph paper and show at least two possible ways he could build the pen. Rectangles ($6 \times 6$ or $3 \times 12$) are easiest. $18 \times 2$ wouldn't be practical.

Then tell how many feet of fence Mr. Smith needs for the pen. Answers will vary.

Fencing is expensive. Mr. Smith can buy a ready-made pen measuring 4 feet by 9 feet for $64.99. Or he can buy fencing and build the pen himself. Fencing costs 60¢ per foot. The four posts he needs cost $5.09 each. He will need a gate too.
(He can't lift the dog into the pen.)
The gate comes complete with posts and all hardware. The gate costs $12.95.

How can he get the pen for the least amount of money? Build it himself. (If he builds a $6 \times 6$ pen, the cost will be $47.71 or $17.28 less.)

Should Mr. Smith's time be considered as part of the cost? Yes. His time might make the do-it-yourself project more expensive.



157

**goal** Application of area and perimeter

**things** graph paper

**page 157** This is a surprisingly difficult page. Pupils may want to work in pairs. The problem is not easy and many facts must be considered to find out which is the cheapest way to get the pen. Ask for opinions on the last question. Perhaps your pupils would like to get the opinion of someone at home before finally deciding for themselves.

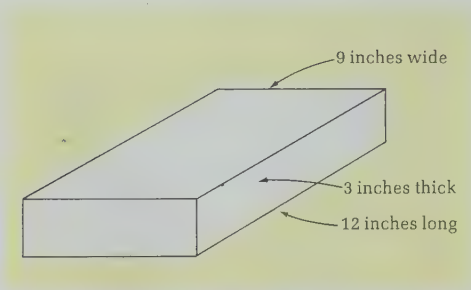
**lesson** Pages 158, 159

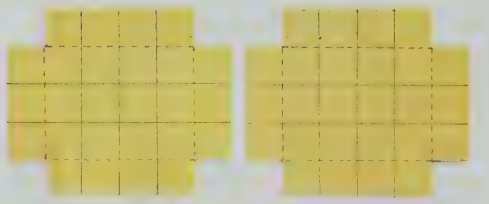
**goal** Relating surface area to making 3-dimensional boxes

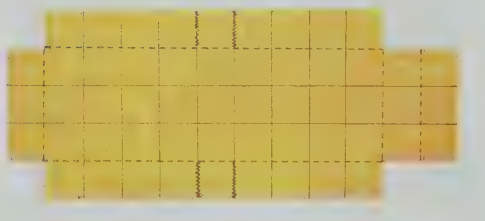
**things** graph paper
scissors
empty boxes
transparent tape

**page 158** Here is an opportunity to talk about careers. A boxmaker must be a good problem solver. He must give the customer what he wants and do it in the least expensive way. He has to think about how to save cardboard. How much waste would there be if a piece of cardboard were cut according to each pattern given on the page? Is there a better way? It is great fun to take empty boxes apart and see what they looked like before they were folded. Try opening up several different kinds of boxes. You'll have fun too.

Have the youngsters verify that the patterns really work by making congruent figures on graph paper, then cutting them out. The work involved in assembling the boxes will help them decide on an answer to the last question.





## Today you are a boxmaker.

Your job is to figure out the size and shape the box you are going to make should be. You have to save cardboard, of course. Extra cardboard costs money. You plan by making models. You start with a flat pattern. Then you cut it out and make the box.

Your first customer comes to you with a book. He plans to mail 10,000 of the books. He needs a box to put each one in.

One box you might pick has a lid and a base.
The box is cut from flat cardboard.
The pattern looks like this.

Let one square show about 3 inches.

The lid would be about $\frac{1}{16}$ larger. Why?
So that it will fit over the bottom of the box

A one-piece, fold-up box pattern would look like thi

~~~~ shows cut marks

- - - - - - shows fold marks

Which box uses less cardboard? Box at top

Which box would you suggest using for the books?
Why? There is no right answer.
The box at top uses less cardboard, but may be more expensiv
the corners must be taped together. It would be handier. The
box would probably be stronger.

158
~~~~

You know one pattern for a square box looks like this when it is laid out flat. This kind of box would be taped together.

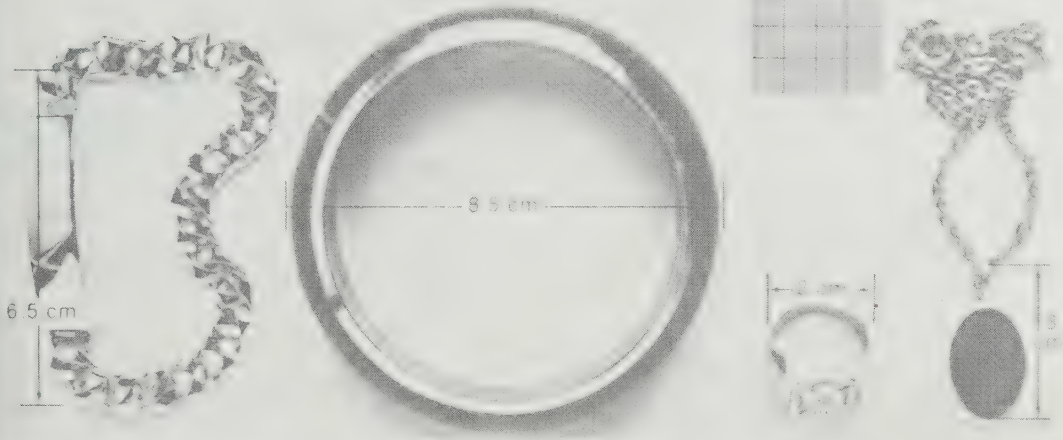
1 square represents 1 square centimetre ($1\ cm^2$)

This could hold something round like a ball.

It could be used for packing a fancy cup you are giving as a gift.

It could hold another box.

Use graph paper. Let each square be 1 cm long and 1 cm wide. Plan a box for the following objects. Show what each box would look like before it was folded together. Make the box actual size.




Answers will vary. Accept and encourage all reasonable solutions

159

**goal** Planning boxes to hold various objects

**things** graph paper
scissors
transparent tape

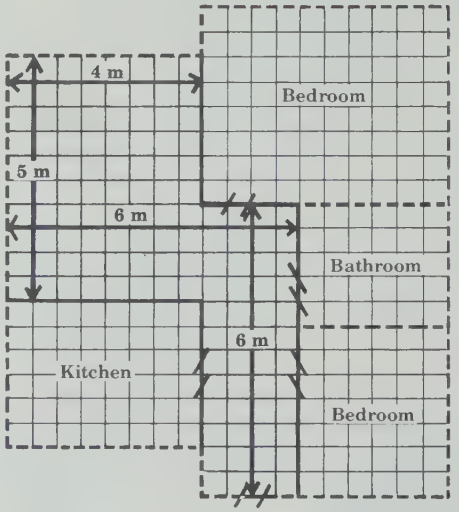
**page 159** A partner will prove helpful when trying to tape a corner—pupils will want to work in pairs.

**lesson** Pages 160, 161

**goal** Application of area

**page 160** Some pupils may not know that carpeting comes on rolls and is sold by area. It comes in several different widths. There could be lots of waste or you might have to make a seam. Maybe that is the reason the man in the problem is buying carpet squares instead. Get the problem solving started by asking how wide the hall is. How long is it? The pupils need to understand that to easily compute the area, they must divide the floor space into several rectangles, find the area of each, and then add the areas together.

A rug salesman does lots of things other than compute area and cost, but that is one of his main jobs. He must try to figure out ways to keep the cost of the carpeting reasonable. Otherwise his customers will go to another store. The option to talk about comparative shopping should be taken if time allows. Consumer education is important enough to be introduced whenever possible.




## Today you are a rug salesman.

1. A man comes in with this floor plan. He says he wan to buy carpeting for the living room and the hallway (the shaded part). He says this is an accurate drawin

   You really want to sell him the carpeting.

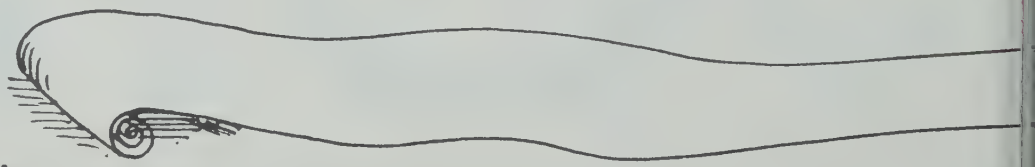
   **a** Can you figure out how many square metres of carpet he needs? 32 m²

   **b** Do you suppose you can talk him into buying some for the big bedroom, too? How many square metres in the big bedroom? 20 m²

The man decides he wants to put down carpet squa
They measure 25 cm² . He could do that himself.
You have three different prices for carpet squares:

| | | Living room and hall | Livi hall ar |
|---|---|---|---|
| Solid color, flat texture | 38¢ each | | |
| Two-tone, textured | 59¢ each | $194.56 | $ |
| Thick shag pile | 75¢ each | $302.08 | $ |
| | | $384.00 | $ |

He wants you to figure all three costs.
Remember — it's your job.



160

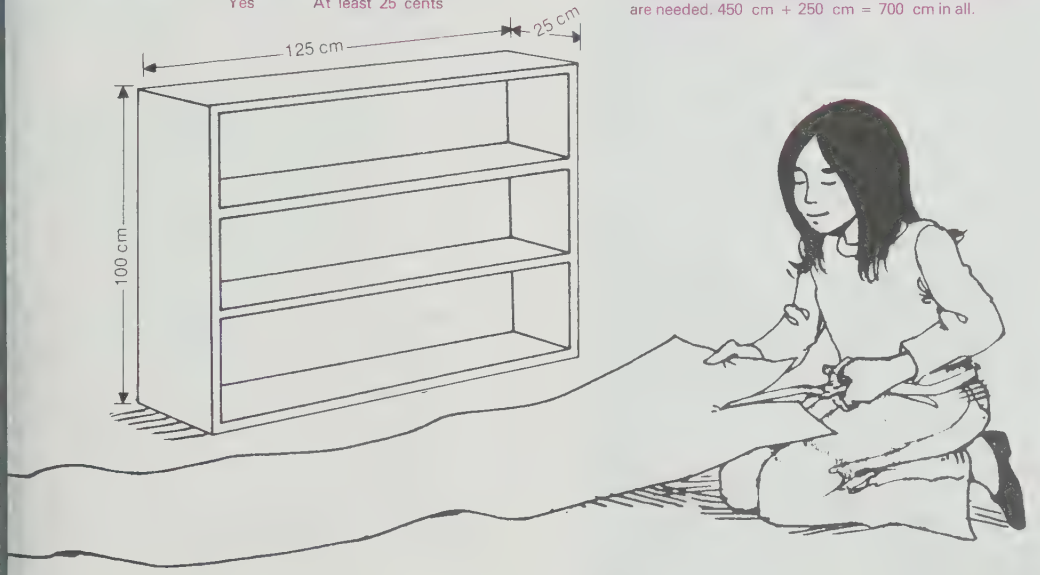
See activity 2, page 165a.

You have an old bookcase. You think it would look great if you covered it with self-stick paper. You have to figure out how much paper it would take.

You have to cover the top and sides. You have to put paper on the inside and on the shelves too. About how much paper 50 cm wide will you need to buy? (Assume outside back and bottom of shelves are not covered.)

Minimum needed: exterior, 100 cm + 125 cm + 100 cm = 325 cm; interior sides, 100 cm + 100 cm = 200 cm; 3 shelves, each 125 cm = 375 cm; Total is 900 cm, but bookcase is only 25 cm deep. So 900 ÷ 2, or 450 cm, plus 2 widths 125 cm long for interior back are needed. 450 cm + 250 cm = 700 cm in all.

You have only $5. The paper cost 75¢/m .
Do you need more money? How much more?
Yes At least 25 cents




161

**goal** Application of area

**page 161** Youngsters from a do-it-yourself family may have experience working with self-stick paper. They'll be far ahead in solving this problem. There's lots to think about and to compute. Capable students could work independently in pairs. Consider solving the problem as a group with less capable students. This is certainly a practical problem. Someone should think of splitting the paper in half since the shelves are 25 cm wide and the paper is 50 cm wide. Is a straight cut important? What about a pattern that goes one way—could this affect the quantity needed?

**lesson** Pages 162, 163, 164

**goal** Applications of area

**things** graph paper

**page 162** Some pupils may be unfamiliar with the work of a real estate agent. Others will be quick to point out that some real estate agencies take care of rental property as well as help people buy and sell land. To get across the concept of what an agent does may be impossible. Letting the pupils know that there is someone who helps people find the kind of place they want to live in or the piece of property they want to buy or sell is all that's necessary.

Real estate agents often work on a tight time schedule. They must be ready to serve the people who come to them for help. Investigate to see whether the pupils have any idea how an agent earns his salary.

The page is self-explanatory. You decide how best to handle it with your pupils. Encourage using the graph paper and experimenting to find the answers.

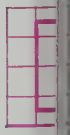
Today you are a real estate agent.

You buy and sell land and buildings. This is a very busy day. You hope to sell a farm to one person. You are going to show another person some land in town where she could build a house.

Land in the country is sold by the hectare. A hectare square unit of land measure. To give you an idea of big it is, think of the size of the field inside an athlet track. That's a lot of land!
The farm you want to sell is 120 hectares (ha).

1. The land costs $920/ha . How much will the far cost? When you tell the man this, he says he wa time to think it over. $110 400
2. Use squared paper. Let 1 square represent 5 ha (24 squares will represent all of the land.) What r be the shape of the land in the farm? 4 X 6; 3 X 8
3. The land in town is 30 m across the front. It is 4( depth. How many square metres of land is this? the shape of the land on squared paper. Let the of one square represent 5 m . 1200 $m^2$; drawing sh be 6 X 8
4. The woman wants to build a house that has 300 square metres of living space. Does she have ro build that big a house? Prove your answer on yc graph paper. Show the area of the land and whe house could be built. You sell the land!

Yes she has room (see drawing).



The skills of finding area and perimeter are used in many situations. There are no right answers to the next questions. There are reasonable answers. You have to use some good thinking to come up with any kind of answer.

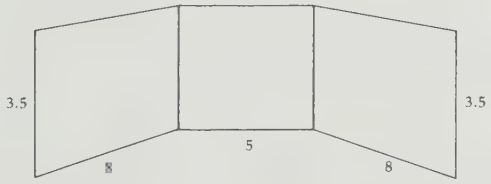
1. Wood moulding is sold by the metre. You want to make a new frame for a window. The window measures 115 cm by 175 cm . About how many metres of moulding should you buy? 6 m

**Think**

2. You want to put wall paper on one wall. The paper is 50 cm wide. There is 12.5 m in a roll. The wall you want to cover is 3.5 m high and 5 m wide. How many rolls of paper do you have to buy? 3 rolls

**Think**

3. It says on the paint tin that its contents, 4 $\ell$, will cover 50 $m^2$. You want to paint the other three walls in the room. It is a rectangular room. You know one of the walls is 3.5 m by 5 m . Each of the other two walls is 3.5 m by 8 m . Will 4 $\ell$ of paint be enough? No; the total area is 73.5 $m^2$ (you'd need more than a litre).




**Think**

4. Do you have to know area or perimeter to buy enough fabric to cover a chair? Area



**things** newspaper real estate ads

Looking at the classified ads in the newspaper is another way to broaden understanding of real estate. Have your pupils check all the different kinds of buildings that are available for sale.

See activity 3, page 165a.

**goal** Applications of area and perimeter

**page 163** Give everyone the opportunity to think about these problems and arrive at some answer. Emphasize that there are no right answers—only reasonable ones. Share their thinking in a discussion. Problems 1 through 3 each involve several considerations:

- Problem 1 — If the window is 115 cm by 175 cm, how much larger must the frame be? How much extra frame is needed for cutting corners that fit?
- Problem 2 — Does the pattern on the wallpaper need to be matched where the strips fit together? How many centimetres before the running pattern begins to repeat again? Does this mean more paper will be needed?
- Problem 3 — Will one coat of paint be enough? Will one thicker coat do the job or should two coats be used? If two coats are used, will the second coat take as much paint? Close to 6 $\ell$ will be needed. Is it cheaper to buy 4 $\ell$ cans, or one 4 $\ell$ can and one smaller can of 2 $\ell$ ?

With the popularity of do-it-yourself jobs, you can expect some of your pupils to think of these questions.

**goal** Covering an area with noncongruent pieces

**things** graph paper
scissors
wood cubes (optional)

**page 164** Fun for pupils of all abilities. Emphasize that the area of each pattern piece should be 5 square units, but no 2 pieces should be congruent. Provide less-capable students with a little help—5 wood cubes that they can manipulate to form the 12 different patterns. These can then be sketched on graph paper so that the pupil can continue with the activity as directed on the page.

# TODAY YOU ARE A STUDENT.

You will need graph paper again.

Your job is to take five squares.

Fit the five squares into twelve different patterns.

At least one side of each square must match another side.

Each side touches at least one other side.

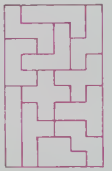
After you have found your twelve different patterns, cut each out.

See 12 patterns below.

Make a rectangular puzzle board from graph paper. One side should be 6 squares high. The other side should be 10 squares wide. 60 squares in all.

Now all you have to do is fit your twelve pieces onto the playing board.

Here is one way:

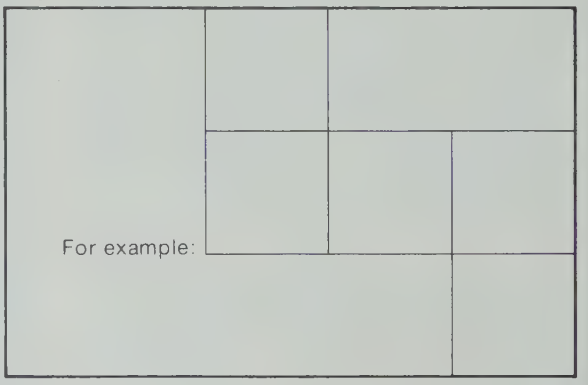


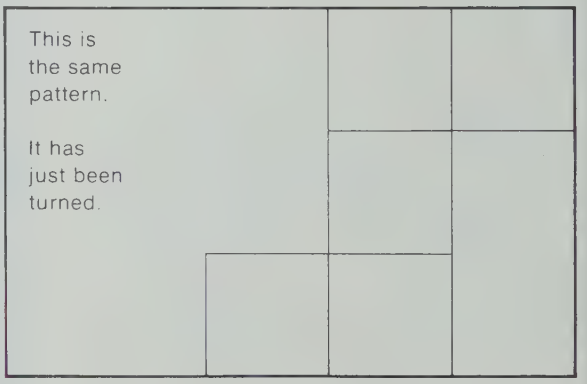
There are others.

GOOD LUCK!

164

This is HARD!

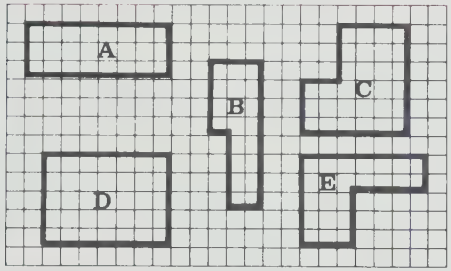


# CHECKOUT

Skill: Computing perimeter

1. Pretend the pictures below show garden plots. Pretend the side of each small square equals 1 m. You want to fence one plot. Give the letter name of the plots that could be enclosed with 24 m of fencing. A, B, C, D, E (With A and B, you'd have 2 feet left over.)




Skill: Computing area

2. Now pretend that the pictures above show parts of a city park that need grass seed. You have enough to cover 24 $m^2$. Give the letter name of the parts that could be sown with seed. A, B, E

Skill: Adding 2- and 3-digit addends

Add.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 3. | 27 | 354 | 426 | 577 | 675 |
| | +64 | + 23 | +152 | +128 | +238 |
| | 91 | 377 | 578 | 705 | 913 |

Skill: Multiplying 1-, 2-, and 3-digit factors

Multiply.

| | | | | | |
|---|---|---|---|---|---|
| 4. | 37 | 53 | 67 | 123 | 687 |
| | × 4 | ×70 | ×23 | ×213 | ×254 |
| | 148 | 3710 | 1541 | 26 199 | 174 498 |

165

See activity 4, page 165b.

See activity 5, page 165b.

**lesson** Page 165

**goal** **Checkout** – computing perimeter and area; addition and multiplication skills

**page 165** Pupils may use either counting or computation as their method for finding area and perimeter. Some may compute first and then verify their computation by counting. With pupils who make errors, you'll want to check first on the method used to arrive at the incorrect answer. Most likely errors are:

- Miscounting
- Computation error
- Mixing up the words area and perimeter

Check pupils who make addition errors. This may be just plain carelessness. Anyone still having a problem needs individual help. Look for these weaknesses:

- Addition facts
- Concept of renaming
- Concept of place value

Whether additional help and practice are needed for pupils who miss problems **4d** and **4e** is your decision. This may not be a mastery objective for your school at this level. Weaknesses to check on:

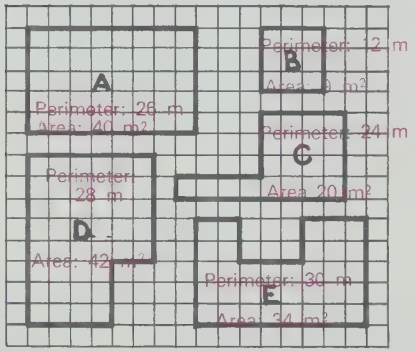
- Multiplication facts
- Understanding the algorithm for 3-digit multipliers
- Addition errors when adding the partial products

# RESOURCES

## another form of evaluation

**for checkout**—page 165

1. Find the perimeter and area of each figure below. Pretend the side of each small square equals 1 m.




2. Add.

| (a) | (b) | (c) | (d) | (e) |
|---|---|---|---|---|
| 37 | 642 | 581 | 364 | 593 |
| +58 | + 36 | +217 | +238 | +167 |
| 95 | 678 | 798 | 602 | 760 |

3. Multiply.

| (a) | (b) | (c) | (d) | (e) |
|---|---|---|---|---|
| 24 | 43 | 37 | 246 | 824 |
| × 6 | ×80 | ×62 | ×264 | ×657 |
| 144 | 3440 | 2294 | 64 944 | 541 368 |

## activities

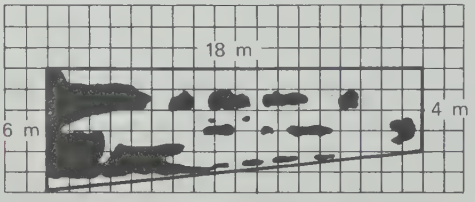
**1.** Independent activity. (Provide each pupil with these directions.)

- Measure the distance around your bicycle wheel.
- Tie a piece of string around the rim and over the tire of a wheel, or mark the tire with a chalk line.
- Ride or walk your bicycle once around the perimeter of the school playground.
- Count how many times the mark on the tire passes a certain point on the frame of your bicycle.
- Use the information you have so far to estimate the perimeter of the playground.
- Use this same technique to estimate the perimeter of the block on which you live.
- Can you use this technique to estimate the distance from your home to the library?

**2.** Independent activity for pairs of pupils. (Provide each pair with the following.)

HOW MUCH SALT?

Today you have a job to do for the owner of a small area used for parking cars. The area is sketched below. It has been covered with ice, and some of the ice has melted. You are to remove the remaining ice by applying salt.




Total area for parking: 90 m²
Shaded areas represent area free of ice.
Each square represents 1 m².

The directions on the bag of salt you are given say to spread 1 cup of salt over a square metre.

1. How much of the parking area (in square metres) is free of ice?
2. How much of the driveway (in square metres) is covered with ice?
3. If you do not salt the areas that are free of ice, how many cups will be needed?

Note: The answers to questions 1, 2, and 3 must be obtained by estimation. No exact answers are possible. Let these answers be a guideline to reasonable results:

1. About 26 m²
2. About 64 m²
3. 64 cups

You may want to discuss the methods used by each pair of pupils in making their estimates.

**3.** Individual activity. (Provide the pupil with the following.)

Would you need to know **area** or **perimeter** to solve the following problems?

1. How much paint is needed to paint the bedroom? (Area)
2. How much fencing is needed to fence the lot? (Perimeter)
3. How much carpeting is needed for the bedroom? (Area)
4. How much edging is needed for the flower garden? (Perimeter)
5. How much baseboard trim is needed for the bedroom? (Perimeter)
6. How much grass seed is needed to seed the lawn? (Area)
7. How many cement blocks are needed to pave the driveway? (Area)
8. How much panelling is needed for the storage room? (Area)
9. How big a cover is needed for the swimming pool? (Area)
10. How much fertilizer is needed for the garden? (Area)

Note: This activity may be extended with the following discussion questions:

- What would be some appropriate units of measure to use in solving each of the above problems?

- Can you think of more situations such as the ones above that require the use of perimeter and area? If so, what are they? (Answers will vary.)

**4.** Individual activity. (Provide the pupils with the following directions.)

Use graph paper. Each square is 1 unit of area. Draw rectangular regions having the following perimeters and areas.

| | *Perimeter* | *Area* | |
|---|---|---|---|
| 1. | 12 | 8 | (2 by 4) |
| 2. | 18 | 8 | (8 by 1) |
| 3. | 14 | 12 | (4 by 3) |
| 4. | 16 | 12 | (6 by 2) |
| 5. | 26 | 12 | (12 by 1) |
| 6. | 20 | 9 | (9 by 1) |
| 7. | 20 | 16 | (8 by 2) |
| 8. | 20 | 21 | (7 by 3) |
| 9. | 20 | 24 | (6 by 4) |
| 10. | 20 | 25 | (5 by 5) |

11. Draw three rectangular regions having the same area and different perimeters. Use different regions.
12. Draw three rectangular regions having the same perimeter and different areas. Use different regions.

Note: As a result of this activity some pupils may discover the formulas for computing area and perimeter. That's great, but this is not the time to develop these formulas. Emphasis should be on the distinction between area and perimeter.

**5. things** graph paper

Individual activity. (Prepare a spirit master as shown.)

Each region shown below has an area of 3 square units. Can you show 3 more?

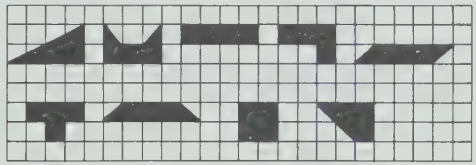
The figures shown above are all polygons. You remember polygons, don't you? If not, better look them up before you go on.

Use graph paper and think of each square as 1 unit of area. Draw **at least** 20 polygons that have an area of 4. Use your imagination!

Note: Answers will vary. Some possible solutions are given below.

Regions of area 3

Regions of area 4



## additional learning aids

### geometry—chapter objective 1

**SRA products**

*Mathematics Learning System*,
*Activity Masters, level B*, SRA (1974)
Spirit masters: G-2, 6, 7; M-9, 10
*Mathematics Involvement Program*, SRA (1971)
Cards: 384, 46, 17
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 42, 43

**other learning aids** (described on page 216h)—Geoboard Activity Cards (intermediate set), Geoboard Kit, Geoboards and Motion Geometry

### measurement—chapter objectives 2, 3

**SRA products**

*Mathematics Learning System*,
*Activity Masters, level B*, SRA (1974)
Spirit masters: M-7, 8, 11, 13
*Math Applications Kit*, SRA (1971)
Appeteasers cards: 6, 8
Science cards: 6, 22, 37
Occupations cards: 11, 13, 15, 16, 18, 50
Everyday Things cards: 4, 7
*Mathematics Involvement Program*, SRA (1971)
Cards: 195, 76, 236, 246, 67, 227
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 52, 53

**other learning aids** (described on page 216h)—Geoboard Activity Cards (intermediate set), Geoboard Kit, Geoboards and Motion Geometry, Geometric Ruler, Good Time Mathematics, Paper and Pencil Geometry

### operation—chapter objective 4

**SRA products**

*Mathematics Learning System*,
*Activity Masters, level B*, SRA (1974)
Spirit masters: W-25, 29, 30, 34, 36
*Computational Skills Development Kit*,
SRA (1965)
Addition cards: 14–17
Multiplication cards: 14–25
*Cross-Number Puzzles* (*Whole Numbers*),
SRA (1966)
Addition cards: 15–21
Multiplication cards: 10–17
*Diagnosis: an instructional aid—Mathematics Level B*, SRA (1972)
Probes: M-1, 3
*Skill Modes in Mathematics*, SRA (1974)
Level I, Molecules: A, C
Level II, Molecules: A, C
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 9, 17, 24, 25

# 7 FRACTIONS x

**before this chapter the learner has —**

1. Renamed a fraction in simplest form
2. Found a common denominator for two fractions having unlike denominators
3. Renamed a mixed number as a fraction; renamed appropriate fractions as mixed or whole numbers
4. Added and subtracted any two fractions
5. Multiplied fractions, whole numbers, and mixed numbers

**in chapter 7 the learner is —**

1. Reviewing all forms of renaming for fractions and applying these skills in computation
2. Reviewing addition and subtraction of fractions with unlike denominators
3. Mastering multiplication of fractions, including mixed numbers

**in later chapters the learner will —**

Apply renaming skills to division of fractions

# Notes & Things

This chapter on multiplication of fractions begins with a review of the concept itself and all the ways it is used. Factors must also be briefly revisited but not in the same old way. Factoring is put into the gamelike situations that our grandparents used. They insist that it was really fun to have factor contests, and to see the gleam in their eye means that it may be a joy for this generation too.

The factor review is necessary to launch the comprehensive study of renaming. The youngsters will participate in a "how to do it" review. All the renaming skills are presented – renaming in simplest form, finding equivalent fractions needed to get a common denominator, renaming mixed numbers as fractions, and the reverse renaming also. Finally the pupils are ready to use most of their skills with the operation of multiplication.

Discuss the unique pages that start on page 181. Make sure each learner understands that separate multiplication skills are checked out with a few carefully selected problems. Then the pupils can check their own answers. Stress that pages like this will help them identify what they know and what they don't know. You may want to give the appropriate pep talk to assure them that they are really the best persons to discover and correct their errors. Depending on the number of errors made, each pupil will get some extra help and practice or be directed to another page. You will find that some pupils breeze through this part of the chapter. There will be some challenging sidetrack pages for the sharpies, but please don't forget to enlist these pupils' help as peer tutors.

The amount of independent study you use in this chapter is your decision. The organization of the pupil pages will allow you freedom of choice.

## things

counters
baseball cards or squares of paper

For the extra activities you will want to have these things available:
- sets of colored plastic sticks
- markers
- wood cubes (or sugar cubes)

**goal** Think about and explore ideas through a picture clue

**page 166** Exploring the history of mathematics is an interesting study. Some of your more able students may find this photograph of Egyptian art interesting enough to start them on an independent research project. Suitable references for children are available. There are appropriate books published by the National Council of Teachers of Mathematics and more good history books can probably be found in your school library.

Organized number systems were used as early as 5000 B.C. Some believe that mathematics was well developed in China considerably before then. Archaeologists' findings in the area of the Indian, Babylonian, and Egyptian civilizations cannot, however, be disputed. People needed numbers. They figured out a way to use them. And they did use them to improve the quality of their lives.

Can your pupils see anything that resembles mathematics in the photograph? Encourage intelligent guessing about this 5000-year-old record.

Where did the idea of fractions come from? Symbols for fractional parts were in use in Egypt a long time ago. The Egyptians already had numbers that told "how many." But they also needed numbers that told about parts of a whole or a unit. They had to keep records, so they also made up symbols for fractions.

This is how the Egyptians wrote $\frac{1}{4}$:

←This symbol meant "a part."

←These marks represent the number 4.

**There are times when you must operate with fractions, whether you like it or not. They really aren't that hard. Your goal is to check out your skills in multiplication with fractions.**



**lesson** Pages 167, 168, 169

**goal** Establishing the learning goal of the chapter

**page 167** Strictly for discussion. Emphasize that whole numbers tell **how many**, while fractions tell **how much**.

History buffs will probably want to do more research on how fractions were written in past civilizations.

**goal** **Survey**—writing fractions to tell how much

**memo** Pages 168 through 179 review and reinforce the basic concepts and skills necessary to continue working successfully with fractions. A few of the ideas will be new; most of them will be review. The responses of your students will determine the pace at which you move and whether some of them are ready to skip parts of this review and go on.

**page 168** Use this page as an informal survey of the abilities of your class. The three models shown on the page are used to illustrate fractions throughout the program. You'll want to make sure each pupil knows how to read them.

Written or oral? You decide.

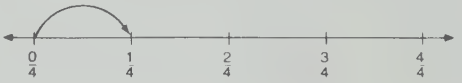
Before leaving the page, you may want to discuss which of these situations could have confronted the Egyptians.

Our symbols for fractions aren't too much different.

**the numerator is a number** → **1** ← **it tells how many parts you're thinking of**

**the denominator is a number** → **4** ← **it tells the number of parts in a unit**

The number line, the region, and the set are the models that can help us think fractions.



1. We use fractions to describe real situations, too. What fractional part is—

   **a** 3 months out of 12 months? $\frac{3}{12}$ or $\frac{1}{4}$

   **b** 30 minutes out of 60 minutes? $\frac{30}{60}$ or $\frac{1}{2}$

   **c** 3 out of 4 questions wrong on a test? $\frac{3}{4}$

   **d** 10 cents out of 1 dollar? $\frac{10}{100}$ or

   **e** 25 cents out of 50 cents? $\frac{25}{50}$ or $\frac{1}{2}$

   **f** 1 out of 3 parts of a pie? $\frac{1}{3}$

   NOW LOOK OUT FOR THESE.

   **g** 1 month out of 1 year? $\frac{1}{12}$ year

   **h** 1 day out of 1 week? $\frac{1}{7}$ week

   **i** 1 hour out of 1 day? $\frac{1}{24}$ day

   **j** 5 cents out of 1 dollar? $\frac{5}{100}$ or $\frac{1}{20}$ dollar

   **k** 6 hours out of 1 day? $\frac{6}{24}$ or $\frac{1}{4}$ day

   **l** 1 minute out of 1 hour? $\frac{1}{60}$ hour

   **m** 1 year out of an 80-year-old man's life? $\frac{1}{80}$

   **n** 1 year out of the years you have lived?
   Answers will vary depending on pupil's age. His age is the denominator of the fraction.

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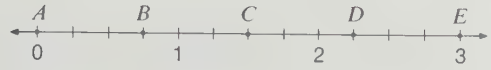
Don't expect correct answers on all of these.

1. How many equal segments from 0 to 3? 12

   **a** This number line shows three units. Each unit is divided into equal-size parts. Point $B$ could be named a fraction. What fraction? $\frac{3}{4}$

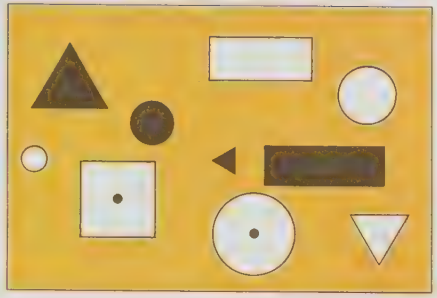
   
   

   **b** What fraction does point $C$ name? point $D$? point $E$? $\frac{12}{4}$ or 3 $\frac{6}{4}$ or $1\frac{2}{4}$ or $1\frac{1}{2}$ $\frac{9}{4}$ or $2\frac{1}{4}$

2. Draw a number line to show the fraction $\frac{7}{4}$.

3. The next questions refer to the set of shapes on the right. Name the fractions.

   **a** What part of the set are triangles? $\frac{3}{10}$

   **b** What part of the set are rectangles? $\frac{3}{10}$

   **c** What part of the set are shaded figures? $\frac{4}{10}$ $(\frac{2}{5})$

   **d** What part of the set are figures with dots? $\frac{2}{10}$ $(\frac{1}{5})$

   **e** What part of the set are circles? $\frac{4}{10}$ $(\frac{2}{5})$

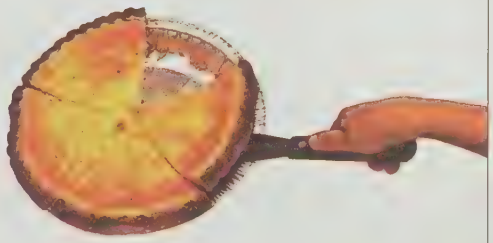
   **f** What part of the set are figures with straight sides? $\frac{6}{10}$ $(\frac{3}{5})$

   **g** What part of the set are hexagons? $\frac{0}{10}$

4. Which would you rather have—
   $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, or $\frac{5}{10}$ of a dollar? All are same amount.

5. Bill looked at the place where the piece of pie had been. He said, "I would like to have had 6 times more than the piece I had." How much pie would Bill like to have had in all? $\frac{6}{5}$ or $1\frac{1}{5}$

6. His sister said she would be satisfied with $\frac{1}{2}$ of the pie that was left. How much did she want? $\frac{2}{5}$



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**goal** **Survey**—naming fractional parts of a number line, set, or region

**page 169** This page will complete your informal survey of pupil ability. Problems 1 and 2 review mixed and whole numbers written as fractions. What signals that a fraction is equal to one or that it is greater than one?

Watch for answers to problem **3g**. The question requires a fraction answer, not a whole number, since it asks "what part."

The last two problems can be solved mentally (sufficient for now) by using multiplication computation.

The following chart will help you meet individual needs.

| Trouble with problems | Additional help on pages |
|---|---|
| 1, 2 | 180 |
| 3, 4 | 177–179 |
| 5, 6 | 181–187 |

**lesson** Pages 170, 171, 172, 173

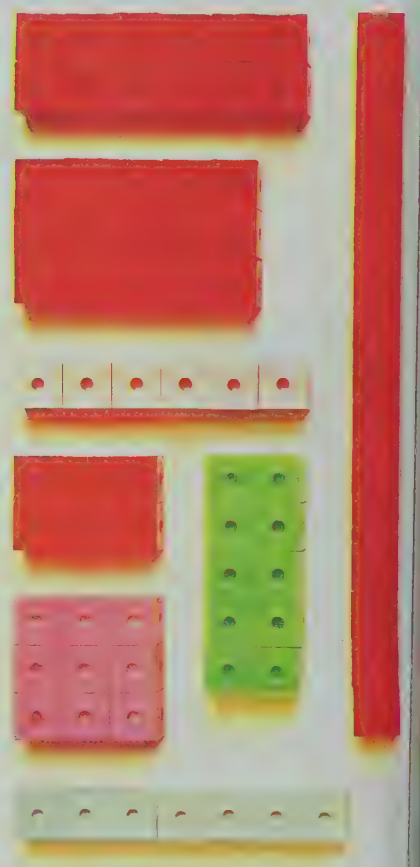
**goal** Introduction to factoring products

**things** counters
baseball cards or squares of paper

**page 170** The box problem serves two purposes—to introduce the concept of factoring and to review multiplication facts. This is an excellent technique to use with pupils who need additional practice with the facts.

The possible box arrangements (forming an array) can be found quickly by using counters, baseball cards, paper squares, or whatever you have available. You will certainly want to use manipulatives with less capable students. Models are shown in the photographs. Use these models to extend the learning on the page. They will help you review the terms **factor** and **product** in case anyone has forgotten these words. Is someone not ready to accept the last line on the page? If so, try these numbers: 11, 12, 13, 15, 21, 30, 31, 36. Have the manipulatives handy.

Pretend that you just invented a lotion that grows hair. You decide to sell your wonderful lotion. You plan to ship it in boxes of 12. You know that you will put only one layer of bottles in each box, and you'll have to have dividers so that the bottles won't break. How many differently shaped boxes could you use? 3



1. Pretend you decided not to sell boxes of 12. Describe the boxes (how many rows, how many in each row) you might use to ship a total of— See answers below.

| | | | | | |
|---|---|---|---|---|---|
| **a** | 6 | **b** | 4 | **c** | 14 |
| **d** | 2 | **e** | 9 | **f** | 3 |
| **g** | 27 | **h** | 15 | **i** | 5 |
| **j** | 20 | **k** | 16 | **l** | 18 |
| **m** | 7 | **n** | 10 | **o** | 1 |

We can call the *total number* in each box a *product*.
To get a product, we multiply the number of rows times the number in each row.
The numbers we multiply are called factors.

**p** Can any of the products named above be named by other factors? Which ones? Yes Answers are given below.
What are the other pairs of factors that can name each product? See answers below.

Every counting number can be named as the product of at least two factors.

Examples: a 1 row of 6, 2 rows of 3 b 1 row of 4, 2 rows of 2 c 1 row of 14, 2 rows of 7 d 1 row of 2 e 1 row of 9, 3 rows of 3

f 1 row of 3 g 1 row of 27, 3 rows of 9 h 1 row of 15, 3 rows of 5 i 1 row of 5 j 1 row of 20, 2 rows of 10, 4 rows of 5

k 1 row of 16, 2 rows of 8, 4 rows of 4 l 1 row of 18, 2 rows of 9, 3 rows of 6 m 1 row of 7 n 1 row of 10, 2 rows of 5 o 1 row of 1

Answers will vary. (But please note that 2 rows of 3 and 3 rows of 2 would be the same box.)

1. Which numbers between 1 and 18 can be named by only *one* pair of factors? 2, 3, 5, 7, 11, 13, 17

2. The number 1 can be used as a factor for any counting number. A number that can be named only by the factors 1 and another number are so special that they have a special name. They are called prime numbers. You found the prime numbers between 1 and 18 in the first problem. Do you suppose there are prime numbers greater than 18? What about 19?

   $2 \times \underline{?} = 19$ No.
   $3 \times \underline{?} = 19$ No.
   $5 \times \underline{?} = 19$ No.
   Why wasn't 4 as a factor investigated? 4 is a multiple of 2.
   Why wasn't 6 as a factor investigated? 6 is a multiple of 2 and 3.
   $7 \times \underline{?} = 19$ No.
   Why can the questions stop with the factor 7?
   Is 19 a prime number? Yes
   Discuss: 8 is a multiple of 2, 9 is a multiple of 3, $10 \times 2 = 20$, $20 > 19$.

3. Is 20 a prime number? 21? 22? 23? 29? No No No Yes

4. Is 31 a prime number? 33? 37? 39? Yes No Yes No

5. Some numbers can be expressed as a product of three or more factors.

   For example: $8 = 2 \times 2 \times 2$

   **a** Express 12 as a product of three factors. $12 = 2 \times 2 \times 3$
   Is each of the factors a prime number? Yes

   **b** Express 18 as a product of three factors. $18 = 2 \times 3 \times 3$
   Is each of the factors a prime number? Yes



**goal** Factoring a number to primes

**memo** Prime and composite numbers were explored in level 5 of this program, but prime numbers may be a new idea for some of your pupils.

**page 171** The definition for a **prime number** is given on the page. For the curious, numbers that are not prime are called **composite numbers**. The number 1 is the exception—it has only one factor, itself—and therefore is neither prime nor composite.

Factoring is fatiguing work for youngsters who are struggling with multiplication and division facts, but the practice does reinforce these facts.

Some pupils may need to use parentheses when factoring so that they can see where the prime factorization came from.
For example: $8 = 2 \times 4$
$= 2 \times (2 \times 2)$

**things** for each pupil: 25 counters

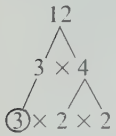
Use whatever you have available as counters—dried beans, plastic chips, baseball cards, and so on. Challenge the youngsters to form as many arrays as possible with 20 counters. (1-by-20, 2-by-10, 4-by-5) Continue with:

- 12 counters (1-by-12, 2-by-6, 3-by-4)
- 19 counters (1-by-19)
- 11 counters (1-by-11)

Can anyone find another number that will form only one array? (Any prime number) By making arrays, can anyone tell whether a number is a prime or a composite?

**goal** Developing a method for factoring to primes

**page 172** Certainly there is more than one way to factor. The method presented on pages 172–173 features division. You may be familiar with the factor tree or splitting method which is dependent on multiplication.




Factors have a tendency to get lost with this method when not brought down from the line above. Note the circled 3.

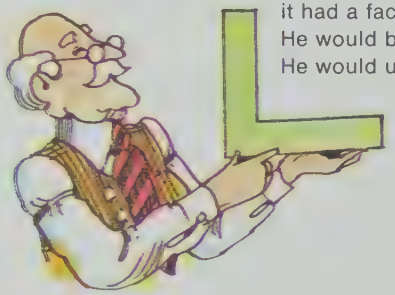
Introduce this method if you wish, but not at the same time as the division method. Two methods at once can only result in confusion.

Someone is sure to ask why the only primes you need to remember are 2, 3, 5, and 7 when factoring numbers less than 100. The square of 7 is 49. What is the next prime? (11) What is the square of 11? (121) And that's greater than 100.

You'll want to talk about the special algorithm used. The focus is on mental computation.

If you were very lucky, you would have a great-great-grandfather around. And if you were even luckier, he would remember his days in school when he was about your age. Factoring a number was almost a game. Everyone had learned the prime numbers. It became a race to see who could factor a large number the fastest.

Our work will not deal with large numbers. But maybe it can be almost like a game. The primes you'll need to remember are 2, 3, 5, and 7.
(If you were going to work with numbers greater than 100, you'd need more.)



Your great-great-grandfather would have factored this way: He would pick a number—60, for example. He would see it was an even number. He would know it had a factor of 2. He would start taking the prime factors "out" of 60.
He would begin with 2. The factors would be taken out through division.
He would use a special symbol for this special division.

| | |
|---|---|
| $2\overline{)60}$ | ⌊ is an upside-down division symbol. |
| $2\overline{)30}$ | He would take "out" all the 2-factors. |
| $3\overline{)15}$ | Then he'd take "out" the 3-factors. |
| $5\overline{)5}$ | No more 3s. But there's a 5-factor. |
| 1 | His job would be finished. |

Check his method.
Does $2 \times 2 \times 3 \times 5 = 60$? Yes

Here is the factoring of 28:

$2\overline{)28}$
$2\overline{)14}$ (No 3s. No 5s.)
$7\overline{)7}$
1

Does $2 \times 2 \times 7 = 28$? Yes

172

Try to factor 11.

?|11 No 2-factors. No 3-factors.
No 5-factors. No 7-factors.
11 must be a prime number!

Factor 50.

2|50
5?|25 No more 2-factors. No 3-factors.
5?|?|5 What's next? 5
1
$50 = 2 \times 5 \times 5$

**1.** Factor 72.

2|72
2|36
2 ?|18
3 ?|?|9
3 ?|?|3
? 1
$72 = 2 \times 2 \times 2 \times 3 \times 3$

**2.** Factor 100.

2|100
2 ?|50
5 ?|25
5 ?|?|5
? 1
$100 = 2 \times 2 \times 5 \times 5$

You can imagine the kind of race there might have been if the number to be factored were as large as 395. For those who try this challenge: $395 = 5 \times 79$

**3.** How many prime factors in 8?
1 prime used 3 times ($2 \times 2 \times 2$)

**4.** How many prime factors in 12?
2 primes, 1 used twice ($2 \times 2 \times 3$)

**5.** How many prime factors in 18?
2, 1 used twice ($2 \times 3 \times 3$)

**6.** How many prime factors in 15? 2 ($3 \times 5$)

**7.** How many prime factors in 22? 2 ($2 \cdot 11$)

**WATCH OUT!**

***8.** What are ALL the factors of 24?
1, 2, 3, 4, 6, 8, 12, and 24!

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**goal** Practice in factoring to primes

**page 173** Everyone who feels confident on his own. Those who are unsure should work with you. If multiplication skills seem stronger than division skills, try the $2 \times \underline{?} = 50$ approach.

Problem 8 can turn into fun for all—the set of factors is so large. Other challenging numbers include: 30, 36, 60, 64, and 81.

**lesson** Pages 174, 175, 176

**goal** Finding all of the factors of a number

**page 174** Note that the sets of factors result from the multiplication facts. For example:

$8 - \overset{\frown}{1,2,4,8}$ $\quad 9 - \overset{\frown}{1,3,9}$ $\quad 11 - \overset{\frown}{1,11}$

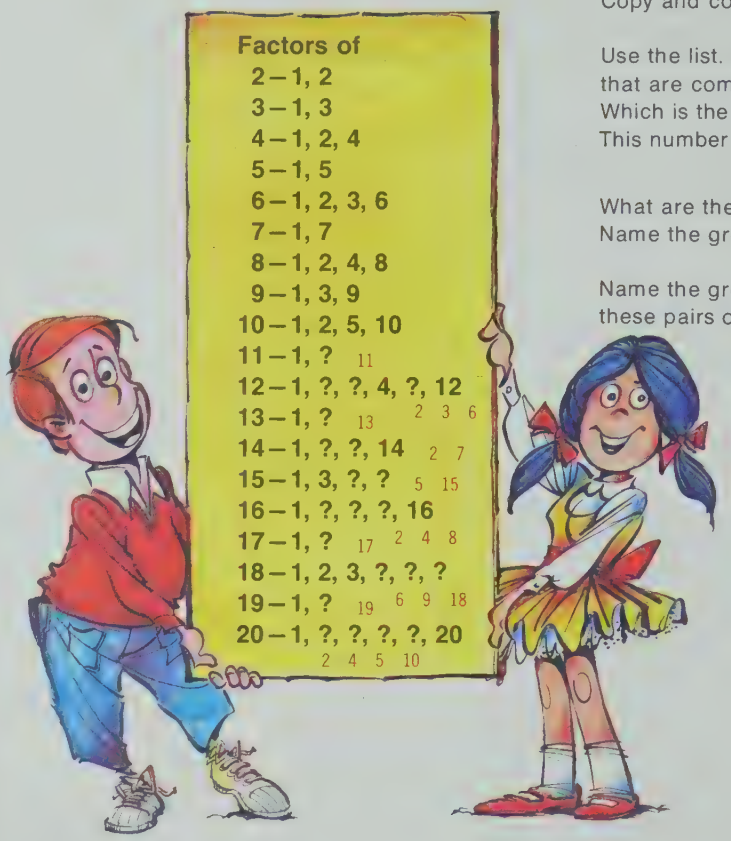
$8 = 1 \times 8$ $\quad 9 = 1 \times 9$ $\quad 11 = 1 \times 11$

$= 2 \times 4$ $\quad = 3 \times 3$

Each factor is listed only once. Examine the factors of 9.

Interesting patterns appear in these sets of factors. Examine the prime numbers. (Have only 2 factors) Look for all numbers with only 3 factors. (Prime numbers squared)

Emphasize finding **common** factors, and then the **greatest common** factor. The latter will be used to rename a fraction to its simplest name.




174

Now think about ALL possible factors of a number. Copy and complete the list.

Use the list. What are the factors that are common to 4 and 8? 1, 2, 4
Which is the largest factor common to 4 and 8? 4
This number is called the *greatest common factor.*

What are the common factors of 10 and 20? 1, 2, 5
Name the greatest common factor of 10 and 20.

Name the greatest common factor of each of these pairs of numbers. Use your list for help.

**a** 5 and 10 5
**b** 3 and 12 3
**c** 4 and 16 4
**d** 6 and 12 6
**e** 2 and 8 2
**f** 5 and 15 5
**g** 6 and 18 6
**h** 9 and 15 3
**i** 10 and 18 2
**j** 12 and 18 6

1. The factors of 12 are 1, 2, 3, 4, 6, 12.
The factors of 15 are 1, 3, 5, 15.
What factors are common to both numbers? 1 and 3

2. What are the common factors of 12 and 16? 1, 2, and 4

12 1, 2, 3, 4, 6, 12

16 1, 2, 4, 8, 16

What is the greatest common factor of 12 and 16? 4

3. What is the greatest common factor for each pair of numbers? The list on the right will help.

**a** 14 and 18 2 **b** 8 and 10 2
**c** 20 and 10 10 **d** 8 and 20 4

4. What is the greatest common factor for the numerator and denominator of the fraction $\frac{8}{20}$? 4
Can both the numerator and the denominator be divided by the same number? What is the greatest common factor of 8 and 20? Yes 4

$$\frac{8}{20} = \frac{8 \div 2}{20 \div 2} = \frac{4}{10} \qquad \frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}$$

common factor greatest common factor

Which factor results in the simplest fraction name for the number? 4, greatest common factor

**FACTORS**

8 ➡ 1, 2, 4, 8

10 ➡ 1, 2, 5, 10

14 ➡ 1, 2, 7, 14

18 ➡ 1, 2, 3, 6, 9, 18

20 ➡ 1, 2, 4, 5, 10, 20

175

**goal** Using the greatest common factor to rename a fraction

**page 175** Problems 1 through 3 are self-explanatory. Everything the pupil needs to answer the questions is given on the page, but you'll probably want to talk about problem 4. When is a fraction in simplest form? (When the numerator and denominator have no **common factor** other than 1) Do 4 and 10 have a common factor greater than 1? Then $\frac{4}{10}$ is not the simplest name for $\frac{8}{20}$. Repeat the question for $\frac{2}{5}$. Emphasize that $\frac{4}{10}$ is another name for $\frac{8}{20}$, but it is not the **simplest name**.

**goal** Finding the simplest name for a fraction

**page 176** Dividing both the numerator and the denominator by their greatest common factor is the fastest way to find the simplest name for a fraction. Assure your pupils that dividing by any common factor is O.K. It simply means that it is necessary to continue dividing until the only common factor is 1.

Fractions written for problem 4 should make much use of prime numbers. Which of the fractions written for problem 5 have a simpler name? (All of them) What is the greatest common factor for these fractions that have a simpler name?

**1.** Complete each set using a common factor and the greatest common factor as indicated.

| | A common factor | Greatest common factor |
|---|---|---|
| **a** | $\frac{12}{18} = \frac{12 \div 2}{18 \div 2} = \frac{?}{?}$ $\frac{6}{9}$ | $\frac{12}{18} = \frac{12 \div ?^{6}}{18 \div ?} = \frac{?}{?}$ $\frac{2}{3}$ |
| **b** | $\frac{16}{24} = \frac{16 \div 2}{24 \div 2} = \frac{?}{?}$ $\frac{8}{12}$ | $\frac{16}{24} = \frac{16 \div ?^{8}}{24 \div ?} = \frac{?}{?}$ $\frac{2}{3}$ |
| **c** | $\frac{20}{24} = \frac{20 \div 2}{24 \div 2} = \frac{?}{?}$ $\frac{10}{12}$ | $\frac{20}{24} = \frac{20 \div ?^{4}}{24 \div ?} = \frac{?}{?}$ $\frac{5}{6}$ |

**2.** Use the greatest common factor to find the simplest fractions.

**a** $\frac{15}{35}$ $\frac{3}{7}$ **b** $\frac{20}{60}$ $\frac{1}{3}$ **c** $\frac{16}{28}$ $\frac{4}{7}$ **d** $\frac{27}{36}$ $\frac{3}{4}$ **e** $\frac{48}{18}$ $2\frac{2}{3}\left(\frac{8}{3}\right)$ **f** $\frac{42}{36}$ $1\frac{1}{6}\left(\frac{7}{6}\right)$ **g** $\frac{25}{40}$ $\frac{5}{8}$ **h** $\frac{21}{35}$ $\frac{3}{5}$

**3.** Which of these fractions have a numerator and a denominator that have 1 as the greatest common factor?

**a** $\frac{3}{7}$ **b** $\frac{3}{8}$ **c** $\frac{4}{6}$ **d** $\frac{5}{12}$ **e** $\frac{2}{9}$ **f** $\frac{6}{12}$ **g** $\frac{3}{12}$ **h** $\frac{7}{11}$

**4.** Write six fractions using numbers in both the numerator and the denominator that have 1 as the greatest common factor. Answers will vary. Example: $\frac{2}{3}, \frac{5}{7}, \frac{7}{11}$, and so on

**5.** Write six fractions using numbers in the numerator and the denominator that have a common factor greater than 1. Answers will vary. Example $\frac{2}{4}, \frac{3}{6}, \frac{12}{16}$, and so on

**6.** In 4 and 5 did you use prime numbers for both the numerator and denominator of any fraction? What is the greatest common factor of any two prime numbers? Answers will vary. 1

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**1.** Which of these fractions is in simplest form?

**a** $\frac{1}{2}$ **b** $\frac{7}{14}$ **c** $\frac{11}{12}$ **d** $\frac{22}{24}$ **e** $\frac{3}{4}$ **f** $\frac{6}{12}$

**2.** Does $\frac{1}{2}$ name the same fractional part as $\frac{7}{14}$? Yes

**a** Does $\frac{11}{12}$ name the same fractional part as $\frac{22}{24}$? Yes

**b** Does $\frac{3}{4}$ name the same fractional part as $\frac{6}{12}$? No

**3.** If two or more fractions name the same fractional part, they are equivalent.

**a** Sometimes you need an equivalent fraction with a numerator and a denominator larger than those of the original fraction.

For example: $\frac{1}{2} + \frac{1}{4} = \underline{?}$

You need a common denominator.
You need an equivalent fraction.
The denominator must be 4.

Here's a "how to do it" review.

$\frac{1}{2} = \frac{?}{4}$ $\frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4}$

Had you needed a denominator still larger, this is what you'd do.

**Think** $\frac{1}{2} = \frac{?}{12}$
└—2 times what factor = 12? $\frac{1}{2} = \frac{1 \cdot 6}{2 \cdot 6} = \frac{6}{12}$

**b** Sometimes you need an equivalent fraction with a numerator and a denominator smaller than those of the original.

For example: $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ You need to rename the answer.
└—You need an equivalent fraction.

Here's another "how to do it" review.

**Think** $\frac{4}{6} = \frac{?}{?}$
└—What is the common factor of 4 and 6? $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

177

**lesson** Pages 177, 178, 179

**goal** Review of methods for renaming fractions

**page 177** Renaming must get pretty confusing at times for the youngsters. They have had lots of practice in earlier levels, but this review should help everyone. There is really only one thing to remember—whatever is done to the numerator must also be done to the denominator. You might even call this the golden rule for fractions. Everything the pupil needs for this review of renaming is right on the page.

**goal** Using renaming to add and subtract fractions with unlike denominators

**page 178** In the addition and subtraction of fractions with unlike denominators, the pupil is required to rename at least one of the fractions from one denominator to another specific denominator. The example illustrates this prerequisite renaming skill. Encourage performing this skill mentally if possible.

Use problem 1 to help you identify pupils who need more help and practice, and use problem 2 for that practice.

Some pupils may need a quick review of addition and subtraction with unlike denominators. Some youngsters will make mistakes because they failed to watch the operation symbols. Look for **why** a wrong answer was given. My, but working with decimals was nice!

When writing an equivalent fraction with a denominator greater than the one given, compare the denominators.

By what number has the denominator been multiplied?

$\frac{5}{6} = \frac{?}{12}$ $\quad$ $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

6 times what number gives 12? 2

Take that number times the numerator.

**1.** Complete.

**a** $\frac{2}{5} = \frac{?^{4}}{10}$ **b** $\frac{3}{4} = \frac{?^{9}}{12}$ **c** $\frac{5}{10} = \frac{?^{50}}{100}$ **d** $\frac{4}{5} = \frac{?^{8}}{10}$

**e** $\frac{3}{5} = \frac{?^{12}}{20}$ **f** $\frac{2}{3} = \frac{?^{10}}{15}$ **g** $\frac{3}{4} = \frac{?^{6}}{8}$ **h** $\frac{5}{6} = \frac{?^{20}}{24}$

**2.** More practice. Complete.

**a** $\frac{1}{4} = \frac{?^{5}}{20}$ **b** $\frac{1}{2} = \frac{?^{5}}{10}$ **c** $\frac{2}{3} = \frac{?^{8}}{12}$ **d** $\frac{3}{8} = \frac{?^{9}}{24}$

**e** $\frac{4}{7} = \frac{?^{8}}{14}$ **f** $\frac{1}{12} = \frac{?^{2}}{24}$ **g** $\frac{3}{4} = \frac{?^{12}}{16}$ **h** $\frac{2}{3} = \frac{?^{6}}{9}$

**3.** Apply this skill: Addition and subtraction problems require renaming. Watch the operation symbols.

**a** $\frac{2}{5} + \frac{3}{10}$ $\frac{7}{10}$ **b** $\frac{5}{6} + \frac{1}{12}$ $\frac{11}{12}$ **c** $\frac{5}{10} - \frac{1}{100}$ $\frac{49}{100}$ **d** $\frac{4}{5} - \frac{1}{10}$ $\frac{7}{10}$

**e** $\frac{3}{5} - \frac{1}{4}$ $\frac{7}{20}$ **f** $\frac{2}{3} - \frac{1}{5}$ $\frac{7}{15}$ **g** $\frac{3}{4} - \frac{3}{8}$ $\frac{3}{8}$ **h** $\frac{1}{6} + \frac{1}{4}$ $\frac{5}{12}$

**i** $\frac{1}{4} + \frac{1}{10}$ $\frac{7}{20}$ **j** $\frac{1}{2} - \frac{2}{5}$ $\frac{1}{10}$ **k** $\frac{2}{3} - \frac{1}{4}$ $\frac{5}{12}$ **l** $\frac{3}{8} + \frac{1}{6}$ $\frac{13}{24}$

178

To find the simplest name for any fraction, *divide* both numerator and denominator *by the same number.*

## THINK

What is the GREATEST common factor of numerator and denominator?

$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$ — 2 is the greatest common factor.

$\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$ — 5 is the greatest common factor.

1. Write the simplest name for each fraction.

a $\frac{4}{8}$ $\frac{1}{2}$ b $\frac{3}{6}$ $\frac{1}{2}$ c $\frac{6}{10}$ $\frac{3}{5}$ d $\frac{10}{10}$ 1 e $\frac{6}{15}$ $\frac{2}{5}$ f $\frac{16}{28}$ $\frac{4}{7}$ g $\frac{15}{30}$ $\frac{1}{2}$ h $\frac{12}{16}$ $\frac{3}{4}$

2. More practice. Write the simplest name for each fraction.

a $\frac{15}{18}$ $\frac{5}{6}$ b $\frac{10}{24}$ $\frac{5}{12}$ c $\frac{18}{20}$ $\frac{9}{10}$ d $\frac{8}{10}$ $\frac{4}{5}$ e $\frac{10}{16}$ $\frac{5}{8}$ f $\frac{16}{20}$ $\frac{4}{5}$ g $\frac{12}{18}$ $\frac{2}{3}$ h $\frac{9}{15}$ $\frac{3}{5}$

## PROGRESS CHECK

Apply this skill: Rename each answer in these addition and subtraction problems.

Skills: Adding and subtracting fractions with like denominators; renaming answers

(1.) $\frac{7}{8} - \frac{1}{8}$ $\frac{3}{4}(\frac{6}{8})$ (2.) $\frac{5}{6} - \frac{2}{6}$ $\frac{1}{2}(\frac{3}{6})$ (3.) $\frac{9}{10} - \frac{3}{10}$ $\frac{3}{5}(\frac{6}{10})$ (4.) $\frac{3}{10} + \frac{7}{10}$ $1(\frac{10}{10})$ (5.) $\frac{2}{5} + \frac{3}{5}$ $1(\frac{5}{5})$ (6.) $\frac{3}{4} + \frac{3}{4}$ $1\frac{1}{2}(\frac{6}{4})$

Skills: Adding and subtracting fractions with unlike denominators; renaming answers

You'll need a common denominator on these. Use two renaming skills.

(7.) $\frac{1}{2} + \frac{3}{9}$ $\frac{5}{6}(\frac{15}{18})$ (8.) $\frac{3}{8} + \frac{1}{6}$ $\frac{13}{24}$ (9.) $\frac{2}{4} + \frac{2}{5}$ $\frac{9}{10}(\frac{18}{20})$ (10.) $\frac{6}{10} - \frac{1}{5}$ $\frac{2}{5}(\frac{4}{10})$ (11.) $\frac{5}{6} - \frac{1}{3}$ $\frac{1}{2}(\frac{3}{6})$ (12.) $\frac{1}{2} - \frac{1}{10}$ $\frac{2}{5}(\frac{4}{10})$

179

See activity 1, page 193a.

See activity 2, page 193a.

**goal** Practice in finding the simplest name for a fraction; **Progress Check** – adding and subtracting fractions, renaming answers

**page 179** Renaming a fraction with its simplest name is used basically with answers. To be truthful, one name is as good as another (as long as the names are equivalent), but writing answers in simplest form makes communication (and checking) far easier. This renaming skill should be performed mentally.

Encourage youngsters who have trouble naming the greatest common factor for the numerator and the denominator of a fraction to use the chart on page 174. Again use problem 1 to identify ability and problem 2 for extra help and practice. Those who have no trouble with problem 1 can go directly to the Progress Check. The skills reviewed and developed thus far in the chapter are used in computation in the Progress Check.

In problems 1 through 6, watch for the following computational error:

$\frac{7}{8} - \frac{1}{8} = \frac{6}{16}$ or $\frac{3}{10} + \frac{7}{10} = \frac{10}{20}$

These pupils need individual help.

The focus is on renaming the answer in simplest form. Trouble? Provide more practice in finding the greatest common factor and dividing.

Problems 7 through 12 have two trouble spots:

- Renaming with a common denominator
- Renaming answer in simplest form

Identify the difficulty and assign a peer tutor.

**goal** Review of renaming mixed numbers as fractions and fractions as mixed or whole numbers

**page 180** Two prerequisite skills to review and check out before going to multiplication of fractions. Most pupils will have met these skills several times since the third or fourth level. Many will have caught on to the shortcuts. Great! But let shortcuts be their discovery.

One row in each set of problems will check on ability. Reserve the remaining problems for additional help and practice if needed.

Whole numbers need not be renamed as fractions for multiplication; however, some fractions can be renamed as whole numbers. This can happen in renaming products. Problem **2c** checks this special skill.

When you rename a mixed number as a fraction, focus on the denominator of the fraction part.

**THINK** I must rename the whole number in the same number of fractional parts as the denominator of the fraction.

$1\frac{3}{4} = \frac{4}{4} + \frac{3}{4}$ or $\frac{7}{4}$
Rename 1 as $\frac{4}{4}$, and add the $\frac{3}{4}$ already there.

$2\frac{2}{3} = \frac{6}{3} + \frac{2}{3}$ or $\frac{8}{3}$
Rename the 2 as $\frac{6}{3}$, and add the $\frac{2}{3}$ already there.

**1.** Rename as a fraction.

**a** $1\frac{1}{3}$ $\frac{4}{3}$ **b** $5\frac{1}{2}$ $\frac{11}{2}$ **c** $2\frac{3}{5}$ $\frac{13}{5}$ **d** $4\frac{3}{4}$ $\frac{19}{4}$ **e** $1\frac{3}{10}$ $\frac{13}{10}$

**f** $3\frac{7}{8}$ $\frac{31}{8}$ **g** $4\frac{4}{5}$ $\frac{24}{5}$ **h** $6\frac{5}{8}$ $\frac{53}{8}$ **i** $2\frac{5}{6}$ $\frac{17}{6}$ **j** $2\frac{7}{10}$ $\frac{27}{10}$

**k** $2\frac{1}{5}$ $\frac{11}{5}$ **l** $2\frac{1}{2}$ $\frac{5}{2}$ **m** $4\frac{1}{3}$ $\frac{13}{3}$ **n** $1\frac{3}{8}$ $\frac{11}{8}$ **o** $3\frac{1}{4}$ $\frac{13}{4}$

When you have a fraction with a numerator that is larger than the denominator, focus on the denominator. That's your start.

**THINK** Take out any whole numbers, and the parts that are left are a fraction.

$\frac{7}{4} = 1\frac{3}{4}$
How many four-fourths? Just 1, and $\frac{3}{4}$ remain.

$\frac{22}{5} = 4\frac{2}{5}$
How many five-fifths? There are 4, and $\frac{2}{5}$ remain

**2.** Rename as a whole or a mixed number.

**a** $\frac{3}{2}$ $1\frac{1}{2}$ **b** $\frac{10}{3}$ $3\frac{1}{3}$ **c** $\frac{12}{4}$ 3 **d** $\frac{15}{8}$ $1\frac{7}{8}$ **e** $\frac{13}{4}$ $3\frac{1}{4}$

**f** $\frac{5}{3}$ $1\frac{2}{3}$ **g** $\frac{11}{4}$ $2\frac{3}{4}$ **h** $\frac{17}{10}$ $1\frac{7}{10}$ **i** $\frac{15}{4}$ $3\frac{3}{4}$ **j** $\frac{15}{6}$ $2\frac{1}{2}$

180

See activity 3, page 193a and activity 4, page 193b.

The main application of renaming mixed numbers as fractions and renaming fractions as mixed numbers is found in multiplication.

Multiply. Write each product in simplest form.

**1.** $3 \times \frac{2}{7}$ $\frac{6}{7}$ **2.** $2 \times \frac{6}{8}$ $1\frac{1}{2}$ **3.** $5 \times \frac{3}{16}$ $\frac{15}{16}$ **4.** $7 \times \frac{2}{3}$ $4\frac{2}{3}$

**5.** $1 \times \frac{5}{6}$ $\frac{5}{6}$ **6.** $4 \times \frac{1}{4}$ $1$ **7.** $4 \times \frac{3}{4}$ $3$ **8.** $6 \times \frac{2}{3}$ $4$

The answers are on page 193. Check your work.

Barry got paid for working $\frac{2}{3}$ of a day for 3 days each week. How many full days of pay did he get?

You can solve by adding. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2 + 2 + 2}{3} = \frac{6}{3}$ or 2 days of pay

Or you can solve by multiplying. $3 \times \frac{2}{3} = \frac{3 \times 2}{3} = \frac{6}{3}$ or 2 days of pay

**1.** Solve these by adding. Write the simplest name for the sum.

**a** $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = ?$ $2\frac{1}{4}$ **b** $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = ?$ $1\frac{1}{7}$ **c** $\frac{5}{8} + \frac{5}{8} = ?$ $1\frac{1}{4}$

**2.** Solve these by multiplying. Write the simplest name, please.

**a** $4 \times \frac{2}{7} = \frac{4 \times 2}{7} = ?$ $1\frac{1}{7}$ **b** $2 \times \frac{5}{8} = ?$ $1\frac{1}{4}$ **c** $3 \times \frac{3}{4} = ?$ $2\frac{1}{4}$

1a with 2c
1b with 2a
1c with 2b

**3.** Find matching problems in 1 and 2. Do your answers match too?

Go on to the next page.



**lesson** Pages 181, 182

**goal** Diagnosis of ability and help in multiplying a fraction by a whole number

**memo** Pages 181 through 188 are self-directive. Pupils have worked with this sort of page organization before. Check to make sure that everyone follows the chart correctly.

**page 181** Check learners who made errors on problems 1 through 8. If the error was made in renaming to simplest form, practice this skill rather than multiplication. For example:

$4 \times \frac{3}{4} = \frac{4 \times 3}{4} = \frac{12}{4}$ Computation correct

$\frac{12 \div 2}{4 \div 2} = \frac{6}{2}$ Renaming incomplete

**goal** Practice in multiplying a fraction by a whole number

**page 182** Adjust the assignment for pupils who were directed to additional help. Problems 2 through 6 provide a challenge for pupils who are ready to apply their skills.

**1.** Multiply.

**a** $5 \times \frac{1}{6}$ $\frac{5}{6}$ **b** $4 \times \frac{2}{9}$ $\frac{8}{9}$ **c** $3 \times \frac{3}{11}$ $\frac{9}{11}$ **d** $2 \times \frac{3}{10}$ $\frac{3}{5}$

**e** $2 \times \frac{2}{3}$ $1\frac{1}{3}$ **f** $3 \times \frac{4}{5}$ $2\frac{2}{5}$ **g** $8 \times \frac{2}{9}$ $1\frac{7}{9}$ **h** $5 \times \frac{7}{10}$ $3\frac{1}{2}$

You should be getting good. Try these problems. Write the simplest name for your answers.

**2.** Mike always spent $\frac{3}{4}$ of his weekly pocket money of $1.60. How much did he spend in February? $4.80

**3.** Sleepy spent $\frac{13}{24}$ of every day sleeping. During 2 days he slept a total time equal to how many days? $1\frac{1}{12}$ days

**4.** Kathy spent $\frac{1}{4}$ of an hour every day on her music practice. How many hours did she spend on music practice during 4 days? 1 hour

**5.** Mary spent $\frac{1}{4}$ of an hour walking to school each morning, and the same time in going home in the afternoon. How much time did she spend each week going to and from school? $2\frac{1}{2}$ ho

**6.** The bakery sold pies whole or by the portion. In one day the bakery sold five $\frac{1}{2}$-portions and nine $\frac{1}{4}$-portions. How many pies did they sell in all?

$4\frac{3}{4}$ pies

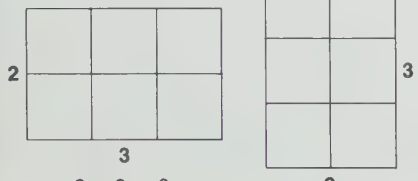
182

Multiply. Write each product in simplest form.

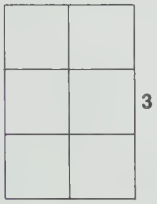
**1.** $\frac{1}{8} \times 5$ $\frac{5}{8}$ **2.** $\frac{2}{5} \times 2$ $\frac{4}{5}$ **3.** $\frac{2}{3} \times 6$ 4 **4.** $\frac{2}{3} \times 3$ 2 **5.** $\frac{1}{2} \times 5$ $2\frac{1}{2}$ **6.** $\frac{3}{7} \times 3$ $1\frac{2}{7}$ **7.** $\frac{7}{8} \times 5$ $4\frac{3}{8}$ **8.** $\frac{3}{4} \times 6$ $4\frac{1}{2}$

The answers are on page 193. Check your work.

You know this.



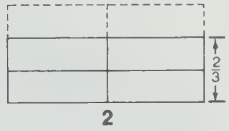

$2 \times 3 = 6$




$3 \times 2 = 6$

Does the order of factors change the product when you operate with fractions?

$2 \times \frac{2}{3} = \frac{2 \times 2}{3} = \frac{4}{3}$ or $1\frac{1}{3}$ square units




$\frac{2}{3} \times 2 = \frac{2 \times 2}{3} = \frac{4}{3}$ or $1\frac{1}{3}$ square units

Multiply. Write each product in simplest form.

**1.** $3 \times \frac{1}{7} = \frac{3 \cdot 1}{7} = \frac{?}{} \; \frac{3}{7}$ **2.** $\frac{1}{7} \times 3 = \frac{1 \cdot 3}{7} = \frac{?}{} \; \frac{3}{7}$ **3.** $5 \times \frac{2}{9} = \frac{5 \times 2}{9} = \frac{?}{} \; 1\frac{1}{9}$ **4.** $\frac{2}{9} \times 5 = \frac{2 \cdot 5}{9} = \frac{?}{} \; 1\frac{1}{9}$

**5.** $2 \times \frac{1}{2} = \frac{2 \times 1}{2} = \frac{?}{} \; 1$ **6.** $\frac{1}{2} \times 2 = \frac{1 \cdot 2}{2} = \frac{?}{} \; 1$ **7.** $4 \times \frac{2}{3} = \frac{4 \times 2}{3} = \frac{?}{} \; 2\frac{2}{3}$ **8.** $\frac{2}{3} \times 4 = \frac{2 \times 4}{3} = \frac{?}{} \; 2\frac{2}{3}$

Go on to the next page.



**lesson** Pages 183, 184

**goal** Diagnosis of ability and help in multiplying a whole number by a fraction

**page 183** Again check errors on the diagnostic check. Make sure the pupil who makes mistakes works on the correct skill:

- Multiplication error
- Renaming error

**goal** Practice in multiplying a whole number by a fraction

**page 184** Adjust assignments to meet individual needs. Once the pupil is confident of his skill, have him apply it to some of the word problems.

# Multiply

**1.** **a** $\frac{3}{4} \times 2 = \frac{3 \times 2}{\square} = ?$ $1\frac{1}{2}$ **b** $\frac{2}{3} \times 7 = \frac{\overset{2}{\square} \times \overset{7}{\triangle}}{3} = ?$ $4\frac{2}{3}$ **c** $\frac{7}{8} \times 5$ $4\frac{3}{8}$

**d** $\frac{9}{10} \times 3$ $2\frac{7}{10}$ **e** $\frac{7}{6} \times 4$ $4\frac{2}{3}$ **f** $\frac{11}{10} \times 10$ 11 **g** $\frac{1}{5} \times 5$ 1 **h** $\frac{2}{5} \times 5$ 2

**i** $\frac{3}{5} \times 5$ 3 **j** $\frac{4}{5} \times 5$ 4 **k** $\frac{1}{10} \times 5$ $\frac{1}{2}$ **l** $\frac{3}{10} \times 5$ $1\frac{1}{2}$ **m** $\frac{7}{10} \times 5$ $3\frac{1}{2}$

**n** $\frac{9}{10} \times 5$ $4\frac{1}{2}$ **o** $\frac{1}{3} \times 6$ 2 **p** $\frac{2}{3} \times 6$ 4 **q** $\frac{1}{4} \times 8$ 2 **r** $\frac{3}{4} \times 8$ 6

Continue. Write the simplest name for your answers.

**2.** Kay had \$3. She spent $\frac{1}{3}$ of it. How much did she spend ? \$1

**3.** Chris had 12 problems to do. She completed $\frac{3}{4}$ of them before her favorite TV show came on. How many problems had she done? 9 problems

**4.** Steve ran the race in 7 minutes last year. This year he ran it in $\frac{3}{4}$ of that time. How many minutes did he run the race in this year? $5\frac{1}{4}$ minutes

**5.** Four people each worked $\frac{3}{4}$ of an hour overtime. How many hours of overtime must the company pay for? 3 hours

**6.** She worked $\frac{1}{2}$ an hour overtime every day for 6 days. How many hours overtime did she work? 3 hours

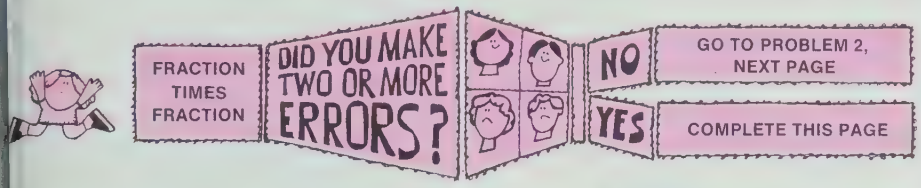
184

Multiply. Write each product in simplest form.

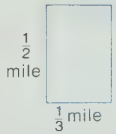
1. $\frac{1}{4} \times \frac{1}{4}$ $\frac{1}{16}$ 2. $\frac{1}{2} \times \frac{1}{2}$ $\frac{1}{4}$ 3. $\frac{1}{4} \times \frac{3}{4}$ $\frac{3}{16}$ 4. $\frac{3}{4} \times \frac{3}{4}$ $\frac{9}{16}$

5. $\frac{2}{3} \times \frac{1}{3}$ $\frac{2}{9}$ 6. $\frac{1}{4} \times \frac{2}{3}$ $\frac{1}{6}$ 7. $\frac{2}{5} \times \frac{2}{3}$ $\frac{4}{15}$ 8. $\frac{3}{5} \times \frac{4}{5}$ $\frac{12}{25}$

The answers are on page 193.
Check your work.

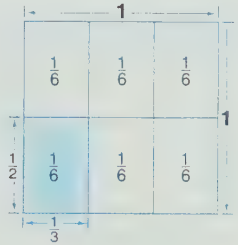



The area of this field is equal to the product of $\frac{1}{2} \times \frac{1}{3}$.
The unit of measure is a mile. The field pictured is part of a piece of land 1 mile long and 1 mile wide.



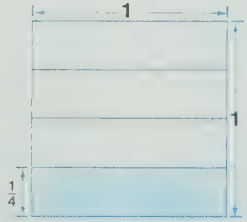

The area of the whole field is 1 mile × 1 mile, or 1 square mile. The entire field has been divided into 6 fields of equal size. Each field is $\frac{1}{2}$ mile × $\frac{1}{3}$ mile.

Therefore the area of one of the six fields is $\frac{1}{2} \times \frac{1}{3}$ or $\frac{1}{6}$ square mile.




What is the area of a rectangle $\frac{1}{4} \times \frac{2}{3}$?
Get out a sheet of paper.

Step 1. Draw a unit square.
Step 2. Divide it into 4 equal parts. Then shade 1 of the 4 parts. This shows $\frac{1}{4} \times 1$.




Go on to the next page.



**lesson** Pages 185, 186

**goal** Diagnosis of ability and help in multiplying two fractions

**page 185** Check pupils' errors. Has someone confused the addition and subtraction algorithm with the algorithm for multiplication?

$\frac{1}{4} \times \frac{3}{4} = \frac{4}{4}$ Pupil added, did not multiply.

$\frac{1}{4} \times \frac{2}{3} = \frac{3}{12} \times \frac{8}{12} = \frac{24}{12}$ Pupil changed to a common denominator, then worked with numerators only.

These pupils need your help!

Pupils who made multiplication errors should work on the whole-number facts. Those who multiplied correctly but renamed the products in simplest form incorrectly should work on renaming only.

Be ready to help interpret the models if necessary. Examine the denominator of the product. The greater the denominator, the smaller the pieces. Does the model verify this? As the number of pieces increased, what happened to the size of the pieces? The final model is on page 186.

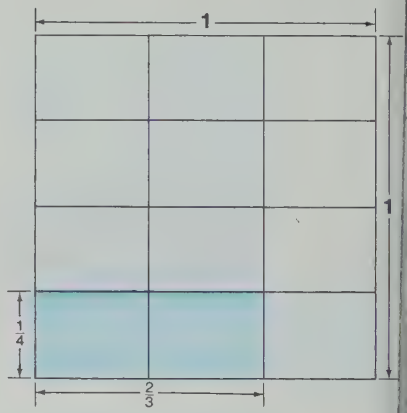
**goal** Practice in multiplying two fractions

**page 186** Some of your pupils probably are able to compute mentally and write only:

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$ That's GOOD!

Others who use this shorter form may make errors simply because they are not ready for it. You will know who needs to write all the steps as shown on the page. The extra step helps focus attention on the operation and on multiplying both the numerator and the denominator. Maybe after the practice they will be ready for the shortcut.

Step 3. Divide the square vertically into 3 equal parts.
Then shade 2 of the 3 parts.




The part of the square with double shading is $\frac{1}{4} \times \frac{2}{3}$.
Its area is 2 of the 12 parts or $\frac{2}{12}$ of the unit square.

So $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$ ← Look again at the equation.

Does $\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12}$? Yes. Can $\frac{2}{12}$ be renamed? Yes — as $\frac{1}{6}$

What is the area of a region $\frac{2}{3} \times \frac{3}{5}$?

$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \underline{?}$ $\frac{6}{15}$

**1.** Multiply. Write each product in simplest form.

**a** $\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \underline{?}$ $\frac{3}{10}$

**b** $\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \underline{?}$ $\frac{1}{3}$

**c** $\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \underline{?}$ $\frac{1}{8}$

**d** $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \underline{?}$ $\frac{1}{2}$

**e** $\frac{4}{7} \times \frac{1}{3} = \frac{4 \times 1}{\square \times \triangle} = \underline{?}$ ($\square = 7$, $\triangle = 3$) $\frac{4}{21}$

**f** $\frac{3}{5} \times \frac{5}{8} = \frac{\square \times \triangle}{5 \times 8} = \underline{?}$ ($\square = 3$, $\triangle = 5$) $\frac{3}{8}$

**g** $\frac{6}{7} \times \frac{1}{4} = \underline{?}$ $\frac{3}{14}$

**h** $\frac{4}{5} \times \frac{5}{4} = \underline{?}$ 1

**i** $\frac{1}{6} \times \frac{3}{4} = \underline{?}$ $\frac{1}{8}$

**2.** Half of the cake was eaten. He ate $\frac{1}{2}$ of that.
What part of the whole cake did he eat? $\frac{1}{4}$ of the cake

**3.** She decided it would take her $\frac{3}{4}$ of an hour to get there.
She spent $\frac{1}{2}$ of that time waiting for a bus.
What fractional part of an hour did she wait? $\frac{3}{8}$ of an hour

186

Multiply. Write each product in simplest form.

**1.** $\frac{2}{3} \times 1\frac{1}{4}$ $\frac{5}{6}$ **2.** $2\frac{1}{3} \times \frac{2}{7}$ $\frac{2}{3}$ **3.** $1\frac{1}{4} \times 2\frac{1}{2}$ $3\frac{1}{8}$ **4.** $1\frac{1}{3} \times 2\frac{1}{4}$ $3$ **5.** $2\frac{1}{5} \times 1\frac{1}{2}$ $3\frac{3}{10}$

**6.** $4 \times 1\frac{1}{3}$ $5\frac{1}{3}$ **7.** $1\frac{1}{10} \times \frac{10}{10}$ $1\frac{1}{10}$ **8.** $1\frac{1}{3} \times \frac{3}{4}$ $1$ Check your answers on page 193.

The action of multiplying mixed numbers is exactly the same as it is for multiplying fractions. Change the mixed number to a fraction and go!

$1\frac{1}{2} \times \frac{1}{2} = ?$ *Rename* $1\frac{1}{2}$ as a fraction: $\frac{3}{2}$ $\quad \frac{3}{2} \times \frac{1}{2} = \frac{3 \cdot 1}{2 \cdot 2} = \frac{3}{4}$

Look at another problem.

$2\frac{2}{3} \times 1\frac{1}{4} = ?$ *Rename* both mixed numbers as fractions.

$\frac{8}{3} \times \frac{5}{4} = \frac{8 \cdot 5}{3 \cdot 4} = \frac{40}{12}$ or $3\frac{4}{12}$ or $3\frac{1}{3}$

## Be prepared

The renaming sometimes takes longer than the multiplication itself. Multiply. Write each product in simplest form.

**1.** $1\frac{1}{4} \times 2\frac{1}{3}$ $2\frac{11}{12}$ **2.** $3\frac{1}{9} \times 1\frac{3}{4}$ $5\frac{4}{9}$ **3.** $2\frac{2}{3} \times 1\frac{1}{2}$ $4$ **4.** $3\frac{1}{4} \times 1\frac{3}{4}$ $5\frac{11}{16}$ **5.** $1\frac{1}{2} \times 1\frac{4}{5}$ $2\frac{7}{10}$

**6.** $2 \times 5\frac{5}{6}$ $11\frac{2}{3}$ **7.** $3\frac{1}{3} \times 2\frac{1}{6}$ $7\frac{2}{9}$ **8.** $2\frac{3}{8} \times 2\frac{1}{2}$ $5\frac{15}{16}$ **9.** $6\frac{1}{5} \times 1\frac{2}{7}$ $7\frac{34}{35}$ **10.** $1\frac{3}{4} \times \frac{1}{8}$ $\frac{7}{32}$



**goal** Diagnosis of ability and help in multiplying fractions and mixed numbers

**page 187** Check errors on the diagnostic check. Look particularly for three types of errors:

- Renaming a mixed number as a fraction (mastery of page 180 shows up here)
- Multiplication facts
- Renaming the product in simplest form

Provide the appropriate type of practice. The page will help with the algorithm itself and will provide additional practice. Adjust the assignment to meet individual needs.

**lesson** Pages 188, 189

**goal** Practice with fractions

**page 188** This is a fun page. Everyone should be able to handle it, but pupils can be assigned to work in pairs to increase the fun if they wish. Competition among the sharper pupils may become quite keen, to see who gets the solution first.

Pupils who miss words will be able to see their own errors very clearly, and perceive why they were wrong.

Go right on to the next page.

# sidetrack

To try these riddles on your friends, print the word in capital letters as you answer each sentence.

Riddle:

**a** The last $\frac{3}{4}$ of the word SHOW.

**b** The first $\frac{3}{5}$ of the word CANNY.

**c** The first $\frac{3}{8}$ of the word YOURSELF.

**d** The first $\frac{5}{6}$ of the word LEAVEN.

**e** The middle $\frac{3}{7}$ of the word HOTHEAD.

**f** The last $\frac{4}{5}$ of the word BROOM.

**g** The first $\frac{1}{2}$ of the word WITHDRAW.

**h** The middle $\frac{3}{7}$ of the word OUTWORK.

**i** The $\frac{1}{1}$ of the word LEGS.

**j** The middle $\frac{3}{5}$ of the word HANDY.

**k** The first $\frac{2}{3}$ of the word COMELY.

**l** The last $\frac{4}{7}$ of the word OUTBACK.

**m** The first $\frac{1}{2}$ of the word WITHERED.

**n** The first $\frac{3}{8}$ of the word SIXTYISH.

Answer:

**a** The last $\frac{1}{3}$ of the word SHABBY.

**b** The last $\frac{8}{10}$ of the word UPBRINGING.

**c** The middle $\frac{1}{5}$ of the word CHAIR.

**d** The first $\frac{5}{8}$ of the word CHAIRMAN.

**e** The first $\frac{1}{2}$ of the word BACKBONE.

Go on to the next riddle.

188

Riddle:

**a** The first $\frac{1}{2}$ of the word WHATEVER.

**b** The first $\frac{4}{5}$ of the word MUSTY.

**c** The first $\frac{3}{5}$ of the word YOURS.

**d** The middle $\frac{3}{5}$ of the word PADDY.

**e** The first $\frac{1}{2}$ of the word TOPS.

**f** The first $\frac{1}{2}$ of the word NINETEEN.

**g** The last $\frac{1}{2}$ of the word INTO.

**h** The first $\frac{4}{9}$ of the word MAKESHIFT.

**i** The second $\frac{1}{4}$ of the word FLITTING.

**j** The first $\frac{3}{5}$ of the word SIXTY.

Answer:

**a** The first $\frac{3}{6}$ of the word THEORY.

**b** The first $\frac{2}{3}$ of the word LETTERING.

**c** The last $\frac{1}{6}$ of the word SEVENS.

**d** The first $\frac{3}{7}$ of the word FORTUNE.

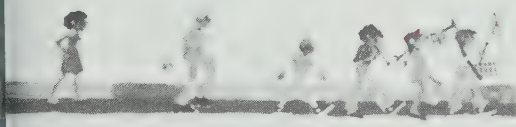
**e** The middle $\frac{1}{7}$ of the word HOUSING.

**f** The first $\frac{4}{5}$ of the word PLUSH.

**g** The middle $\frac{2}{4}$ of the word PIXY.

**h** The last $\frac{5}{7}$ of the word REMAKES.

**i** The first $\frac{3}{7}$ of the word SIXTEEN.



## PROGRESS CHECK

Multiply. Write the simplest name for each product.

**1.** $5 \times \frac{2}{5}$ $\left(\frac{10}{5}\right)$ 2

**2.** $\frac{3}{10} \times 5$ $\left(\frac{15}{10}\right)$ $1\frac{1}{2}$

**3.** $\frac{3}{4} \times \frac{2}{3}$ $\left(\frac{6}{12}\right)$ $\frac{1}{2}$

**4.** $1\frac{1}{2} \times \frac{4}{5}$ $\left(\frac{12}{10}\right)$ $1\frac{1}{5}$

**5.** $1\frac{1}{3} \times 2\frac{1}{4}$ $\left(\frac{36}{12}\right)$ 3

**6.** $2\frac{1}{2} \times 1\frac{3}{4}$ $\left(\frac{35}{8}\right)$ $4\frac{3}{8}$

**7.** $\frac{2}{5} \times 2\frac{1}{2}$ $\left(\frac{10}{10}\right)$ 1

**8.** $2\frac{1}{5} \times 2\frac{1}{2}$ $\left(\frac{11}{2}\right)$ $5\frac{1}{2}$

**9.** $3\frac{1}{6} \times 1\frac{1}{4}$ $\left(\frac{95}{24}\right)$ $3\frac{23}{24}$

**10.** $2\frac{3}{5} \times 1\frac{2}{3}$ $\left(\frac{13}{3}\right)$ $4\frac{1}{3}$

**11.** $3\frac{2}{3} \times 1\frac{1}{2}$ $\left(\frac{11}{2}\right)$ $5\frac{1}{2}$

**12.** $1\frac{1}{2} \times 4\frac{5}{6}$ $\left(\frac{29}{4}\right)$ $7\frac{1}{4}$

189

See activity 5, page 193b.

See activity 6, page 193b.

**goal** Practice with fractions

**Progress Check** — multiplying fractions, renaming products

**page 189** More of the same fun with riddles and fractions. Pupils may have trouble with the answer in this case, as it involves Roman numerals. The longer the frustrating period before they see the key to the solution, the greater will be their joy when they finally do see. Some may need to have it pointed out.

Note the range of factors used in the Progress Check.

| Problem | Type of factors |
|---|---|
| 1 | Whole number × fraction |
| 2 | Fraction × whole number |
| 3 | Fraction × fraction |
| 4 | Mixed number × fraction |
| 5,6 | Mixed number × mixed number |

Separate multiplication errors from renaming errors. Pupils who incorrectly multiply problems 1 and 2 may be having trouble handling a whole-number factor. Check problems 4, 5, and 6 for errors in renaming the mixed numbers.

(4) $\frac{3}{2} \times \frac{4}{5}$ (5) $\frac{4}{3} \times \frac{9}{4}$ (6) $\frac{5}{2} \times \frac{7}{4}$

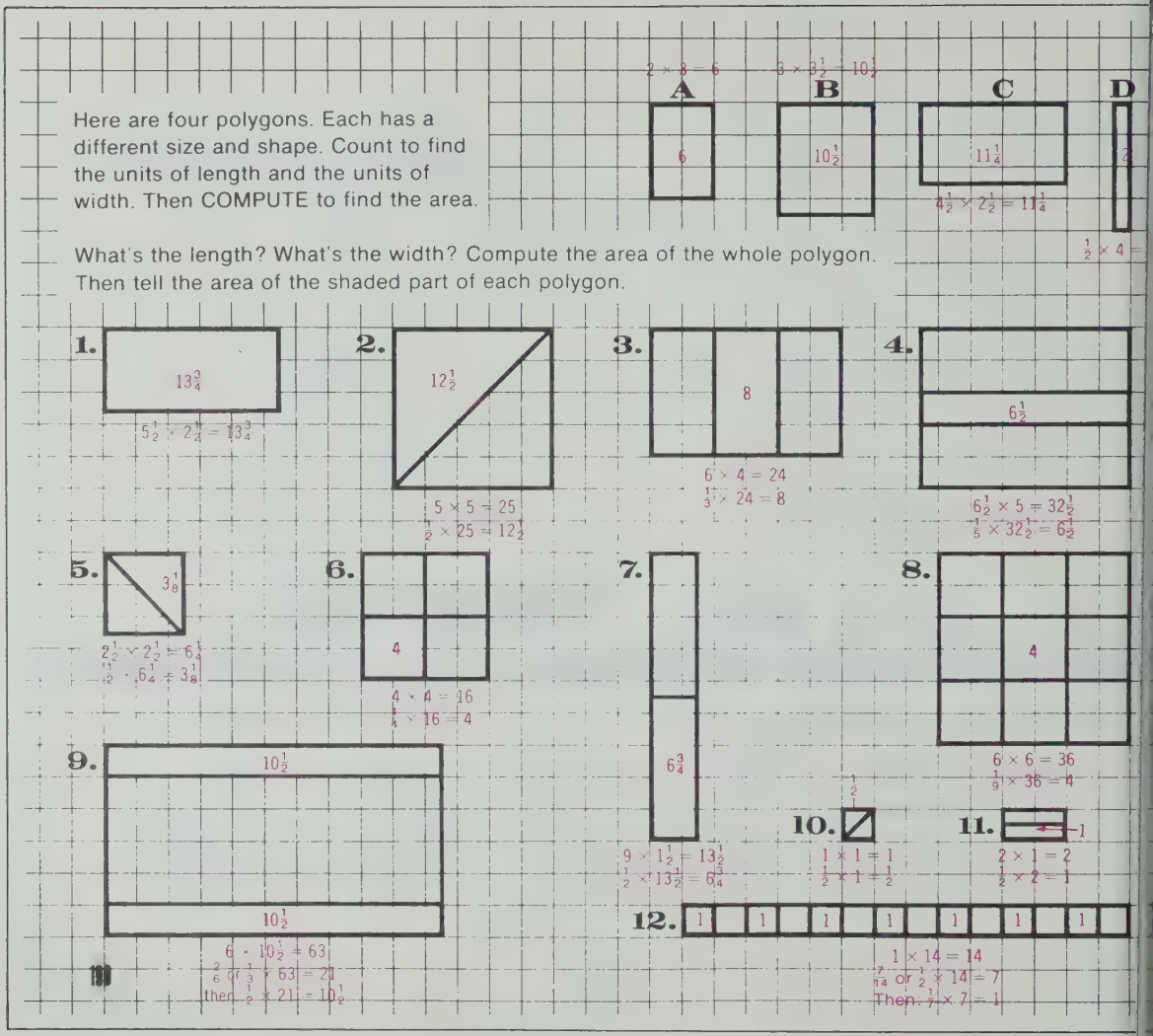
Nobody needs to do all the questions. The first row should be enough for most; your very ambitious pupils may try both rows.

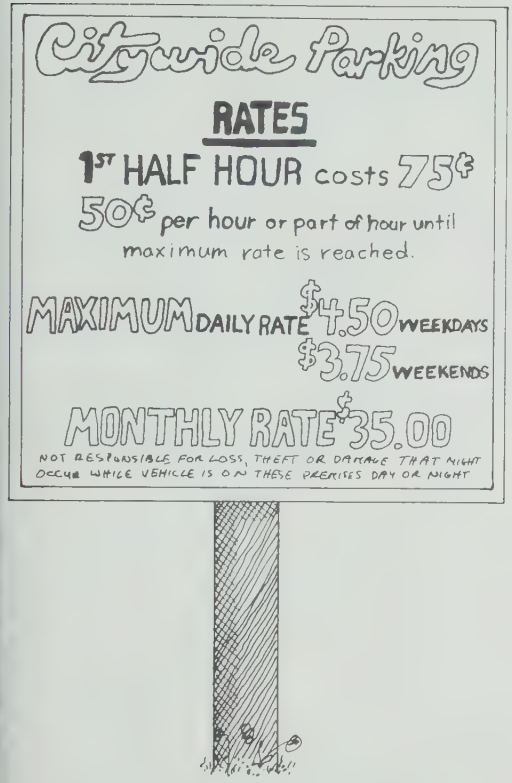
**lesson** Pages 190, 191

**goal** Computing the area of polygons

**page 190** Certainly the answers can be found by counting, **but** the directions say to **compute** the area. A review of the area algorithm may be necessary.

Finding the area of only part of a figure should not be too hard. If $\frac{1}{2}$ of the figure is shaded, some pupils will divide the area of the whole polygon by 2—that's O.K. Some sharpie may associate finding a half with multiplying the area by $\frac{1}{2}$—that's great! Challenge everyone to solve the problems in their own way, then discuss the various problem-solving methods.




Citywide Parking
RATES
1ST HALF HOUR costs 75¢
50¢ per hour or part of hour until maximum rate is reached.
MAXIMUM DAILY RATE $4.50 WEEKDAYS
$3.75 WEEKENDS
MONTHLY RATE $35.00
NOT RESPONSIBLE FOR LOSS, THEFT OR DAMAGE THAT MIGHT OCCUR WHILE VEHICLE IS ON THESE PREMISES DAY OR NIGHT

1. To park a car in a large city is quite expensive. Fortunately Ms. Bakuska has the monthly rate. If she works an average of 20 days a month, how much does she pay per day? $1.75

2. How much would you pay for 20 weekdays at the daily rate? $90

3. If you parked for 3 h on Saturday, how much would parking cost you? Are you under or over the maximum rate? $2.25 Under

4. Compute the parking costs for these people.
   - **a** Mr. Lei: $5\frac{1}{4}$ h on Tuesday. $3.25
   - **b** Miss Thibaudeau: $1\frac{3}{4}$ h on Friday. $1.75
   - **c** The McConnell family: 6 h on Wednesday. $3.75
   - **d** Mrs. Shipton: $6\frac{1}{4}$ h on Sunday. $3.75
   - **e** Mr. and Mrs. Logan: $8\frac{1}{2}$ h on Monday. $4.50
   - **f** Ms. Westington: Wednesday, Thursday, and Friday for $3\frac{1}{4}$ h each day. $6.75

5. Why do parking costs vary?
   Discuss all answers

191

**goal** Applying multiplication of fractions

**page 191** There is much computation on this page. Some of the questions look simple, but they actually involve two or three operations. Your most independent workers may manage on their own, but many students will need help with sorting out the sequence of operations.

Before they try on their own, be sure the pupils understand how to handle the higher price for the first half hour, and that they must calculate the amount of time left to be paid at the regular rate. They will also have to realize that people sometimes have to pay for a period longer than they actually park.

This page is a good demonstration of the fact that in applied mathematics one has to **think** as well as compute. Question 5 gives the opportunity for a different kind of thinking.

**lesson** Page 192

**goal** **Checkout**—multiplying fractions, renaming products

**page 192** Each column contains a different type of problem. Going across any row checks the pupil's ability on all four types of problem. There are far more problems than are needed to check out ability. Be selective. Reserve the remaining problems for additional practice or to check on progress.

Have pupils label each problem on the Checkout with the appropriate letter. This will help determine whether a pupil's weakness is one specific type of problem and will help direct him to additional practice.

Check for two types of errors:
- Multiplication facts
- Renaming skills

Self-directive help was given on the pupil page for each type of problem on the Checkout.

| Column | Pages |
|---|---|
| A | 181–182 |
| B | 183–184 |
| C | 185–186 |
| D | 180, 187 |

# CHECKOUT

Skills: Multiplying fractions; renaming answers

Pick any five rows of problems to complete. Multiply. Write the simplest name for each product.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | a | $3 \times \frac{2}{3}$ | 2 | b | $\frac{4}{5} \times 6$ | $4\frac{4}{5}$ | c | $\frac{2}{3} \times \frac{1}{4}$ | $\frac{1}{6}$ | d | $5\frac{3}{5} \times 2$ | |
| 2. | a | $4 \times \frac{3}{4}$ | 3 | b | $\frac{2}{3} \times 4$ | $2\frac{2}{3}$ | c | $\frac{5}{6} \times \frac{7}{5}$ | $1\frac{1}{6}$ | d | $2\frac{4}{5} \times 3$ | |
| 3. | a | $6 \times \frac{6}{7}$ | $5\frac{1}{7}$ | b | $\frac{8}{9} \times 3$ | $2\frac{2}{3}$ | c | $\frac{5}{12} \times \frac{4}{5}$ | $\frac{1}{3}$ | d | $3\frac{1}{3} \times 1$ | |
| 4. | a | $9 \times \frac{5}{9}$ | 5 | b | $\frac{2}{11} \times 5$ | $\frac{10}{11}$ | c | $\frac{1}{5} \times \frac{2}{3}$ | $\frac{2}{15}$ | d | $1\frac{5}{6} \times 3$ | |
| 5. | a | $5 \times \frac{4}{5}$ | 4 | b | $\frac{10}{11} \times 5$ | $4\frac{6}{11}$ | c | $\frac{4}{5} \times \frac{3}{8}$ | $\frac{3}{10}$ | d | $\frac{3}{8} \times 4\frac{1}{4}$ | |
| 6. | a | $7 \times \frac{2}{7}$ | 2 | b | $\frac{3}{16} \times 4$ | $\frac{3}{4}$ | c | $\frac{6}{10} \times \frac{2}{3}$ | $\frac{2}{5}$ | d | $\frac{5}{6} \times 3\frac{1}{9}$ | |
| 7. | a | $10 \times \frac{7}{10}$ | 7 | b | $\frac{2}{5} \times 6$ | $2\frac{2}{5}$ | c | $\frac{1}{7} \times \frac{3}{3}$ | $\frac{1}{7}$ | d | $\frac{3}{4} \times 2\frac{2}{5}$ | |
| 8. | a | $8 \times \frac{7}{8}$ | 7 | b | $\frac{3}{4} \times 12$ | 9 | c | $\frac{1}{10} \times \frac{2}{7}$ | $\frac{1}{35}$ | d | $\frac{2}{3} \times 4\frac{3}{4}$ | |

If you make two or more errors on an **a** problem, do all the other **a**'s for practice. If you make two or more errors on **b**, do other **b**'s for practice. You know the pattern. If you need the practice, do it now.

192

See activity 7, page 193b.

See activity 8, page 193b.

## ANSWERS

### ANSWERS TO PROBLEMS ON PAGE 180.

1. $\frac{6}{7}$ 2. $1\frac{1}{2}$ 3. $\frac{15}{16}$ 4. $4\frac{2}{3}$
5. $\frac{5}{6}$ 6. 1 7. 3 8. 4

### ANSWERS TO PROBLEMS ON PAGE 183.

1. $\frac{5}{8}$ 2. $\frac{4}{5}$ 3. 4 4. 2
5. $2\frac{1}{2}$ 6. $1\frac{2}{7}$ 7. $4\frac{3}{8}$ 8. $4\frac{1}{2}$

### ANSWERS TO PROBLEMS ON PAGE 185.

1. $\frac{1}{16}$ 2. $\frac{1}{4}$ 3. $\frac{3}{16}$ 4. $\frac{9}{16}$
5. $\frac{2}{9}$ 6. $\frac{1}{6}$ 7. $\frac{4}{15}$ 8. $\frac{12}{25}$

### ANSWERS TO PROBLEMS ON PAGE 187.

1. $\frac{5}{6}$ 2. $\frac{2}{3}$ 3. $3\frac{1}{8}$ 4. 3
5. $3\frac{3}{10}$ 6. $5\frac{1}{3}$ 7. $1\frac{1}{10}$ 8. 1



page 193

Nothing needs to be said here . . .

# RESOURCES

## another form of evaluation

**for progress check** — page 179

Apply this skill: Rename each answer in these addition and subtraction problems.

1. $\frac{8}{9} - \frac{5}{9}$ $(\frac{3}{9})$ $\frac{1}{3}$ 2. $\frac{7}{10} - \frac{3}{10}$ $(\frac{4}{10})$ $\frac{2}{5}$ 3. $\frac{1}{6} + \frac{5}{6}$ $(\frac{6}{6})$ 1

4. $\frac{3}{8} + \frac{1}{8}$ $(\frac{4}{8})$ $\frac{1}{2}$ 5. $\frac{11}{12} - \frac{5}{12}$ $(\frac{6}{12})$ $\frac{1}{2}$ 6. $\frac{5}{16} + \frac{7}{16}$ $(\frac{12}{16})$ $\frac{3}{4}$

You'll need a common denominator on these. Use two renaming skills.

7. $\frac{1}{3} + \frac{2}{4}$ $(\frac{10}{12})$ $\frac{5}{6}$ 8. $\frac{9}{10} - \frac{2}{5}$ $(\frac{5}{10})$ $\frac{1}{2}$ 9. $\frac{5}{12} + \frac{1}{4}$ $(\frac{8}{12})$ $\frac{2}{3}$

10. $\frac{3}{9} + \frac{1}{2}$ $(\frac{15}{18})$ $\frac{5}{6}$ 11. $\frac{3}{8} - \frac{1}{6}$ $(\frac{10}{48})$ $\frac{5}{24}$ 12. $\frac{5}{6} - \frac{1}{4}$ $(\frac{14}{24})$ $\frac{7}{12}$

**for progress check** — page 189

Multiply. Write the simplest name for each product.

1. $4 \times \frac{3}{4}$ $(\frac{12}{4})$ 3 2. $\frac{7}{8} \times 4$ $(\frac{28}{8})$ $3\frac{1}{2}$ 3. $\frac{3}{5} \times \frac{5}{6}$ $(\frac{15}{30})$ $\frac{1}{2}$

4. $1\frac{1}{3} \times \frac{3}{4}$ $(\frac{12}{12})$ 1 5. $1\frac{1}{2} \times 2\frac{1}{4}$ $(\frac{27}{8})$ $3\frac{3}{8}$ 6. $2\frac{1}{4} \times 1\frac{2}{3}$ $(\frac{45}{12})$ $3\frac{3}{4}$

**for checkout** — page 192

Pick any five rows of problems to complete. Multiply. Write the simplest name for each product.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 1. | $5 \times \frac{3}{5}$ 3 | $\frac{2}{3} \times 6$ 4 | $\frac{1}{6} \times \frac{3}{4}$ $\frac{1}{8}$ | $5\frac{3}{5} \times 3\frac{2}{5}$ $19\frac{1}{25}$ |
| 2. | $7 \times \frac{4}{7}$ 4 | $\frac{8}{9} \times 4$ $3\frac{5}{9}$ | $\frac{5}{12} \times \frac{3}{5}$ $\frac{1}{4}$ | $3\frac{1}{3} \times 2\frac{4}{5}$ $9\frac{1}{3}$ |
| 3. | $4 \times \frac{3}{4}$ 3 | $\frac{2}{11} \times 3$ $\frac{6}{11}$ | $\frac{1}{5} \times \frac{5}{7}$ $\frac{1}{7}$ | $2\frac{3}{8} \times 3\frac{2}{3}$ $8\frac{17}{24}$ |
| 4. | $8 \times \frac{5}{8}$ 5 | $\frac{3}{16} \times 5$ $\frac{15}{16}$ | $\frac{4}{5} \times \frac{2}{3}$ $\frac{8}{15}$ | $4\frac{1}{5} \times 2\frac{1}{3}$ $9\frac{4}{5}$ |
| 5. | $10 \times \frac{9}{10}$ 9 | $\frac{2}{5} \times 4$ $1\frac{3}{5}$ | $\frac{6}{10} \times \frac{3}{4}$ $\frac{9}{20}$ | $1\frac{2}{7} \times 3\frac{3}{4}$ $4\frac{23}{28}$ |
| 6. | $9 \times \frac{4}{9}$ 4 | $\frac{3}{4} \times 6$ $4\frac{1}{2}$ | $\frac{3}{7} \times \frac{1}{4}$ $\frac{3}{28}$ | $4\frac{4}{5} \times 2\frac{1}{4}$ $1\frac{4}{5}$ |
| 7. | $6 \times \frac{5}{6}$ 5 | $4\frac{4}{5} \times 12$ $9\frac{3}{5}$ | $\frac{1}{10} \times \frac{2}{2}$ $\frac{1}{10}$ | $5\frac{5}{8} \times 3\frac{2}{7}$ $2\frac{3}{56}$ |
| 8. | $3 \times \frac{2}{3}$ 2 | $\frac{5}{6} \times 3$ $2\frac{1}{2}$ | $\frac{2}{3} \times \frac{2}{7}$ $\frac{4}{21}$ | $\frac{2}{3} \times 4\frac{2}{5}$ $2\frac{14}{15}$ |

If you make two or more errors on an **a** problem, do all the other **a**'s for practice. If you make two or more errors on **b**, do the other **b**'s for practice. You know the pattern. If you need the practice, do it now.

## activities

**1. things** sets of colored plastic sticks

Assign a different fractional value to the sticks of each color. Different values can be assigned to different sets. For example:

| | *Set A* | *Set B* |
|---|---|---|
| orange | $\frac{1}{2}$ | $\frac{3}{5}$ |
| red | $\frac{3}{9}$ | $\frac{1}{2}$ |
| green | $\frac{1}{6}$ | $\frac{2}{3}$ |
| blue | $\frac{5}{6}$ | $\frac{4}{7}$ |
| yellow | $\frac{4}{9}$ | $\frac{5}{8}$ |
| black | $\frac{1}{5}$ | $\frac{1}{4}$ |

At first you will want to limit the number of sticks in a set.

The player holds all the sticks in the set in one hand and then drops them all together. With the aid of a neutral stick, he continues to pick up sticks until he moves a stick he is not trying to pick up or until he has picked up all the sticks. His score is the sum of the values of the sticks picked up minus the value of the sticks moved (if any). The player with the highest score wins the round.

**2.** Individual activity. (Provide the pupil with the following directions.)

Fun with Factors

1. List all of the factors for these numbers:
   6 → (1, 2, 3, 6)
   14 → (1, 2, 7, 14)
   24 → (1, 2, 3, 4, 6, 8, 12, 24)
2. Add the factors for 6. Do not include 6 itself. Compare their sum to 6. Is it greater than, less than, or equal to 6? (Equal to)
   6 is called a **perfect** number.
3. Add the factors for 14. Do not include 14 itself. Compare their sum to 14. (Less than)
   14 is called a **deficient** number.
4. Add the factors for 24. Do not include 24 itself. Compare their sum to 24. (Greater than)
   24 is called an **abundant** number.
5. What makes a number a perfect number? a deficient number? an abundant number?
6. Find the factors for the numbers from 2 through 30. Find the sum of the factors for each number. Do not include the number itself. Decide whether each is a perfect, a deficient, or an abundant number.

**3. things** game boards; small cards; markers; container

Prepare a list of 15 whole numbers and 15 mixed numbers. Write each number on a card. Mix these in a container. Have each player prepare a 4-by-4 array as his game board. Each player selects any 16 of the 30 numbers listed and writes one in fraction form in each square of his game board. The 16 fractions must all be different.

One card at a time is drawn from the container. If a player has that number written in fraction form in a square on his game board, he covers the square with a marker. The player who gets four markers in a row, column, or diagonal wins.

Variation: Prepare cards with whole and mixed numbers written in fraction form, and game boards containing whole and mixed numbers.

**4. things** small cards

Prepare two sets of 16 cards—one set consisting of whole and mixed numbers, the other of the same numbers written in fraction form. Each set is shuffled separately and laid facedown to form two 4-by-4 arrays.

In turn, a player turns over one card in each array. If the two cards match (name the same number), he removes the cards, keeps them, and takes another turn. If the cards do not match, they are again turned facedown, and play continues with the next player. The player with the most cards wins.

**5. things** wood cubes (or sugar cubes)

Write a mixed number, fraction, or whole number on each face of two wood cubes. Sugar cubes sprayed with enamel can be substituted for wood cubes.

Pair pupils. Players alternate in rolling the two cubes. One point is earned for finding the product of the numbers that land faceup. An extra point is earned if the player gives the simplest name for the product. A player can earn 2 points per round. Players predetermine the number of points needed to win.

**6. things** small cards

Prepare two sets of 16 cards—one set consisting of open multiplication sentences, the other of the products of these sentences written in simplest form. Each set is shuffled separately and laid facedown to form two 4-by-4 arrays.

In turn, a player turns over one card in each array. If the product card completes the multiplication sentence, he removes the cards, keeps them, and takes another turn. If the product card is not the correct solution, the cards are again turned facedown, and play continues with the next player. The player with the most cards wins.

**7. things** spirit master

Prepare a spirit master of grids as shown. Leave the numbers out of several grids so that pupils can supply their own fractions and challenge each other.

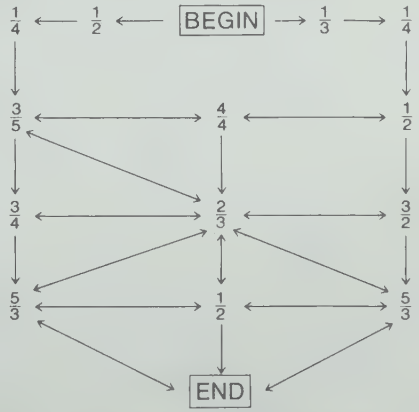
To complete the grid, the pupil multiplies across each row and down each column, writing the products in the squares provided. He then multiplies the products found vertically and those found horizontally. These two products should be the same. This number is recorded in the bottom right-hand square. Each grid is self-corrective.

| | | |
|---|---|---|
| $\frac{1}{2}$ | $\frac{2}{5}$ | |
| $\frac{1}{3}$ | $\frac{1}{4}$ | |
| | | |

**8. things** spirit master

Prepare a spirit master as shown.

Goal: Find the path leading from BEGIN to END that has the **smallest** product when the fractions along that path are multiplied. You may not retrace your path between any two fractions.




Note that the shortest path does not lead to the smallest product.

# additional learning aids

**operation**—chapter objectives 1, 2, 3

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: F-4, 6, 7, 8, 9, 10, 14, 15, 18, 28, 29, 30, 33, 35
*Computapes*, SRA (1972)
Module 5, Lessons: FR 6–8, 10–21
*Computational Skills Development Kit*, SRA (1965)
Use of Fractional Numerals cards: 1–10
Multiplication of Fractions cards: 1–9
*Cross-Number Puzzles (Fractions)*, SRA (1966)
Multiplication cards: 1–20
*Mathematics Involvement Program*, SRA (1971)
Cards: 105, 265, 166, 77, 167, 197
*Skill Modes in Mathematics*, SRA (1974)
Level I, Molecules: A, C
Level II, Molecules: A, C
*Skill through Patterns, level 6*, SRA (1974)
Spirit master: 62
*Visual Approach to Mathematics, Rational Numbers*, SRA (1967)
Visuals: 3, 10, 13

**other learning aids** (described on page 216h)—The Fat Fraction Game, Fraction Bars Student Activity Book (levels I and II), Good Time Mathematics, Mathimagination (book D—Fractions)

# 8 PROBABILITY

**before this chapter the learner has –**

1. Tallied data and made bar graphs to illustrate the data
2. Computed the arithmetical mean for data

**in chapter 8 the learner is –**

1. Mastering using a fraction to express the probability of an outcome
2. Tallying data and making a bar graph to illustrate the data
3. Verifying the probability of an uncertain outcome by repeated trials
4. Expressing the odds for an outcome
5. Determining whether a game is fair
6. Finding the mean and the range for data

**in later levels the learner will –**

1. Construct a sample space to show the possible outcome of a probability experiment
2. State the probability and the odds of any event shown in a sample space

# Notes & Things

Carnival games are used to introduce the learner to sophisticated topics such as probability and odds. Simple experiments will help the pupil come to know about range and sample means. These new vocabulary words will become secondary to the activities themselves.

There are no computation problems listed in the chapter, but opportunities for computation are great.

Learners are led to take an objective look at a number of games of chance. The "fairness" of a game for both the player and the operator is analyzed. All carnival games in real life favor the operator, even if only slightly.

Mastery of learning objectives is not intended with this chapter. The approach stresses verifying predictions by experimentation. The purpose is to explore the concept of probability and to develop some critical thinking skills.

## things

graph paper
cardboard
scissors
wood cubes
wood beads
container for beads
paper cups of several sizes

For the extra activities you will want to have these things available:

transparent tape
spinners from commercial games
1 die
coin
phone models
clock
paper clips of different sizes and shapes
novel

**goal** Think about and explore ideas through a picture clue

**page 194** There is no doubt that your baseball fans will lead the discussion on this page. Start the discussions with a couple of leading questions: *Will the batter hit the ball? Will he strike out in this turn at bat? Will he hit a home run?* Answers to those questions will be wild guesses, but what if that were Babe Ruth or Hank Aaron at bat? Would that change anyone's answers? Why?

It is inevitable that someone will say the word *probable* or *probably*. **Good!**

Then the youngsters are aimed in the right direction. One more question will do the trick. *When you say he probably will hit the ball, what facts do you consider?* Be prepared for your experts to reel off statistical records of times at bat, strikeouts, season batting averages, career batting averages, and on and on and on. The goal of the page has been accomplished. We consider facts, sort them, organize them, and then use this information to make some good guesses. That is the essence of probability.

Would you like to know everything that's going to happen to you tomorrow? Probably not! Most people like surprises. But almost everyone tries to guess about the future. The mathematics of *probability* can help. **Your goal is to learn how probability can help you in the world around you.**

Insurance companies use probability so that they know what to charge for insurance. Scientists use probability to predict what atoms will do. A baseball catcher uses probability to decide what pitch to call for. You use probability too.

**What Are the Chances?** There are no right answers – for discussion only.

that the Baltimore Orioles will win the next World Series?
that you'll get a good grade in math?
that it will rain during the picnic?
that you'll win first prize in the raffle?

**You don't need magic to figure out the chances (or probability) that certain things will happen. A few facts can help you predict more accurately.**

**What Do You Need to Know?**

before you can decide whether the Orioles are likely to win the Series?
before you can feel you'll probably get a good grade in math?
before you can figure the odds of rain during the picnic?
before you know if you have a decent chance to win the raffle?

**lesson** Page 195

**goal** Exploring the role of facts in making predictions

**page 195** Baseball fans will have an opportunity to show what they know about league standings and how positions are determined.

Pupil insight into what is necessary for a good grade in math should be revealing. You may have to straighten out some misconceptions.

Pupils who watch weather forecasts on TV should have a great deal to contribute to the discussion.

The last situation may stop everyone. Consider the total number of tickets in the drawing. Then think about the number of tickets that will be drawn.

Emphasize that PROBABILITY is deciding what the **chances** are of something happening.

**lesson** Pages 196, 197, 198

**goal** Examining the possibilities and making a guess

**memo** Pages 197 and 198 are related to page 196. Use them together, please.

**page 196** Before playing a game of chance, the player must examine the possible results. Break the balloon or miss the balloon; hit the cup with the ball or miss the cup. How many possibilities are there in each game? In guessing the season of the year a person was born, how many possibilities are right? How many are wrong? Pupils are asked to make a guess. Pages 197 and 198 will help them gather facts that will give them more insight into this game of chance.

Have you ever been to a carnival? Carnival games are based on probability. They are also set up to make money for the operators. Players will probably miss the balloon more often than they'll hit it with the dart. They'll probably miss the cup more often than they'll get the ball into it. If a game is too easy, the operator will lose money. If it looks too hard, no one will play.

In order to make money, the operator has to know what he's doing. He has to make sure that his guesses will be correct often enough for him to make a profit. How many seasons of the year are there? Which season would you guess? 4 Answers will vary.

196

**1**

Find how many students in your class have birthdays in each of the four seasons. Your results might look like this:

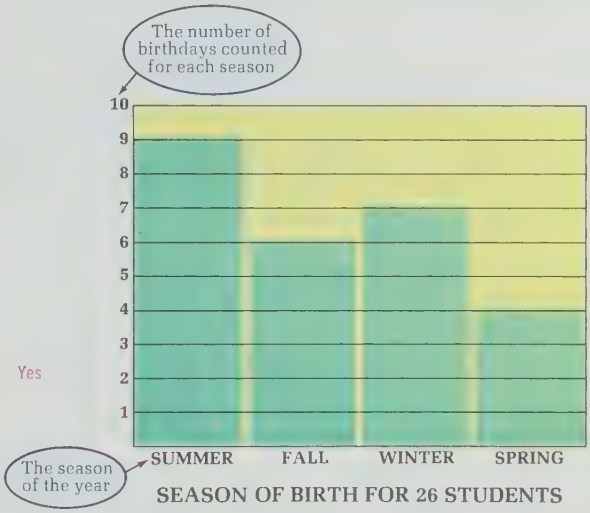
This is called a *tally chart.*

Summer ( June, July, August ) HTT IIII
Fall ( September, October, November) HTT I
Winter ( December, January, February) HTT II
Spring ( March, April, May) IIII

**2**

Make a bar graph showing your results. A bar graph showing the tally above would look like this:

Discuss similarities and differences between class results and these results.




SEASON OF BIRTH FOR 26 STUDENTS

**3**

If the bar graph had been made for the same students a year ago, would it have looked the same? If it's made a year from now, will it look the same? Yes Yes

**4**

Would a bar graph showing the seasons of birth for the class next door look the same? Would it look similar? Not necessarily It might.

Go on to the next page.



**goal** Tallying facts to help make more accurate predictions; showing data on a bar graph

**memo** Tallying data and showing data on a bar graph is developed in level 5 of this program.

**things** graph paper

**page 197** This activity page involves everyone. The tally chart can be made on the chalkboard, but graph paper is needed to make a graph. Remember—a graph needs a title (let pupils make up their own). Both vertical and horizontal scales should also be labeled. Data for the class will be much more interesting than the example shown on the page, but do compare the two graphs. Results from classes organized on annual promotion will differ from results from classes organized on semiannual promotion.

**goal** Examining how someone can win or lose money in a game of chance

**page 198** There's a lot of thinking to be done and ideas to be shared. You might consider what would happen if the barker guessed spring—or either of the other two seasons—for every one of the 26 students shown in the graph on page 197.

Fall—Break even
Winter—$.65 profit
Spring—$1.30 loss

**5** In which season is a person in your class most likely to celebrate his birthday? (Look for the highest bar on the graph.) In which season is a person in your class least likely to celebrate his birthday? Refer to class bar graph.

**a** If the operator guesses just one season for each person in your class, which should it be? Highest bar on class graph.

**b** How many times would he be correct?
Answer should be stated as: x (number of birthdays in that season) out of y (total number of students) times. For the example shown: 9 out of 26 times

**6** Pretend each of the 26 people in the class in our example pays 50¢ for a guess. Each incorrect guess costs the barker 65¢. (He has to pay for the stuffed rabbit.) Suppose the barker guesses summer as the season of birth for each person. How much money would he make on this class?

- —How many guesses? 26
- —What is the cost to the students? 26 × 50¢, or ? $13.00
- —How many correct guesses? ? 9
- —How many incorrect guesses? ? 17
- —What is the cost to the barker? ? $11.05
- —What is the barker's profit? ? − ?, or ? $13.00 − $11.05 or $1.95 profit

Do the same calculations for your own class.

**7** Where could you find more information about the seasons of birth for your community? for the whole country? This information is available through your Bureau of Vital Statistics. Do you think it is likely that the barker studied this information for your community before the carnival opened? Answers will vary.

Discuss.
Community: city or county Dep of Vital Statistics
Country: Census Bureau, U.S. of Commerce, Washi

198

In the next pages you'll play a carnival game called the Wheel of Fortune. Probability will help you determine your chances of winning. (It can't guarantee that you *will* win, however!)

WHEEL I

Here's the first Wheel of Fortune. It's divided into two sections, A and B. These sections are exactly the same size. A spinner is fastened at the center of the wheel. O.K.! The spinner is spun. Round and round it goes. Where it will stop, nobody knows. But we do know that it will stop on section A or section B. (If it stops on the line, it's spun again.)

There are two *outcomes* on this wheel:
The spinner stops on section A.
The spinner stops on section B.
Since sections A and B are the same size, the spinner is as likely to stop on A as on B.
A and B are *equally likely* outcomes.

One spin has 2 equally likely outcomes. The probability of stopping on A is 1 out of 2, or $\frac{1}{2}$. (Probability is usually expressed with a fraction.) What does the numerator of this fraction stand for?* the denominator?** What is the probability of stopping on B? $\frac{1}{2}$

If you were going to bet on the outcome of a spin, would you always bet on A? always on B?*** If you make many bets and always bet on the same outcome, you can expect to win about $\frac{1}{2}$ of the time. If you make only a few bets, however, you can't be sure of winning $\frac{1}{2}$ of the time.

*Outcome A
**All possible outcomes
***It makes no difference.

199

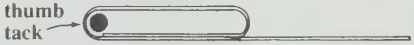
See activity 1, page 216a.

See extension of activity 1, page 216a.

**lesson** Pages 199, 200

**goal** Introduction to writing probability

**memo** The games described on pages 199 through 206 will be more meaningful if actually performed by the pupils. Spinners can be easily constructed, using any small container with a soft plastic top. A pointer can be made by straightening the outside edge of a paper clip. Attach the paper clip to the lid with a thumbtack to form the spinner.




Fit colored paper inside the lid to show parts of the spinner. A plastic straw fastened to a piece of light cardboard with a straight pin will also work.

**page 199** Your pupils should be familiar with the type of spinner being discussed. Spinners are often part of commercially produced games.

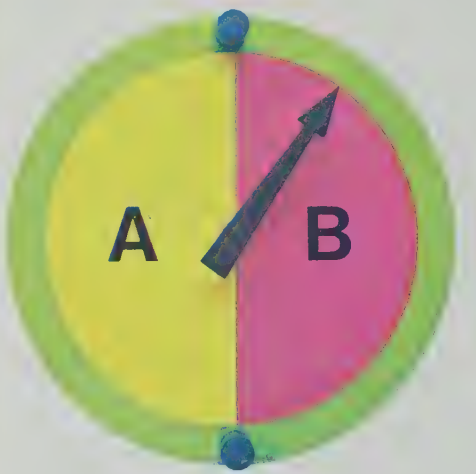
New vocabulary is emphasized on the page. The fraction that expresses the probability is read as usual—one-half.

Emphasize that probability $\frac{1}{2}$ will not guarantee the player's winning one out of every two turns. But it will give the player an idea of what his chances are.

**goal** Examining the cost of playing and the amount that might be won in a game of chance

**page 200** Make sure your students understand that the game wheel may be fair—it may give the player and the operator equal chances of winning—but that the rate at which a win is paid off may make the game unfair. The player must examine how many games must be won before he actually wins money, if ever. In the real world, games of chance always favor the operator, if only slightly. In this specific game, the player is not likely to win money because of the rate of payoff. The problems focus on the cost of playing in contrast to the amount that might be won.

## Get ready to play




WHEEL 1

Here are the rules:
Each bet costs 50¢.
If you win, you get 75¢.
Bets are placed before the spinner is spun.
You may not change your bet after the spinner starts to spin.

Now let's look at an example of how money changes hands in this game.

**1** You bet on A 10 times in a row. This will cost you $10 \times 50¢$, or \$ ? . 5.00

**2** About how many times will the spinner stop on A in 10 spins? Let's say it stops on A 5 times. Then you win $5 \times 75¢$, or \$ ? . 3.75

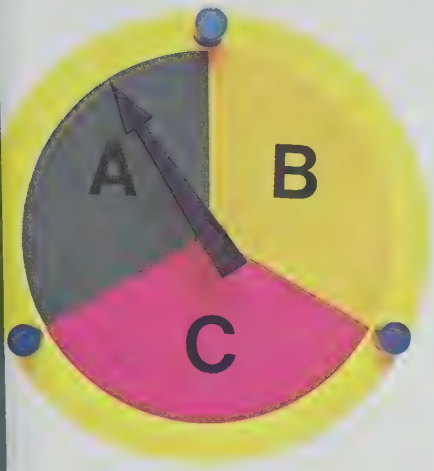
**3** After 10 spins, how much money have you gained or lost? You have paid \$5.00 and won only \$3.75. This means you lost money. How much?
\$5.00 − \$3.75 = ? \$1.25

**You lost!** This game, like all carnival games, favors the operator over the player.

**4** Of course the spinner might have stopped on A more than 5 times. How many times would it have to stop on A in order for you to gain money? 7

**5** Could the spinner stop on B 10 times? Yes
Would this be very likely? No
How much would you lose if this happened? \$5.00

et's look at other Wheels of Fortune to learn more about probability.




WHEEL II

1 Name three outcomes for wheel II. A, B, or C

2 Sections A, B, and C are all the same size. So the probability that the spinner will stop on A is 1 out of 3, or $\frac{1}{3}$. What is the probability that it will stop on B? $\frac{1}{3}$ on C? $\frac{1}{3}$ Are these outcomes equally likely? Yes

3 One section of the wheel is shaded black. The other two are color. What is the probability of stopping on the black section? $\frac{1}{3}$ What is the probability of stopping on either of the color sections? $\frac{2}{3}$ Are the outcomes "black" and "color" equally likely? No

4 A bet on wheel II costs 10¢. For each bet on black you win 20¢ if the spinner stops on A, the black section. You get nothing if it stops on a color section. Suppose you bet on black 12 times and win $\frac{1}{3}$ of the time. How many times do you win? Let's see how much you gain (or lose).

Cost of 12 bets: $12 \times 10¢$, or ? $1.20

Your winnings: $4 \times 20¢$, or ? $.80

Which is more? The cost of 12 bets

How much more? ? − ?, or ?
$1.20 − $.80, or $.40

201

See activity 2, page 216a.

**lesson** Pages 201, 202

**goal** Examining a game in which the outcomes are not equally likely

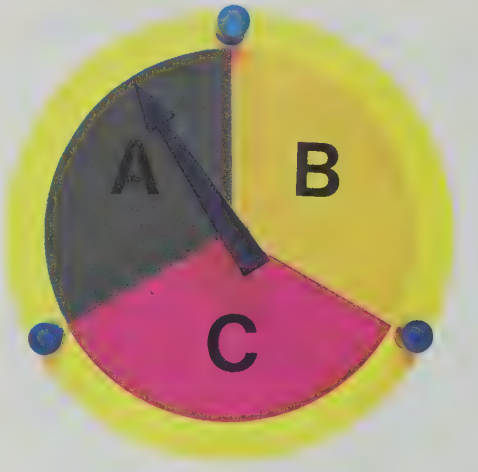
**page 201** The probability that the spinner will stop on A, on B, or on C is equally likely. However, the probability of its stopping on black or on color is not equally likely—one outcome yields black, two outcomes yield color. Only the black section pays off. Does this wheel favor the player or the operator? (Operator)

Pick one of the other colors. What is the probability of stopping on that color?

**goal** Deciding whether a game is fair

**things** cardboard
scissors

**page 202** The pupil has already found that the outcomes of this game are not equally likely. The focus now is on examining the amount that might be won. The needed computations should lead the pupil to realize that the game favors the operator. If you owned this game, would you make money? If you played this game, could you win money? What is most likely to happen as you continue to play? Is this game unfair to the player or to the operator?




WHEEL II

**1** A *fair game* would give you and the operator an equal chance to make money. Do you think the game with wheel II is fair? How much money did you lose after playing 12 games? What do you think would happen if you played longer?

Discu
40¢
You'd probably lose more money.

**2** Each time you bet on black, you pay 10¢. If you play many games, you can expect to win on $\frac{1}{3}$ of the spins. Each time you win, you get 20¢; but it costs 10¢ to play, so you're only 10¢ ahead when you win. If you play many games, you can expect to lose on $\frac{2}{3}$ of the spins. You lose 10¢ $\frac{2}{3}$ of the time. You gain 10¢ $\frac{1}{3}$ of the time.
Is the game fair? Not to the player

**3** Suppose you bet on black 24 times and you win $\frac{1}{3}$ of the time.

| | |
|---|---|
| Cost of 24 bets: | 24 × 10¢, or ? |
| Your winnings: | 8 × 20¢, or ? |
| How much do you lose? | ? − ? , or ? |

$2.40 − $1.60, or $.80

**4** Make a copy of wheel II on a piece of cardboard. Make sure the sections are the same size. Cut a spinner out of cardboard. Fasten it to the center of the wheel (with a pin, pencil point, or similar object) so that it spins freely. Predict the number of times black will win in 12 spins. Then spin it 12 times and compare the results with your prediction.
Discuss variations in results.

202

WHEEL III

1 Name five outcomes for wheel III.
A, B, C, D, E

2 The sections are all the same size.
What is the probability of landing on E? $\frac{1}{5}$

3 Sections B and E are shaded black. A, C, and D are color. Suppose you bet on black. For which outcomes would you win? E or B

4 What is the probability of landing on black? $\frac{2}{5}$

5 A bet on wheel III costs 10¢. For each bet on black, you win 30¢ if the spinner stops on a black section. You get nothing if it stops on a color section. Do you think this is a fair game?
Discuss—it is fair for the player.

6 On wheel III black wins $\frac{2}{5}$ of the time. Black loses $\frac{3}{5}$ of the time. Let's say you bet on black 20 times and win 8 times. How much money will you gain (or lose)?

| | | |
|---|---|---|
| Cost of 20 bets: | 20 × 10¢, or ? | $2.00 |
| Your winnings: | 8 × 30¢, or ? | $2.40 |
| Which is more? | Your winnings | $2.40 − $2.00, or $.40 |
| How much more? | ? − ?, or ? | |

You can see that you have *gained* money by betting as you have. Would you be wise to continue betting on black at wheel III? How much money would you expect to gain if you bet on black 100 times in a row? Yes
Cost of 100 bets: $10
Possible winnings: $12 Gain: $2

7 Do you think the operator would lose money with wheel III? Is this a fair game? Yes Discuss—it's not fair for the operator.



**things** spinners from commercial games

Many games that the youngsters have at home or in school contain spinners. Any of the experiments on pages 201 through 206 can be adapted to spinners. Challenge everyone to think of a strategy to use when trying to win.

**lesson** Pages 203, 204

**goal** Examining whether a game is fair

**page 203** When the fairness of a game is discussed, no inference is being made that the game may be fixed. Rather, the chances for winning or losing are examined together with the rate paid for winning. When the wheel is examined, a game may or may not be fair. The amount paid for winning may be such that it compensates for the wheel's not giving the player an equal chance with the operator. Games may be unfair to the operator as well as to the player.

Computing the cost to play and the potential earnings for the game should lead the player to realize that he is almost certain to win. In fact, the rate for winning is so good that the player will like this game—but what about fairness to the operator?

**goal** Examining whether a game is fair

**things** cardboard

**page 204** The probability of winning is only $\frac{1}{8}$. Whom does the wheel favor? (The operator) Consider taking 8 chances, with the spinner stopping once on each outcome. Do you win or lose money? This should help everyone decide that the rate of pay more than compensates the player for the low probability of stopping on the winning outcome.

As pupils try the game themselves, have them continue to increase the number of spins—being careful to tally the total number of spins and the outcomes. As the number of trials increases, the number of wins should more closely meet the probability of winning.

WHEEL IV

1 What are the possible outcomes for wheel IV?
A, B, C, D, E, F, G, H

2 Which outcome would you want if you bet on black? B

3 The sections are all the same size. What is the probability of landing on B? $\frac{1}{8}$

4 What is the probability of stopping on black? $\frac{1}{8}$

5 What is the probability of stopping on a color section? $\frac{7}{8}$

6 A bet on black costs 10¢. You win $1.00 if the spinner stops on black and nothing if it stops on color. Do you think this is a fair game? No

7 Make a wheel like wheel IV. Predict the number of times black will win in 16 spins. Then spin it 16 times and compare the results with your prediction. Spin it 16 more times and see if you get black a different number of times. 2
Discuss variations in results.

204

Here are two more Wheels of Fortune:

WHEEL V

WHEEL VI

1 On wheel V the probability of stopping on black is $\frac{0}{6}$ or 0.
Would it be smart to bet on black? Why not?
No That outcome would never occur. You couldn't win

2 On wheel VI the probability of stopping on black is $\frac{6}{6}$ or 1.
Would the operator accept bets on black? Why not? No The operator would never win
Would it be smart to bet on color? Why not?
No That outcome would never occur. You'd always lose.

3 It's important to know whether a game is fair.
Ask yourself these questions before you play a game of chance:

How many chances do I have to win?

How many chances do I have to lose?

Is the prize worth the amount I will probably have to pay for it?



**lesson** Pages 205, 206

**goal** Examining a probability of 0 and a probability of 1

**page 205** Zero probability means there is absolutely no possible way for a given outcome to occur, while a probability of 1 indicates there is one and only one possible outcome. The probability of an outcome can never be greater than 1 or less than 0.

**goal** Introduction to the concept of odds

**page 206** The ODDS of a game are stated as the number of chances of winning to the number of chances of losing. Together these two numbers should equal the total number of possible outcomes. In the real world, odds of 2 to 4 may be stated as 1 to 2. For the present, concept is more important than stating odds in simplest form—settle for 2 to 4, unless one of your pros thinks of 1 to 2 on his own.

Compare writing probability to writing odds. Probability is written as a fraction:

$$\frac{\text{number of chances for a given outcome}}{\text{number of possible outcomes}}$$

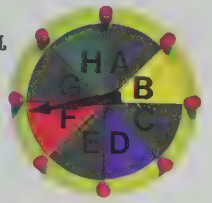
Are the numbers used in the probability the same as those used for the odds?

In each of the following games the sections for each game are all the same size. The payoff is the same, too. If you bet on black and win, you get 50¢. If you bet on color and win, you get 50¢.

WHEEL VII

There are 2 chances for black to win and 3 chances for black to lose. The *odds of winning* are 2 to 3. Is it smart to bet on black? No

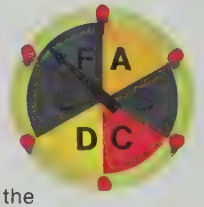
WHEEL VIII




How many chances for black to win on this wheel? 4 How many chances for black to lose? The *odds* of 4 black's winning are 4 to 4. We also say that the *odds are even* that black will win. (That's a funny sentence—"The odds are even!") This is a fair game. It would be smarter to bet on black here than on wheel VII. But could you lose anyway? Yes

When the odds are even, some people say that the odds are 50-50. That means if you played 100 times, the rules of probability say that you can expect to win 50 times and lose 50 times. Of course the actual results could be different.

**1**

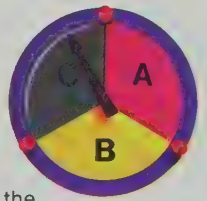



Bet black.

**a** What is the probability of stopping on black? $\frac{2}{6}$

**b** What are the odds that you will win? 2 to 4

**2**

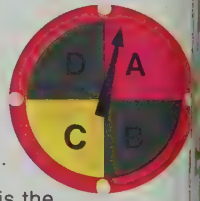



Bet black.

**a** What is the probability of stopping on black? $\frac{1}{3}$

**b** What are the odds that you will win? 1 to 2

**3**




Bet color.

**a** What is the probability of stopping on color? $\frac{2}{4}$

**b** What are the odds that you will win? 2 to 2 (Odds of winning and losing are even)

206

Find a cube.
Write 1 on one face, 2 on another face, 3 on another, 4 on another, 5 on another, and 6 on the last face.

You'll need a sheet of paper to record your tests.

Roll until 3 comes up on top of the cube.

What is the probability that you'll get 3 on the first throw? $\frac{1}{6}$

Record how many throws you need to get 3.

Try again (trial 2).
Throw until you get 3.
Record the number of throws.

Throw for 3 again (trial 3).
Record the number of throws.

Complete 7 more trials.
In each trial you roll until you get 3.

1. Draw a bar graph to show the number of throws needed to get 3 in each trial. Is there more than one way to make the graph? Bar graphs will vary. Yes—can have horizontal or vertical bars
2. What is the most common number of throws? Compare answers.
*3. Find the total number of throws for all 10 trials. Divide the total by 10. According to the rules of probability, your answer should be close to 6. Why? In ten trials there are ten 3s. By the rules of probability, you should get ten 3s in 60 throws. $60 \div 10 = 6$.

207

See activity 3, page 216a.

**lesson** Page 207

**goal** Evaluating the reliability of probability

**things** wood cubes
graph paper

**page 207** How reliable is probability? Remember, it offers no guarantee. Enough trials are necessary so that a pattern becomes clear. A tally chart will be needed to show each trial and the number of throws needed to get a 3 in each trial. Pupils can work in small groups or individually. Graph paper will help in making the bar graphs. Share results. Look for common patterns. Talk about problem 3.

**lesson** Pages 208, 209, 210

**goal** Writing the probability and the odds for an outcome

**memo** The Marble Draw is the basis for all questions on pages 208 through 210. These pages should be used at one time.

**page 208** Color is not important in problem 1; only the numeral written on each marble matters. Since there are no duplicates, the probability and the odds for drawing any given number are the same. This situation is changed in problem 2. Two things must be considered here: (1) color and (2) number of each color.

# The Marble Draw is another game based on probability

1. A jar contains six marbles. Each marble has a number on it. You are blindfolded. Then you draw one marble from the jar.

   **a** What are the possible outcomes? (Name the different possible choices.) 1, 2, 3, 4, 5, 6

   **b** What is the probability of drawing marble 4? $\frac{1}{6}$

   **c** What are the odds of drawing marble 4? 1 to 5

   **d** What is the probability of drawing marble 1? $\frac{1}{6}$

   **e** What are the odds of drawing marble 1? 1 to 5

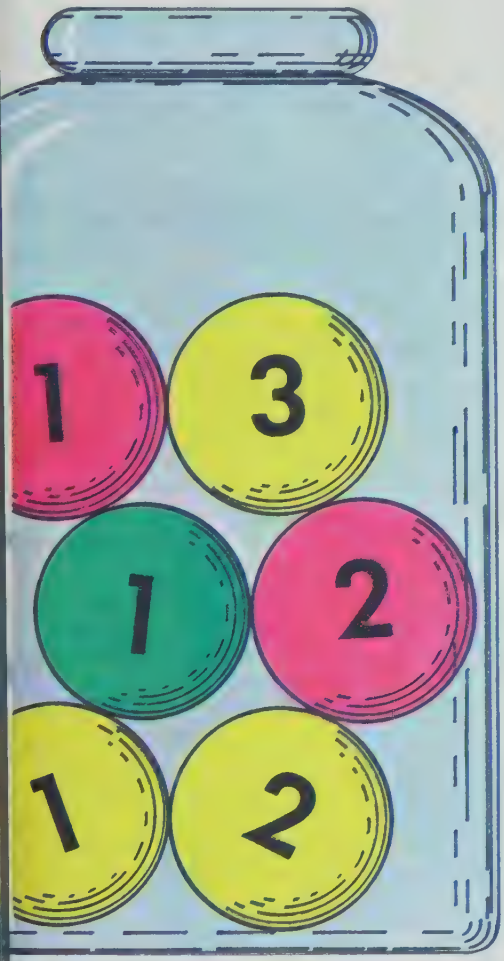
Another jar contains six marbles—two red, three yellow, and one green. Again you draw a marble blindfolded.

Just so we can tell the red marbles apart, let's mark one of them with a 1 and the other with a 2. We do the same for the yellow marbles.

208

**things** container; wood beads

Borrow some wood beads from your friend, the kindergarten teacher. Number them as shown in the activities. Have your pupils verify their answers to the questions by actually performing the experiments.




1. Do "red," "yellow," and "green" describe all possible outcomes? Do these words tell you how many red, yellow, and green marbles there are? What is a better way to describe all possible outcomes?
   Yes
   No
   Red 1, red 2, yellow 1, yellow 2, yellow 3, green 1
   - **a** What is the probability of drawing red 1? $\frac{1}{6}$
   - **b** What is the probability of drawing red 2? $\frac{1}{6}$
   - **c** What is the probability of drawing a red marble? $\frac{2}{6}$ or $\frac{1}{3}$
   - **d** What are the odds of drawing a red marble? 2 to 4 or 1 to 2

2. The probability of drawing a red marble is equal to the probability of drawing red 1 plus the probability of drawing red 2. Let's see what happens with yellow.
   - **a** The probability of drawing yellow 1 is ?. $\frac{1}{6}$
   - **b** The probability of drawing yellow 2 is ?. $\frac{1}{6}$
   - **c** The probability of drawing yellow 3 is ?. $\frac{1}{6}$
   - **d** What is the probability of drawing a yellow marble? $\frac{3}{6}$ or $\frac{1}{2}$

3. What are the odds of drawing a green marble? 1 to 5

4. What are the odds of drawing a yellow marble? 3 to 3 or 1 to 1 (even)

209

**goal** Writing the probability and the odds for equally likely outcomes

**things** wood beads
container for beads

**page 209** Part of the fun of studying a sophisticated topic like probability is in performing the activities to find how simple the whole thing is. Wood beads make excellent marbles—see your friend, the kindergarten teacher. Examine the beads unnumbered first to help everyone see how numbering helps with the description. The numbering can be done with a felt-tip marker.

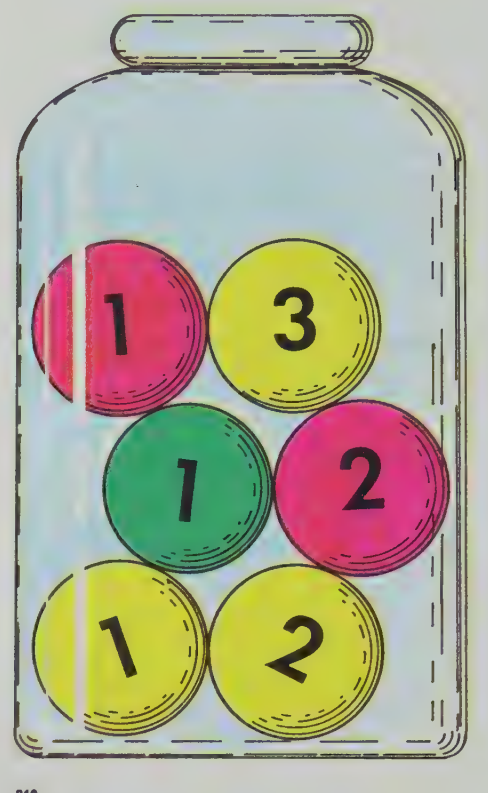
Why is the probability for drawing red 1 different from the probability for drawing red? Red 1 is a specific marble—number **and** color are important. Any red marble is O.K. for drawing red—color only is important. The pupils who see that the probability for drawing red is the same as the probability for drawing red 1 plus the probability for drawing red 2 deserve to be praised. Let this be their discovery.

**goal** Deciding whether a game is fair

**things** wood beads
container for beads

**page 210** The youngsters have been exposed to games that favored the operator and games that favored the player. Now comes a game where the outcomes, together with the rate of pay, offer an opportunity to break even—provided enough chances are taken. Emphasize that the marble is replaced after each drawing so that the jar is full for the next drawing. Here's another opportunity to compare the cost of the ticket to the amount of the winnings. This is an important factor in determining whether the game is fair.

This is one game your pupils are sure to want to play.




210

A ticket to the Marble Draw costs 25¢.
You win 90¢ if you draw a green marble, 30¢ if you draw a red marble, and nothing if you draw a yellow marble.

1. You buy 1 ticket. What is the probability of winning 90¢? $\frac{1}{6}$ of winning 30¢? $\frac{2}{6}$ or $\frac{1}{3}$ of winning nothing? $\frac{3}{6}$ or $\frac{1}{2}$
2. You buy 6 tickets. You will draw from the full jar 6 times.
   - **a** What is the probability of drawing yellow on one try? Then you would probably draw $\frac{3}{6}$ or $\frac{1}{2}$ yellow how many times in 6 tries? 3
   - **b** What is the probability of drawing red on one try? Then you would probably draw red $\frac{2}{6}$ how many times in 6 tries? 2
   - **c** What is the probability of drawing green on one try? Then you would probably draw green how many times in 6 tries? 1
   - **d** How much money are you likely to win on the 6 tickets?

     yellow = 3 × 0¢, or ? 0¢
     red = 2 × 30¢, or ? 60¢
     green = 1 × 90¢, or ? 90¢
     Total = ? + ? + ?, or ?

     0 + 60 + 90, or $1.50
   - **e** How much money does it cost you to buy 6 tickets? $1.50
3. How much money are you likely to win or lose betting 6 times on the Marble Draw? ? − ? = ?

   $1.50 − $1.50 =
4. Is this a fair game? Why?

   Yes You have an equal chance of winning or l

See activity 4, page 216b.

When an outcome is certain to happen, its probability is 1.

When an outcome is impossible, its probability is 0.

When there are only two equally likely outcomes, the probability of either is $\frac{1}{2}$.

$$\text{Probability of a favorable outcome} = \frac{\text{number of favorable outcomes}}{\text{number of all possible outcomes}}$$

Tell whether the probability is 1, 0, or $\frac{1}{2}$ for each of these.

1. You are older today than yesterday. 1
2. You will draw the longer of two straws. $\frac{1}{2}$
3. A tossed coin will land heads up. $\frac{1}{2}$
4. A six-year-old will run a three-minute mile next week. 0
5. The numerals from 1 through 6 are written on the faces of an unmarked cube, one on each face. The cube is tossed, and the face with 7 on it comes up on top. 0

**lesson** Pages 211, 212, 213

**goal** Writing the probability for a favorable outcome

**page 211** Purely for discussion. What is a **favorable** outcome? (An outcome you want)

**goal** Writing the probability for a given outcome

**page 212** Part of the study of probability is to recognize **all** the possible outcomes. This requires critical thinking. You will certainly want to try to duplicate the situations in the book with youngsters who have difficulty seeing the possible outcomes.

You should have some interesting discussions on this page. The situations in 4 and 5 might bring out some teasing. The traffic lights in situation 6 may bring an argument about the length of time the red or green light is on in contrast to the length of time the yellow light is on. If so, you'll know the group is really thinking probability. That's good! You will find out who has faith in the weather forecaster when you talk about the situation in 7. How many different types of weather might there be? Would snow be a possible choice (outcome) for the people in Florida? Probability has to be based on the number of outcomes.

A probability must be at least 0. It can't be greater than 1. But it might be any number from 0 to 1. What is the probability of each of the following?

1. There are 9 cards. Each card is marked with a different numeral—1 through 9. The card you draw is marked with 5. $\frac{1}{9}$

2. You roll one die. When it stops, the face with 6 dots is on top. $\frac{1}{6}$

3. The names of 7 persons are written on 7 cards. (A different name is on each card.) Of these persons, 4 are boys and 3 are girls. The cards are put in a hat, and you draw one without looking. You draw the name of a girl. $\frac{3}{7}$

**Discuss** each of the following situations. How might you calculate the probability of the given outcome?

4. You enter a room with 7 person in it: 4 are boys and 3 are girls. The first person to speak to you is a girl. $\frac{3}{7}$

5. You will be called on to answer the next question in class. total stude

6. The next traffic light you see will be green. Depends on the length of time for each color

7. Tomorrow will be a cloudy day. Depends on the local weath

Can you think of ways probability must be used by a business man to help him make money?

Discuss. Examples: sales predictions, new products, and so on

212

The rules of probability can help you save money on long-distance telephone calls.

You have a choice when you call long distance.
You can dial the call yourself.
You can ask the operator to call person-to-person.

If you dial the call yourself, you must pay for it if someone answers. If the person you want isn't around, you've wasted your money.

If you call person-to-person, you pay only if the person you want to talk to is there.

Dialing a long-distance call costs less, but . . .

Jim has to call Tom twice a week on business. The odds are even that Tom will be near the phone when Jim calls. Should Jim dial direct or call person-to-person? Here are the costs for the calls:

| | |
|---|---|
| Dialed | $1.35 |
| Person-to-person | 3.60 |

If Tom were there both times, which type of call would be cheaper? If Tom were gone both times, which would be cheaper? Dialed

If no one answers Tom's phone, they both cost the same. If someone answers Tom's phone, the person-to-person would be cheaper.

Suppose Jim calls and Tom isn't there. Whoever answers can tell Jim (or the operator) when Tom will be back. If Jim dials the call at that time, Tom will answer. Under these conditions, is it cheaper to make the first call by dialing or calling person-to-person?

Two dialed calls are still cheaper than one person-to-person call

213

**goal** Application of the rules of probability to a real-world situation

**page 213** Some pupils may form the idea that it is never wise to call person-to-person. Consider trying to locate someone who is traveling whose exact location is not known. Several calls may be necessary to locate that person. No charge is made until that person is finally reached. The charge begins when the two parties begin talking. The probability of reaching the person on the first call could be 0 if the caller knows no more than a possible location and that the traveler is constantly on the move.

**things** phone models; clock

Students enjoy being involved in adult situations. Do you have some phone models available? Act out the instructions on the page. Actually time the calls; then compute the phone charges.

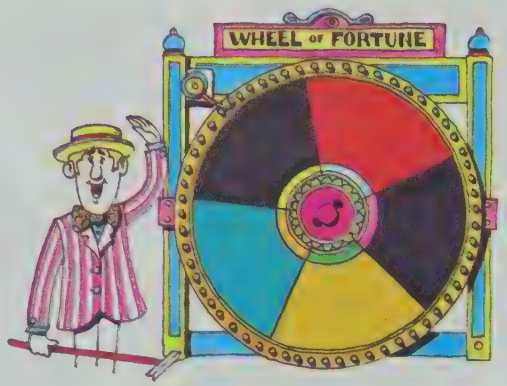
**lesson** page 214

**goal** Examining how physical features may affect probability

**things** paper cups of several sizes

**page 214** Get everyone organized into small groups. Provide paper cups of several different sizes and weights. Compare the results obtained from the experiment by the various groups. There are three possible outcomes. Is the probability for each outcome $\frac{1}{3}$? What do the results of the experiment show?

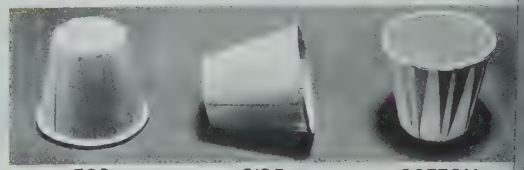
The manner in which the cup is thrown will affect the results of the experiment. Cups hitting on the circular edge will most often topple over. The weight of the bottom of the cup together with the height of the fall will affect the way in which the cup lands. Whether the cup is turned up, down, or sideways when it is tossed should also be considered.




In the games we've been playing you could figure out the probability of an outcome before playing the game. For example, one Wheel of Fortune had 5 sections, 2 of which were black; so the probability of stopping on black was $\frac{2}{5}$.

Sometimes you can know all the possible outcomes but not be able to figure out the probabilities ahead of time. Then you need some facts before you can estimate the probabilities.

1. Find a paper cup with a flat bottom. If you toss the cup onto the floor, it can end up in 3 different positions: on its top, on its side, or on its bottom. Without more information, can you tell what the probability is that it will land on its bottom? on its side? on its top? No No




2. Actually toss a cup 25 times and tally the results.

3. Repeat this experiment 25 more times. Are the results the same as before?

4. Out of the 50 flips, how many times did the cup land on its top? on its side? on its bottom? According to these results, what's the probability that a cup will land on its top? on its side? on its bottom?

5. Suppose you tossed the cup 50 more times. Do you think you'd get *exactly* the same results as for the first 50 tosses? About the same results? No

6. Do you think your results would be true for any kind of cup, or do they depend on the size and shape of the cup?

214

You can often predict what's going to happen by looking at what happened before in a similar situation. You study the facts.

Do an experiment. Discuss results of the experiment.

1. Open a book to any page. Look at the first 30 words on that page. Make a tally chart showing how many words have 1 letter, how many have 2 letters, how many have 3 letters, and so on.
2. Make a bar graph showing your results.
3. Subtract the shortest word length from the longest. This difference will show you the *range* of word length. Compare your results with others'. What is the greatest range of word length that anyone found?
4. Find the average length of the words in your sample. To do this, add the lengths of all the words. Divide their sum by 30. (You're considering a total of 30 words.) This average is called the *mean*. Since we are reporting the mean for a sample of words, we could call this average the *sample mean*.
5. If you were to look at another page in the same book, do you think the range of word lengths would be the same?* What do you think the sample mean would be for the lengths of the first 30 words? Select a page and check your predictions. Were your answers close?
6. Turn to another page in the same book and find the sample mean of the first 30 words. Is it close to the two sample means you calculated previously?

* Not necessarily



**lesson** Page 215

**goal** Finding the mean and the range for data

**memo** Finding an arithmetical mean was introduced in level 5 of this program.

**things** graph paper for bar graphs

**page 215** Encourage the youngsters to select a wide variety of pages from several different books rather than have everyone working on the same sample. This will provide more interesting comparisons and depth of understanding for making the predictions required in problems 5 and 6.

Be careful of the directions for problem 4—"add the lengths of all the words" could have been stated "add the number of letters in each word."

**lesson** Page 216

**goal** **Checkout** – interpreting data, making a bar graph, expressing the probability of an outcome

**things** graph paper for bar graphs

**page 216** To be considered complete, bar graphs should have a title as well as labels for the vertical and horizontal scales.

The objectives of this chapter can be attained only through more experiences. Decide how important these objectives are in your total year's curriculum and whether mastery is desired. Then use the buddy system – assign a peer tutor. Let the pair select various activities from the chapter for additional practice. The change of pace may be a benefit in itself.

# CHECKOUT

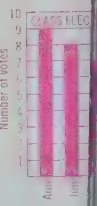
Skills: Interpreting data: making bar graphs

1. Here is a tally chart of votes for the class election.

   **a** Total the votes.

   **b** Complete a bar graph to show the results.

| CLASS ELECTION | | |
|---|---|---|
| Candidates | Votes | Total |
| Ann | 𝍸 IIII | 9 ? |
| Jim | 𝍸 III | 8 ? |
| Sam | 𝍸 II | 7 ? |
| Sue | 𝍸 𝍸 | 10 ? |



Skill: Expressing probability with a fraction

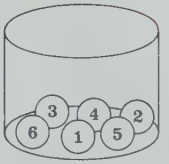
2. **a** Probability of stopping on **A**? on **B**?
   Bet costs 10¢.
   Stop on black – win 25¢.
   Stop on color – win 0¢.

   **b** Outcomes that win? B, E

   **c** Cost of 12 bets? $1.20

   You win 4 times.

   **d** Did you earn any money? No, you lost




3. **a** What is the probability of drawing marble 2? $\frac{1}{6}$

   **b** What is the probability of drawing marble 5? $\frac{1}{6}$

4. A coin is flipped to decide which team starts the game. What is the probability your team will start? $\frac{1}{2}$

5. Six boys apply to be patrol boys. Four will be chosen. What is the probability of being chosen? $\frac{4}{6}$ or $\frac{2}{3}$

216

See activity 5, page 216b.

See activity 6, page 216b.

# RESOURCES

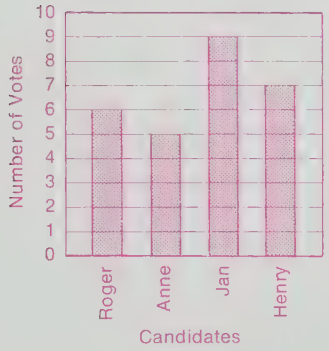
## another form of evaluation

**for checkout**—page 216

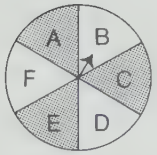
1. Here is a tally chart of votes for the club election.
   a) Total the votes.
   b) Complete a bar graph to show the results. *

| Club Election | | |
|---|---|---|
| Candidates | Votes | Total |
| Roger | 𝍸 I | 6 |
| Anne | 𝍸 | 5 |
| Jan | 𝍸 IIII | 9 |
| Henry | 𝍸 II | 7 |

*

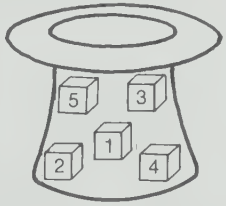



2.




   a) Probability of stopping on B? $\frac{1}{6}$ on E? $\frac{1}{6}$
      Bet costs 5¢
      Stop on black—win 10¢.
      Stop on plain—win 0¢.
   b) Outcomes that win? A, C, E
   c) Cost of 8 bets? 40¢
      You win 3 times.
   d) Did you earn any money? No, you lost 10¢.

3.




   a) What is the probability of drawing cube 3? $\frac{1}{5}$
   b) What is the probability of drawing cube 5? $\frac{1}{5}$
4. There is a traffic light at the corner. What is the probability that it will be red when you get there today? $\frac{1}{3}$
5. A club has 12 members. Three will be elected officers. What is the probability of being elected? $\frac{3}{12}$ or $\frac{1}{4}$

## activities

**1. things** 10 wood cubes

Make two dots on any two sides of 5 cubes. Leave the remaining sides plain. Have the pupils roll the cubes to find all the possible outcomes (number of dots that land faceup). The outcomes should be listed on a tally chart. After several trials, have the pupils decide in what order the outcomes are most likely to happen.

Extension: Mark the other 5 cubes with one dot on any three sides, leaving the remaining sides plain. Repeat the above experiment.

Then mix 3 of the cubes marked with one dot and 3 of the cubes marked with two dots and repeat the experiment.

**2. things** cardboard; transparent tape

Cut 4 congruent equilateral triangles from cardboard. Glue an extra piece or two of the cardboard on the back of one triangle to weight it. Keep this fact a secret. Tape the 4 triangles together to form a tetrahedron. Write a numeral (1, 2, 3, 4) on each face to form a die. Have the pupils roll the die and tally the results. Focus on the number that lands facedown. Do all the outcomes seem equally likely?

Prepare a second die. This time weight a different number. Have the pupils roll both dice and tally the sum of the numbers that land facedown. What sum seems most likely to occur? least likely? Have the pupils find these answers by experience, not by mathematics.

**3. things** 1 die; graph paper

Individual activity. (Provide the pupil with the following directions.)

Make a tally chart for the possible outcomes of rolling 1 die. Now roll the die 36 times and record the results. Prepare a bar graph illustrating your data.

1. Should the results for each number of dots landing faceup be the same?
2. Were the results the same?
3. If you rolled the die 60 times, would each number land faceup exactly 10 times? Try it.
4. If you rolled the die 600 times, how many times would you expect three dots to land faceup?

**4. things** coin

Here is another carnival game. You toss a coin—tails you lose; heads you win $1 and may toss again. If you get tails the second toss, you take your $1 and quit. If you get heads, you win $2 and toss again. You may continue playing as long as you get heads, and each time your winnings double—$1, $2, $4, $8, and so on. As soon as you get tails, you quit and take your winnings.

Play the game 20 times and record how much you win each time. Then find your average winnings. Decide on a fair price to pay to play this game. Now play the game 100 times at that price. Did you win or lose?

**5. things** paper clips of different sizes and shapes

Take a paper clip and spread the inner loop about 30° out from the outer loop. Use this clip to do the following experiment.

1. What are the outcomes possible in tossing this clip? (Landing on side; big loop up; small loop up)
2. Make a tally chart for the outcomes. Toss the clip at least 12 inches in the air or drop it at least 12 inches. Do this 100 times and tally the results.
3. What is the probability that each outcome will occur?
4. Will how you toss the clip make a difference? Experiment.
5. Will how far you bend the inner loop make a difference? Try it.
6. Use a larger paper clip and repeat the experiment. Are the probabilities the same?
7. Use a paper clip of a different shape. Do the probabilities change?

**6. things** novel

Sample a novel rather than a textbook. Count the first 100 consecutive letters on a page. Make a tally chart to show the number of times each letter of the alphabet occurs. Repeat the procedure, using 3 or 4 different pages. Find the average number of times each letter occurred on each of the pages sampled. What letters occur most often? least often?

The sentence below is in code. The code is formed by replacing each letter in the alphabet by the letter that comes a certain number of letters to the right of it. For example, if the code is 2 places to the right then:

A is replaced by C.
B is replaced by D.
Y is replaced by A.

Break the code by using the data you found above.

JZF LCP CTRSE PLNS TD XZGPO PWPGPY!

Answer:

You are right. Each is moved eleven!

# additional learning aids

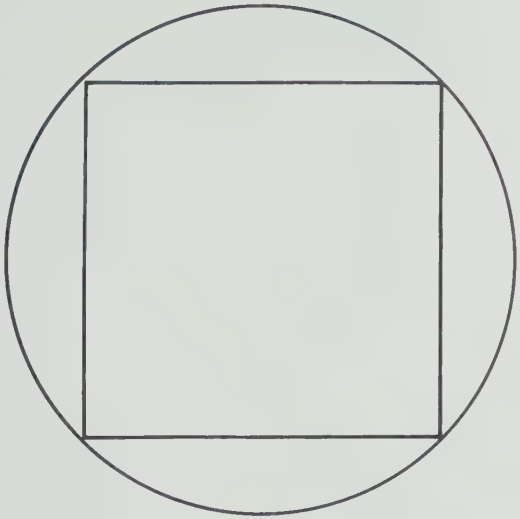
**measurement of events**—chapter objectives 1, 2, 3, 4, 5, 6

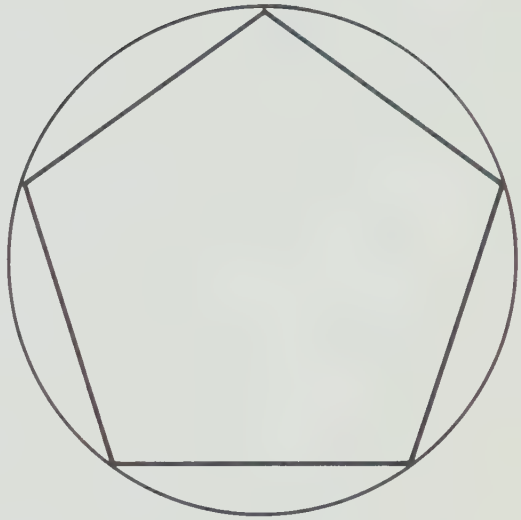
**SRA products**

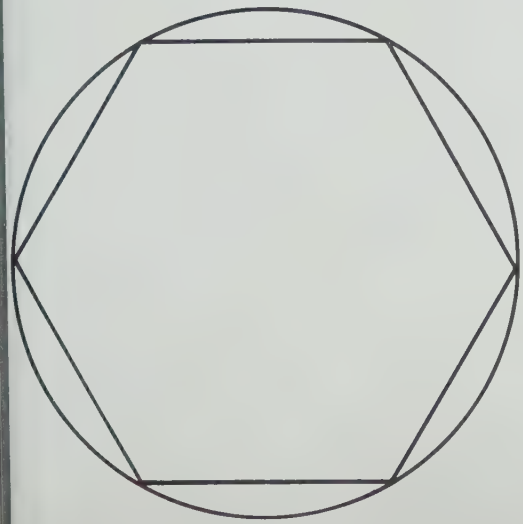
*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: S-1, 2, 6, 7
*Math Applications Kit*, SRA (1971)
Sports and Games cards: 15, 17, 44, 48, 49
*Mathematics Involvement Program*, SRA (1971)
Cards: 364, 375, 266, 366, 317, 357
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 40–41

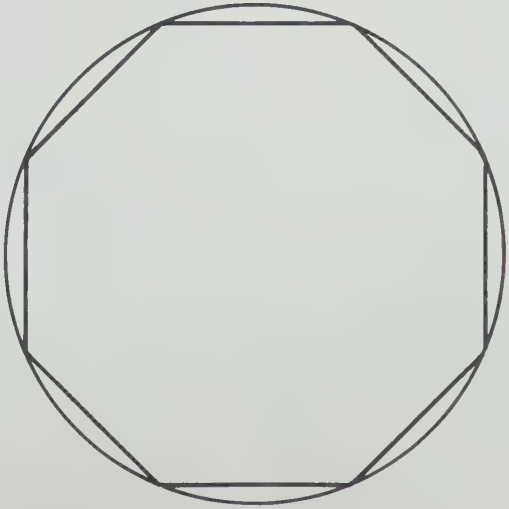
**other learning aids** (described on page 216h)—Block Graph, Good Time Mathematics, Making and Using Graphs and Nomographs, Probability Maze, Probability Set, Towards Probability

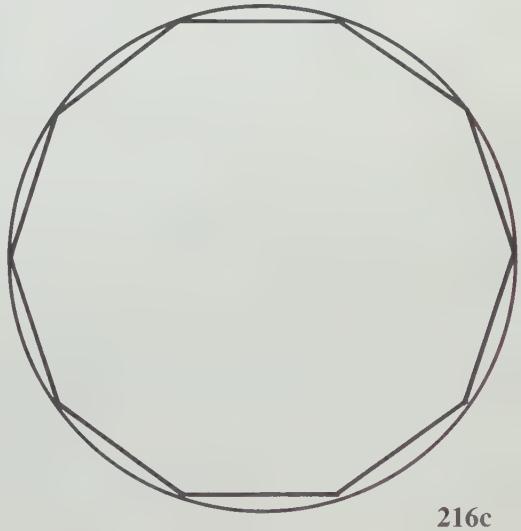
These circular regions fit inside the lid for a disposable coffee cup. You may want to use them for special lessons in the study of fractions or in the study of geometry too.











When people first started keeping track of time and days and years, they noticed three important natural events: day and night; the moon changing; and the seasons changing.

1. Many early calendars were based on 12 lunar (moon) months of $29\frac{1}{2}$ days each. How many days would there be in a lunar year?
2. Soon people began to realize that this didn't match the number of days it takes the earth to go around the sun (the solar year). How many days off were they in a 365 day year?
3. How many days off were they after a half-century?
4. How do you suppose this error affected their seasons?
5. Was February always in the middle of winter?
6. The Egyptians decided to have 12 months of 30 days each. How many days were in their year?

Julius Caesar ruled the Roman Empire over 2000 years ago. He thought the Egyptian calendar was too far off. He made up a new calendar that had a leap year. A leap year has an extra day. It comes once every 4 years. This extra day made up for the extra 6 hours in the solar year. Poor Caesar tried but he still wasn't exactly correct. A solar year is really 365 days, 5 hours, 48 minutes, and 46 seconds. Don't blame him too much. His clocks weren't as good as ours.

1. How far off was the length of the year that Caesar made?
2. About how many years would it take for Caesar's calendar to be off by one whole day?
3. In 1582 Pope Gregory XIII decided that every year divisible by 4 would be a leap year except certain century years. (What's a century year?) Only the century years divisible by 400 would be leap years. Bet he had to work hard to figure that one out!
   a) Was 1776 a leap year?
   b) Was 1914?
   c) Was 1950?
4. Which of the following are leap years? 1100, 1200, 1300, 1900.
5. When is the next leap year? Will the year 2000 be one?
6. How many leap years have there been since Pope Gregory XIII made up the rules we still follow?

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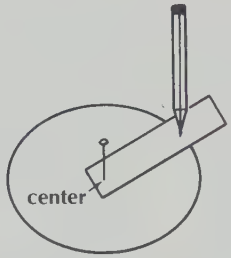
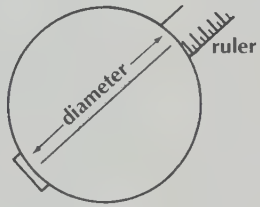
Cut a strip of cardboard about 2 cm wide and about 10 cm long. Make a small pin hole near one end of the strip. Measure 4 cm from the pin hole and make another hole large enough for a pencil point.

Put the strip in the middle of a clean sheet of paper. You can draw a circle with the cardboard strip and a pencil. Do it.




The pin hole is the center of your circle. The greatest distance across the circle is called the ***diameter***. The diameter of the circle you just drew is 8 cm.

What should be the distance from the pin hole to the pencil hole if you wanted a circle with a diameter of 2 cm? 4 cm? 10 cm? 5 cm?



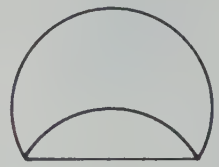

Draw and cut out 4 circular regions with a diameter of 8 cm.

Take one of the circular regions. Fold it so you have 2 equal parts. (Each part would represent $\frac{1}{2}$ of the whole region.) The fold mark also shows the diameter of your circle.

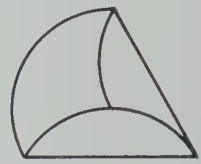
Now fold it again so you can show 4 equal parts. (Each part now represents $\frac{1}{4}$ of the whole region.) You would know where the center of your circle was even though you couldn't see the pin hole. The point where the two fold marks cross marks the center.

Here's one way to fold your circular region so you can mark thirds.

Fold so the rim just touches the center (that's where your pin mark will be).

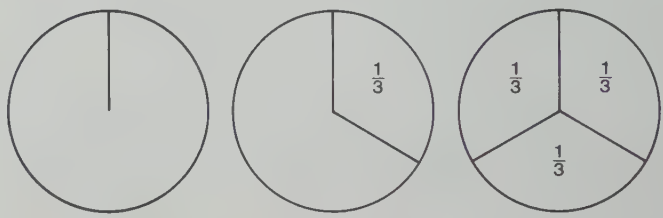


Then fold again. Work from the corner of the first fold.



And fold again. What shape do you have now?

Open your circular region. Use a straightedge. Mark a line from each corner of the triangle to the center. (You can still see the triangle in the fold marks.)




Here's another hard one. Maybe you'll get it. Maybe you won't. But you're on your own on this one. Figure out a way to fold and mark 6 equal parts.

Maybe these clues will help.

How many $\frac{1}{6}$s in $\frac{1}{3}$? Is $\frac{1}{6}$ really the same as $\frac{1}{2}$ of $\frac{1}{3}$?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# 9 ANGLES

**before this chapter the learner has –**

1. Identified square-corner angles in a set of angles
2. Determined by tracing whether two figures are congruent
3. Measured with a centimetre ruler
4. Used decimals with metric measurements
5. Identified curves and straight lines in a set of line drawings

**in chapter 9 the learner is –**

1. Identifying curves and straight lines in a set of line drawings, without error
2. Identifying the sides and vertex of an angle
3. Measuring angles with a protractor
4. Identifying angles that are straight angles, right angles, and greater than a right angle or less than a right angle, using a protractor

**in later levels the learner will –**

1. Master identifying acute, right, obtuse, and straight angles
2. Master measuring an angle of 180 degrees or less using a protractor

# Notes & Things

The learner's knowledge of measurement is extended to include circular measurement. The concept of an angle has been explored in the preceding level, but very few applications were investigated. There was only a hint that angles have size. All that is changed in this chapter.

It is not unusual for experienced teachers to think with horror of the seemingly impossible job of convincing some learners that their store-bought protractor can be read in two different directions. The protractor does present problems. First of all a degree is a unit of circular measure. Yet the youngster's protractor is only half of a circle. Sometimes the mark on the protractor that should be placed on the vertex of an angle is easy to find; sometimes it isn't. And to add insult to injury, Johnny might have a big, fancy expensive one and Junior one that a distant relative used and used and used. It sometimes appears that for every ten youngsters, at least five forgot to bring their protractor and the others all have a different kind.

If you can gather up patience and are willing to spend one hectic day, you'll find that none of those old problems will haunt you.

This chapter explores the measurement of angle by using a full-circle protractor—a homemade one at that. The youngsters won't get the most accurate measurements in the world, but they should come to understand what an angle really is as well as the essence of circular measure before they switch to commercially prepared protractors.

The chapter is full of activities. You want to have this be a "talk together—do together" series of lessons. There are many applications to investigate, and the activities will provide opportunities for the children to learn from one another.

## things

for each pupil: soda straw
masking tape
straightedges
clock
paper for tracing
thin cardboard
paste
scissors
paper clips (optional)
string
transparent tape
protractors
magnetic compass (optional)
centimetre rulers

For the extra activities you will want to have these things available:
- pipe cleaners
- wood rods

**goal** Think about and explore ideas through a picture clue

**page 217** The pupils certainly have an intuitive understanding of angles even though they may not have a mathematical definition. The photograph shows an everyday situation. Once the roof is put on the building, the angle of each of the supports is hidden. Does the carpenter put each of the supports together so that they are the same size and shape? What would happen if they were different? Take time out to think about other kinds of frames. What kinds are there? Surely the pupils will be able to come up with at least the idea of a picture frame and a window frame. Are the corners of each of these frames the same shape and size? What would happen if they weren't? Why not ask for some sketches? Maybe the pupils will come up with some observations about how the size of the angle sometimes affects the length of sides of the frames, or observations about the number of angles in relation to the number of sides.

**Exploration** is the key word. No generalizations are expected now. Measurement of just one angle is the aim in this chapter.

**lesson** Pages 218, 219

**goal** Review of straight lines and curved lines

**memo** Straight and curved lines were introduced in level 5 of this program.

**page 218** Geometry is a very real part of our everyday lives. Less capable learners can benefit from a kinesthetic approach. Let them actually "feel" curves and straight lines on real-world models as illustrated on the page—first with their eyes open, later with their eyes closed. Challenge your students to find examples of curves and straight lines within the classroom, within their homes, and within the community.

It might be interesting to discuss how the other uses of the word **angle** might have originated. "What's your angle?" "He always has an angle on the job," "He angled his way in," "He angled for attention," are just a few examples of the other uses. This discussion may serve to emphasize that the learning goal is to study about angles in mathematics and the use of geometric angles in the real world too.

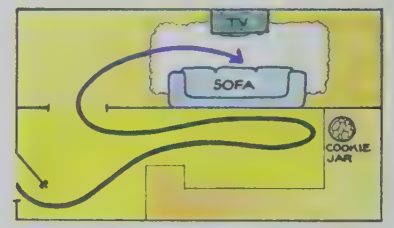
A piece of string or yarn lying on a table would probably look like this.

If you traced the location of the string, would you draw a curved line or a straight line?

You know some geometric shapes are made up of curved lines.

If you look at a map, you could see curved lines used to represent rivers and some roads or streets.

You would probably draw a curve if you traced the path you take after you go in the door at home.




If you looked at the edge of a step into your house or the frame of the door or a window, you would see straight lines. If you were to draw a straight line, would you use something more than pencil and paper?

Yes—straightedge, ruler, or the like

As you pick up objects in the kitchen, you can feel straight and curved lines. You can feel a curve as your fingers go around the top of a can of vegetables or the edge of a bowl, plate, or drinking glass.

You can feel straight lines as you let your fingers touch the edge of a milk carton, a box of cereal, or the edge of a pancake turner.

The angles on these pages are math types. Your goal is to explore what an angle is and what it isn't.

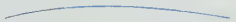
1. Which of the drawings below represent straight lines?

a

b

c

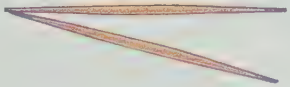
d

e 

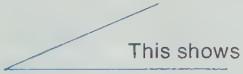
f

2. The focus of this chapter is on angles. An angle must have two straight sides that come together at one point.

Two toothpicks can represent an angle or a bent soda straw can represent an angle.



The kind of angles you will be looking at are formed by straight sides only.



This shows an angle.

This does not show an angle.

a Bend a soda straw so that it looks like this:
The straw takes the shape of an angle.
Does the angle have straight sides? Yes

b Push the sides of your soda straw closer together.
This action makes the angle smaller.

c Let the straw take the shape of a square corner.

A square-corner angle is called a *right angle*.

d Let the straw take the shape of a straight line.
Can you still see the two sides? Yes

This kind of angle is called a *straight angle*.

219

**goal** Introduction to right angles and straight angles

**things** for each pupil: soda straw

**memo** In an earlier level in this program the pupils have tested for square corners with a folded-paper tester. They have found congruent angles by tracing a movable model and have examined angles formed by intersecting lines.

**page 219** The pupil first learned about square corners by tracing a box. As a result of the tracing, the figure on paper then had square corners. Now he will learn that a square-corner angle has another name—RIGHT ANGLE.

A straight angle is not the easiest geometric figure to introduce. The soda straw will be a valuable aid. The bend in the straw will enable the pupil to continue to see the two sides of the angle.

The approach used to explore angles is intuitive. Do not focus on memorizing the technical vocabulary. Rather, through your own correct use of this vocabulary in communicating with your pupils, let the labels become a natural part of each child's vocabulary.

**lesson** Pages 220, 221, 222

**goal** Finding angles greater than, less than, and equal to a straight angle

**things** masking tape
straightedges

**page 220** The focus is on finding angles of various sizes—some greater than a straight angle and some less than a straight angle. This exploratory activity is great for small-group organization. You'll need more than one door. Each group should reach its own generalizations for problems 1, 2, and 3. Unfortunately there are few swinging doors in a school, so problem 4 takes some good thinking. Compare the various findings in a discussion.

For this project you will need masking tape and a door that can be opened.

Open the door a little way. Put down a piece of tape on the floor that shows the location of the open door.

Close the door. Put another piece of tape on the floor to show the location of the door now. Open the door all the way. Do you see an angle formed by the tape? Does the angle have two straight sides? Yes Yes

Now use the tape and the door to make a right angle.

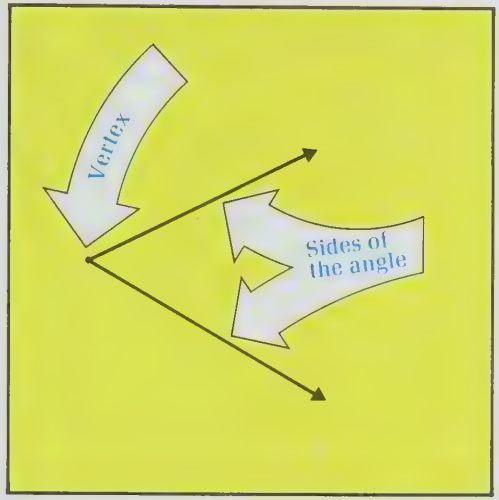
1. Can you use the door to mark a straight angle? Yes
2. Can you use the door to mark an angle larger than a straight angle? Probably not, unless it's a swinging door
3. What kind of door do you need in order to mark an angle larger than a straight angle?
   Swinging or revolving door
4. Think; then draw—
   - **a** angles that can be formed by a door that opens into a room; Any angle ≤ 180°
   - **b** angles that can be formed by a swinging door; Any angle < 360°
   - **c** angles that can be formed by a revolving door. Any angle ≤ 360°

Drawings will vary. Accept—

220

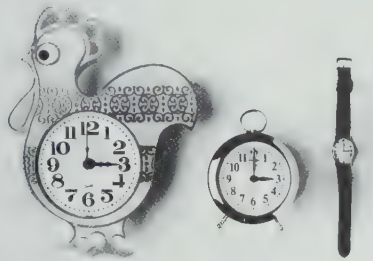
1. An angle is formed by two rays joined at their endpoints. The point where they are joined is called the *vertex* of the angle.
Remember—a ray has one endpoint and continues in a straight line forever.

   The length of the side of an angle in a drawing can be made longer without changing the size of the angle.




2. Look at the hands on your room clock. They show an angle. Each hand forms a side of an angle, and the pin in the center is located at the vertex of the angle.

   The hands of the clocks shown below all show the same angle. The hands are different lengths, but the angles are the same size.



3. Could the same-size angle be shown if the hands pointed to another time? Back up your answer with examples. Be careful. This is tricky.

1:20, 4:35, 3:30, and the like  Manipulation of a real clock will help student's understand that the hour hand moves slightly as the minute hand makes a complete rotation



**goal** Introduction to the definition of an angle; examining the size of angles

**things** clock (optional)

**page 221** The technical definition of an angle is introduced, **not** for memorization but to tie previously learned concepts together. Naming the common endpoint for two rays a VERTEX will be new learning.

Emphasize that the size of an angle has absolutely nothing to do with the length of the two sides—the sides represent rays, and rays go on forever. The size of an angle refers to the opening between the two rays.

A real clock or a geared model clock will be helpful for problems 2 and 3. Students can actually see the movements of the rays as the rotation is performed.

**goal** Examining the size of angles and finding congruent angles

**things** paper for tracing
straightedges

**memo** The student has had no experiences with angles greater than a straight angle (reflex angles). Therefore you need not be concerned about the angle openings not being marked with an arc ∡. Reflex angles are introduced on page 229. The reflex angle will always be marked.

**page 222** Continue to focus on the size of the opening between the two rays. This opening determines size of the angle.

Making a tracing to test for congruency is a familiar technique. Straightedges will help keep the rays straight.

# What makes one angle larger than another?

1. Look at these clocks. Each shows a different hour.

   **a** Is the angle formed by the clock hands in *a* smaller or larger than the angle in *b*?

   **b** Is the angle in *c* larger than the angle in *b*? Yes

   **c** Which of the four angles is largest? d

2. Draw ten angles of your own. Make some less than the size of a right angle. Make some greater. Angles will vary.

   Do any of your angles look as if they might be the same size? One way to find out if they are the same size is to trace one angle and put the tracing on top of another angle. Probably

3. There are three same-size angles below. Trace and match to find them. B, C, D

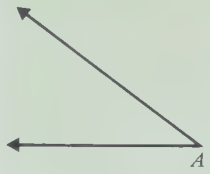
   The length of the sides didn't make any difference, did it? No
   The size is determined by the opening of the sides.
   Angles that have the same opening are congruent.

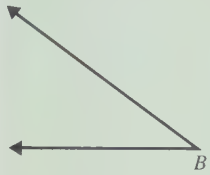
222

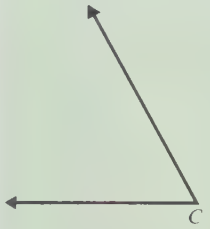
See activity 1, page 238a.

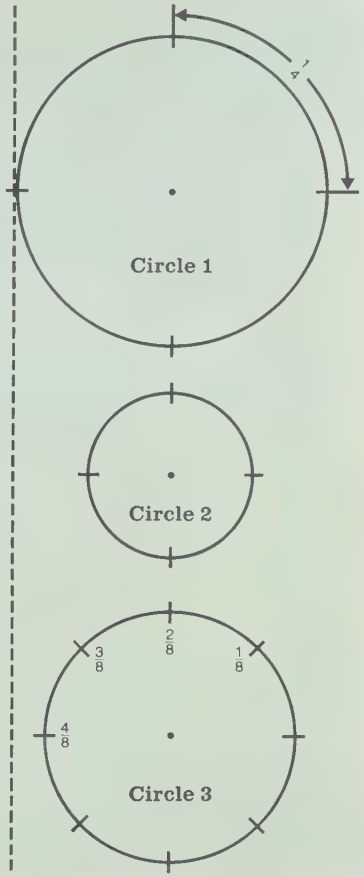
See activity 2, page 238a.

We can see how big an angle is by measuring it with a circle. Circle 1 has been divided into 4 equal parts. We can measure an angle and tell whether it is less than, equal to, or greater than $\frac{1}{4}$ of a circle. Trace all the angles and circles and the broken line, exactly as shown. Be sure your paper doesn't slip.




1. Fold your tracing on the broken line, from right to left. The vertex of each angle will be at the center of a circle if you folded carefully. Look at angle $A$. Is it less than $\frac{1}{4}$ of circle 1? Yes

2. Now look at angle $B$. Measure it with circle 2. Is it less than $\frac{1}{4}$ of a circle? Yes
It should be; it's the same size as angle $A$.

3. Circle 3 is divided in 8 equal parts. Is angle $C$ greater than, less than, or equal to $\frac{1}{8}$ of a circle?

SAVE YOUR TRACING

223

**lesson** Pages 223, 224

**goal** Developing readiness for using a protractor

**memo** The tracings from page 223 are used again on page 224.

**page 223** Having the pupil clip his paper to the text page may help keep the paper from slipping while he makes his tracing. Don't forget that the fold mark must also be traced. The dot in the center of each circle is important too. Each circle should be labeled with its corresponding number and each angle vertex with its corresponding letter. This will free the youngster from the need to refer back and forth to the models on the page while completing his answers to the questions.

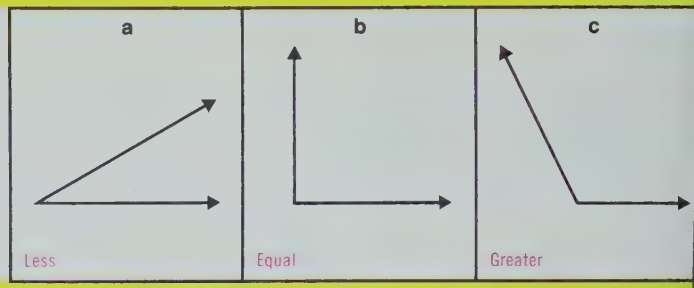
**goal** Readiness for using a protractor

**things** tracings from page 223

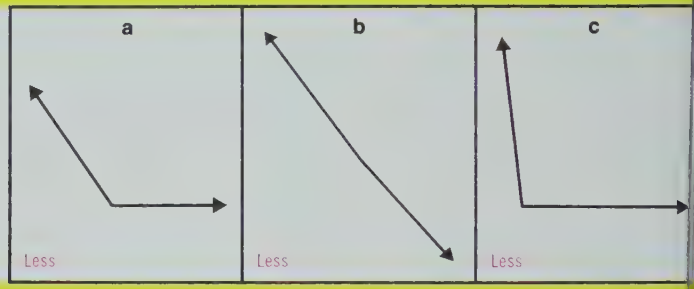
**page 224** The dot in the center of each circle is very important. On page 223, the vertex of the angle should have landed right on the center of the circle after the paper was folded—provided that the tracing was fairly accurate. In this activity the pupil must match the center of the circle with the vertex of the angle to be measured.

Be careful with problems **2b** and **3c**. There could be some argument. If two people disagree, ask them to **prove** their answers.

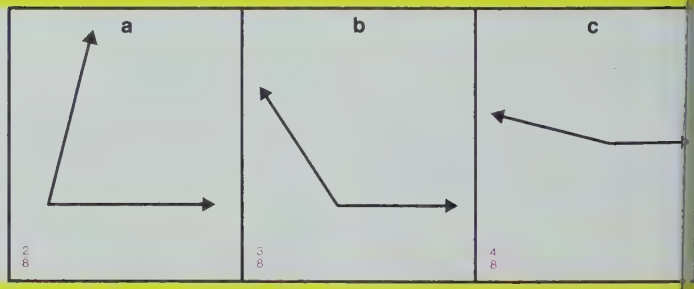
1. Use your tracing of circle 2 from the preceding page to measure each of these angles. Tell whether they are greater than, less than, or equal to $\frac{1}{4}$ of a circle. Be sure the center of the circle is at the vertex of the angle.




2. Use your tracing of circle 2 and tell whether these angles are greater than, less than, or equal to $\frac{2}{4}$ of a circle.




3. Use your tracing of circle 3 to measure these angles. Write the name of the fraction of a circle their size comes closest to.




224

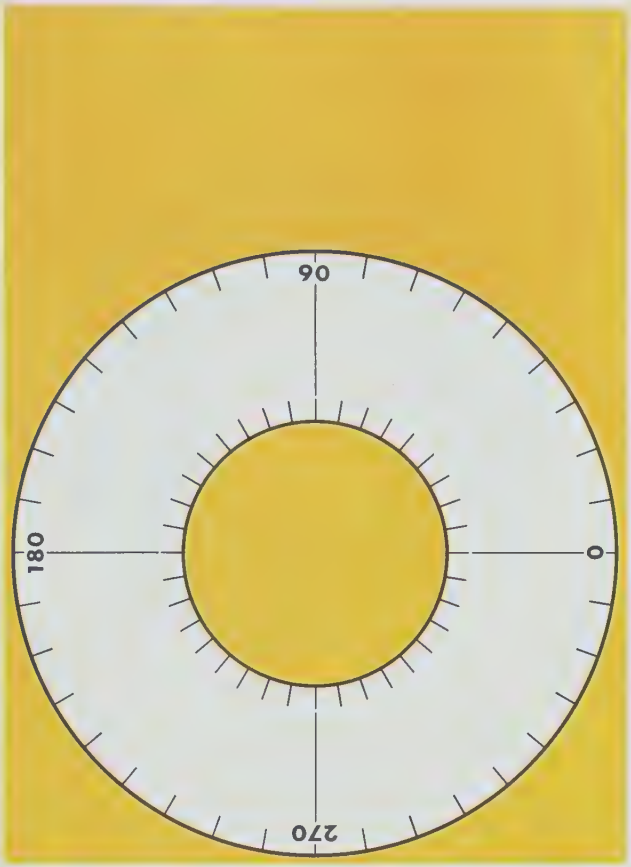
See activity 3, page 238b.

Many, many years ago it was decided to divide a circle into 360 equal parts. An angle that has the same size as one part of the 360 parts looks like this:

It was agreed that the size of this angle would be called 1 degree. (That name does not mean the same as the temperature measure called degree. It's too bad somebody didn't pick another name.)

The instrument used to measure an angle opening is called a protractor. Make one of your own. Trace this.

Cut out your tracing and paste it on thin cardboard. Then cut the cardboard around the outside and center of your tracing.




225

**lesson** Pages 225, 226

**goal** Introduction to degree as the unit of measure for angle measurement and protractor as the measuring device

**memo** You may choose to prepare a spirit master rather than have each pupil trace the protractor as directed on the page. Or the pupils could use commercially made protractors. But please appreciate that the full-circle protractor will yield a better understanding of this type of measurement.

**things** paper for tracing
thin cardboard
paste
scissors
paper clips (optional)

**page 225** Paper clips may help steady the paper while the tracing is being made. The majority of finished products will be messy, but no accurate measurement is expected and each pupil will have an understanding of what a protractor really is.

**goal** Completing construction of a protractor

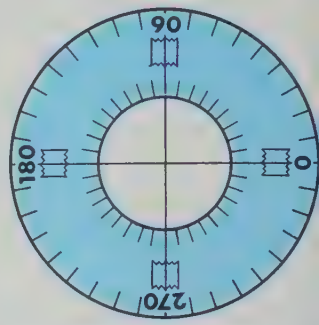
**things** string
transparent tape

**page 226** The pupils will probably have to help one another put the two strings in place. Keep reminding yourself that this is a valuable learning experience.

# Have patience.

## 1.

If you put your protractor on top of an angle now, you would not have a way to match the center of the protractor and the vertex of the angle.




**a** Stretch a piece of string across the hole. Tape it down so that one end of the string is on the 0-degree mark and the other is on the 180-degree mark.

**b** Tape another piece of string so that one end is on the 90-degree mark and the other end is on the 270-degree mark.

Your measuring instrument is nearly finished.

**c** Look at the hash marks around the edge. How many between 0 and 90? 8

**d** What does each mark stand for? 10 degrees

**e** Label each on your protractor.

## 2.

How many marks should be between 0 and 10? 9

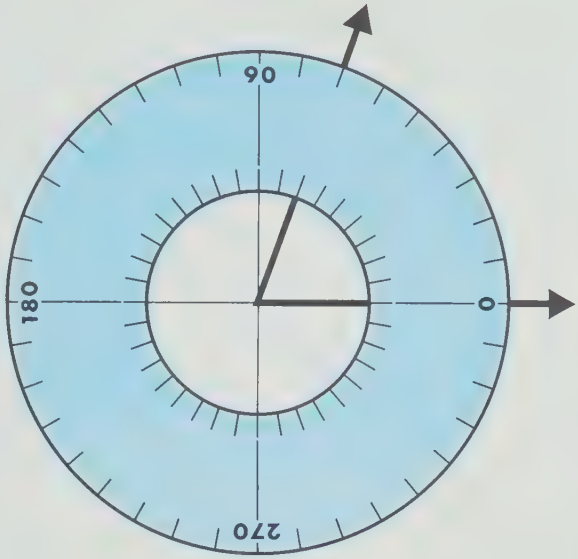
**a** Try to put these on your protractor too. There isn't enough room to label them.

**b** If you made the 1-degree marks around the entire edge, how many marks would you have? 360

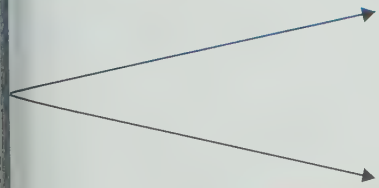
Now you are ready to do some measuring!

226

Get ready to measure an angle with a protractor. Place the protractor over the angle so that the vertex of the angle is at the center and one side of the angle is at 0. Then, read up from zero to where the other side of the angle crosses the edge of the protractor. That number tells you the number of degrees in the angle. In the diagram at the right, the angle measures about 70 degrees.




# MEASURE THIS ONE



Did you get 25 degrees? Depends on accuracy of protractor

227

**lesson** Pages 227, 228, 229, 230

**goal** Introduction to measuring with a protractor

**memo** Handmade protractors are not likely to be 100 percent accurate. The pupils are asked to measure various angles. Since these measuring instruments are crude and the youngsters have had no previous experience with protractors, allow leeway of at least 2 degrees in either direction from the answers given in the answer key.

**things** protractors

**page 227** At last, after all that work, an angle to be measured. Have the pupils put their protractors down on top of the protractor in the book. Then they will appreciate that the point at which the strings intersect must match the vertex of the angle. Remind them that one side of the angle must also match the zero-degree mark.

If someone says the angle measures 20 to 30 degrees, he is either positioning his protractor incorrectly or his protractor is really too crude to use at all.

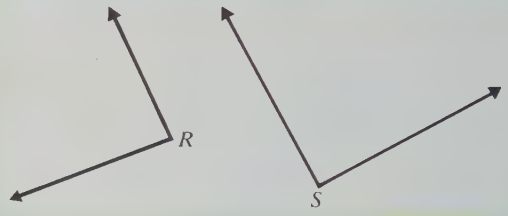
**goal** Practice in measuring with a protractor

**things** protractors

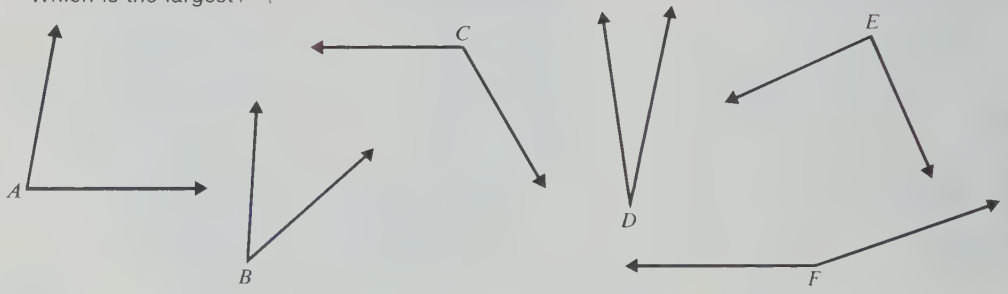
**page 228** Everyone may find it helpful to join with a line segment the marks on the inside edge of his protractor to the corresponding mark on the outside edge. This will allow the pupil to see the angle when the protractor is placed over the angle.

Look out! The angles are shown in different positions. If you think there will be trouble distinguishing the inside angle to be measured from the outside angle, you may want to draw on the chalkboard a facsimile of angles *A* and *B* of problem 2 and point out the inside of each angle.

**1.** Measure these two angles.
Which is larger? S
How many degrees larger? 5




**2.** Measure these angles.
Which is the largest? F




**3.** Can you find the right angle above? E
How many degrees in a right angle? 90

**4.** Complete. Use the relation symbols $>$ and $<$.

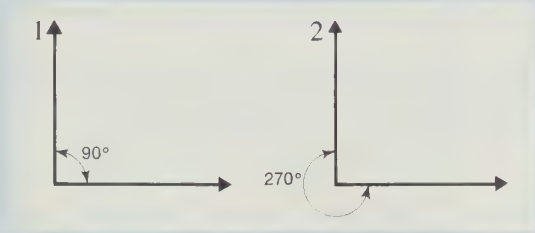
**a** angle $A$ $\overset{>}{?}$ angle $B$ **b** angle $C$ $\overset{<}{?}$ angle $F$ **c** angle $B$ $\overset{<}{?}$ angle $E$

**d** angle $D$ $\overset{<}{?}$ angle $B$ **e** angle $C$ $\overset{>}{?}$ angle $E$ **f** angle $E$ $\overset{>}{?}$ angle $A$

228

See activity 4, page 238b.

Have you noticed that there are actually two angles in one? Look at the angles at the right. The arrows show the two ways that you can measure an angle.




In this chapter measure the smaller angle unless an arrow is pointing to the larger angle.

Name the angles that measure—

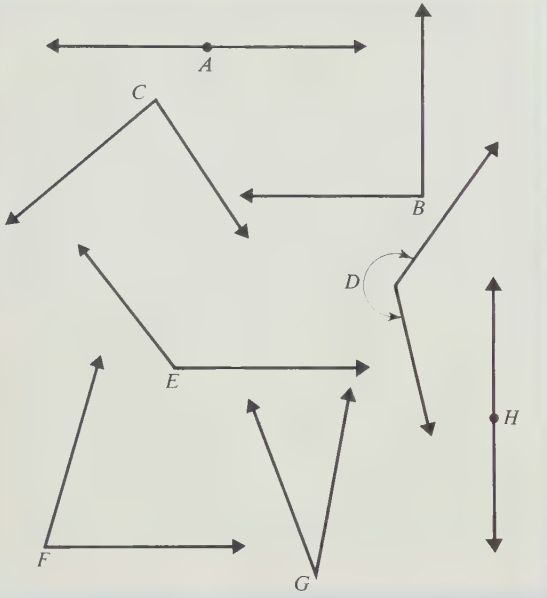
1. less than 90 degrees. G, F, C
   These angles are called *acute angles*.

2. 90 degrees. B
   These angles are called *right angles*.

3. 180 degrees. H, A
   These angles are called *straight angles*.

4. more than 90 degrees
   BUT less than 180 degrees. E
   These angles are called *obtuse angles*.

5. more than 180 degrees. D
   These angles are called *reflex angles*.




229

See activity 5, page 238c.

**goal** Introduction to acute, obtuse, and reflex angles

**things** protractors

**page 229** There are lots of names introduced. Go nice and easy. For now, memorization of vocabulary isn't as important as knowing that various-sized angles do have special names.

Angles *A* through *H* will need to be measured in order to answer the questions for problems 1 through 5.

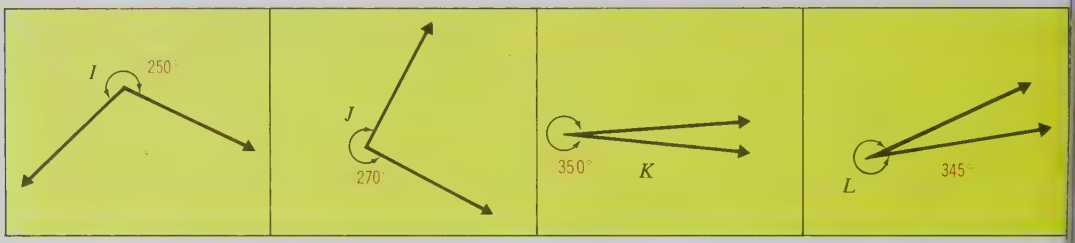
**goal** Measuring more angles

**things** protractors

**page 230** Note that for problem 1 the pupils are directed to measure the reflex angles. They will have real trouble if a full-circle protractor is not used.

Be prepared for a sharpie who says that he doesn't have to measure the two angles in either problem 3 or problem 4 because he can see that the sum of the angles in each problem is a straight angle. He's right!

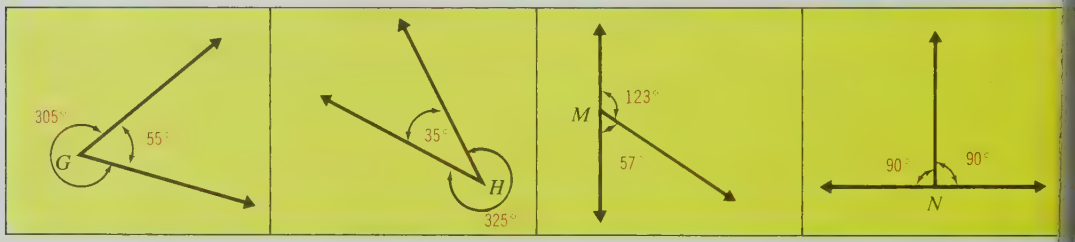
**1** Measure these angles.
Then tell what kind of angle it is.
All are reflex angles.




**2** Measure both angle $G$ and angle $H$.
- **a** Measure the acute angle.
- **b** Measure the reflex angle.
- **c** How many degrees in the sum of the two angles? 360°

**3** Measure and find how many degrees in the sum of these two angles. 180°

**4** How many degrees in the sum of these two angles?




230

See activity 6, page 238c.

Use your protractor. Measure and label angles you see around you. Are any two the same size?
There should be many right angles.
Which are the largest angles you found?
Which are the smallest? Answers will depend on angles chosen.

Use the clock in your room to help you answer these questions.

1. How large an angle are the hands forming?
   Answers will vary, depending on time question is answered
2. What angle is formed at two o'clock? About 60°
   at ten o'clock? About 60°
3. At what hours do the hands form right angles? With the minute hand at 12, the hands form right angles at 3:00 and 9:00.
4. At what hours do the hands form two angles greater than 90 degrees? (with minute hand at 12)
   4:00, 5:00, 6:00, 7:00, 8:00

Teacher note: On problems 3 and 4 above, if the minute hand is not restricted to 12, there are many other possible answers.

*5. Look for angles formed by these intersecting lines.

   a How many acute angles? 10

   b How many right angles? 4

   c How many obtuse angles? 10

   d How many reflex angles? 24



**lesson** Page 231

**goal** Exploration of real-world angles

**things** clock
protractor

**page 231** Angles are everywhere—on faces of boxes, formed by an open book, a folded paper, the movement of fingers. They can be seen in the construction of a jungle gym or other playground equipment, on bridges, and in building construction. The purpose of this activity is twofold:

- To develop an awareness of angles in real-world situations
- To provide more practice in measuring with a protractor

Watch for problems 3 and 4. Problem 3 has the obvious answers—3:00 and 9:00. But there are at least 20 additional specific hand positions that form right angles. Problem 4 also has many possible specific answers. These two problems make interesting challenges for young mathematicians. Have them verify one another's findings.

Problem 5 is a real challenge and should be an independent activity for your most capable students. A group discussion with the other youngsters will help you keep them from forming erroneous ideas.

**lesson** Pages 232, 233

**goal** Introduction to the half-circle protractor and the degree symbol

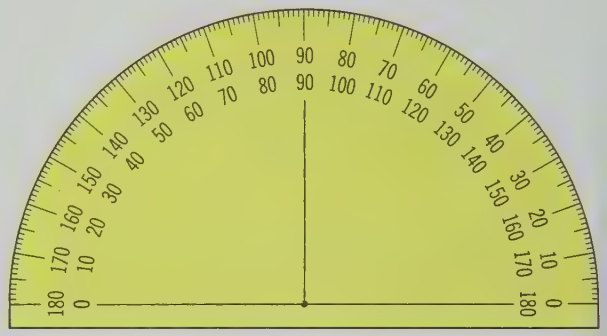
**things** commercially made or pupil-made protractors

**page 232** If you have a set of commercially made protractors, now is the time for hands-on experience. Your pupils may still not be proficient in measuring with this more accurate instrument. Therefore, continue to allow at least a 1-degree error in either direction from the answer given in the answer key.

Writing the word *degree* takes time. There is a symbol for the word. It is a small raised circle:

90° means ninety degrees.

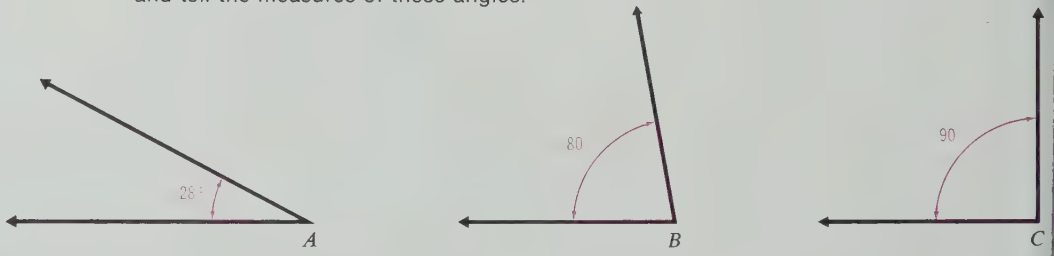
The protractor you made shows 360° in all. Most protractors that you buy in a store show only 180°. They are shaped like a half circle.



They might be larger or smaller than the one pictured above. There is one difference between the way the numerals are marked on the ones purchased from a store and the way they are marked on your handmade one. What is the difference? Mark your protractor the same way.

The inside scale goes in a direction opposite the outside scale.

1. Use your new 0 location, read the inside scales, and tell the measures of these angles.

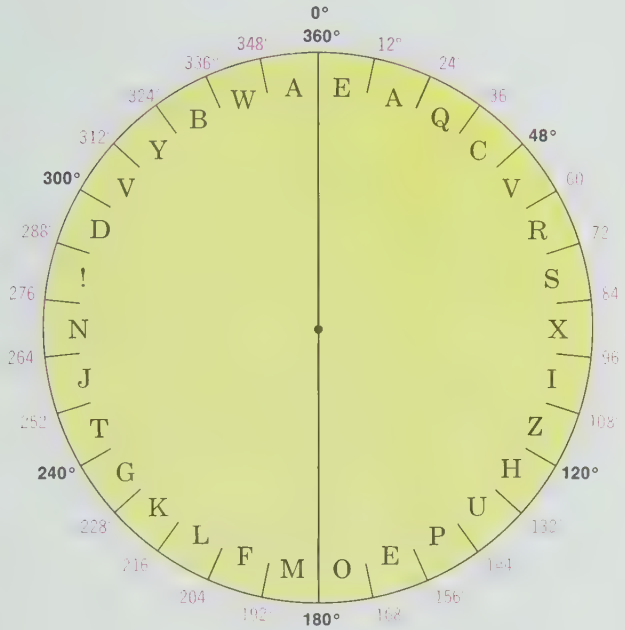



232

You've stumbled across a secret code. And you have finally found the key to it. It is a circle divided into 30 equal sections. Each section measures 12°. And each section has a special meaning.

First trace this special protractor. Then write the degree measure for each mark. You may use your protractor for this.

Now try to crack the code!
Each degree measure can be replaced by a letter. Can you figure out the message?




| Y | O | U | | H | A | V | E | |
|---|---|---|---|---|---|---|---|---|
| 323° | 177° | 143° | * | 130° | 15° | 310° | 8° | * |

| T | H | E | | I | D | E | A |
|---|---|---|---|---|---|---|---|
| 242° | 128° | 166° | * | 100° | 293° | 6° | 350° |

## Get it?

| G | O | | T | O | | I | T | ! |
|---|---|---|---|---|---|---|---|---|
| 238° | 173° | * | 243° | 169° | * | 98° | 242° | 284° |

Now make up some coded messages of your own.
Can your friends figure them out?

Answers can vary. Example:

| S | E | C | R | E | T | * | C | O | D | E | S | * | A | R | E | * | F | U | N | ! |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 75° | 159° | 40° | 65° | 3° | 245° | | 47° | 179° | 290° | 10° | 80° | | 350° | 70° | 157° | | 200° | 143° | 267° | 280° |



**goal** Measuring degrees of a circle

**things** protractors

**memo** This page is meant just for fun.

**page 233** The pupil is free to use computation or his protractor to figure out the number of degrees for each section. He may begin with the protractor, see what is happening, then switch to computation. Let this be his decision.

Don't be surprised when some youngsters volunteer to make up coded messages of their own. These can be exchanged among classmates.

**lesson** Pages 234, 235, 236, 237

**goal** Application of angle measurement

**things** protractors
magnetic compass (optional)

**page 234** Scouts who have used a magnetic compass may already know about HEADINGS. If you or a scout has a compass, you might want to compare reading this instrument with the protractor technique presented on the page. Discuss the need for accuracy in measuring a heading.

# IT'S A LONG, LONG TRAIL TO ANGLE

Tommy Trailblazer gave the Explorers Club directions to get from Forest Service Camp to Bright Angle Lodge. He told them what headings they should follow. A *heading* is a degree measure that tells how far a given direction is from north.

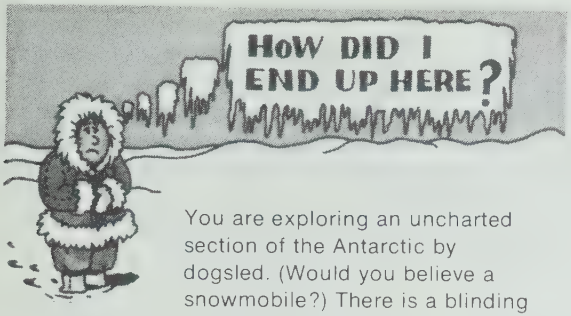
He wanted to make sure everyone knew exactly where they were going, so he gave them headings to checkpoints. He said, "The heading to follow from the Camp to Old Flat Top is ? degrees east of north."

1. Use your protractor. Find the heading Tommy gave to get from the camp to Old Flat Top. 55° east of north

2. Then Tommy told them to go on the heading ? degrees west of north to get to Lost Ledge Mine. What heading did they follow? 35° west of north

3. On the last leg of the trip Tommy told them to follow a heading of ? degrees east of north. What was their heading from Lost Ledge Mine to Bright Angle Lodge? 45° east of north

234




You are exploring an uncharted section of the Antarctic by dogsled. (Would you believe a snowmobile?) There is a blinding snowstorm. You head straight south towards your camp, but your compass is off by 10°.

1. You travel 2 km . Notice on the map that you are already more than 0.3 km off course.

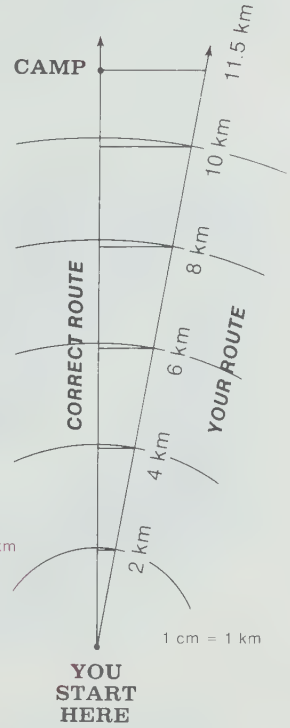
2. You travel 2 km farther. You are farther off course than before. About how far? About 0.7 km

3. Are you farther off course after 6 km? Yes
   At least a kilometre off? Yes

4. How far off course are you after travelling 8 km? About 1.4 km
   After 10 km? About 1.8 km

5. After travelling 11.5 km on the wrong path, you are straight across from your camp, but quite far from it. Measure and record this distance. About 2 km

A 10° mistake in the Antarctic is serious, isn't it?




235

**goal** Application of angle measurement

**things** centimetre rulers

**page 235** The focus is on the need for using an accurate measurement instrument. Notice the map scale. Your students will need their centimetre rulers to measure the distance off course. Computations will be performed, using decimals.

When the page is completed, take time for some role playing. How would you feel finding yourself 2 kilometres off course in the Yukon? Could such a thing happen? How could it be avoided?

**goal** Application of angle measurement

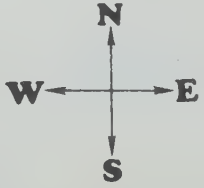
**things** protractors

**page 236** Strictly for discussion. Consider what **could** happen, but reassure the nervous that this is not likely to happen with large commercial airlines. On the other hand, consider the independent pleasure flyer. His error may land him in a cornfield.

# I WON'T RIDE WITH YOU AGAIN

Planes use very complicated electronic equipment to keep on course. Imagine what would happen if the equipment were not so accurate. Suppose a plane were *only* 1° off course. If that plane were flying from Montreal to Vancouver, it might end up in the Coast Mountains some 55 km away.

Use your protractor to see if the incorrect course shown on the map is indeed 1° off. Yes




Do you see how important a 1° error can be? This plane flew only about 3235 km, and it was almost 55 km off course. Can you guess what would happen if a plane flew 1° off course for many thousands of kilometres.

Yes. Discuss. Flying from Montreal or New York City to Honolulu. You could be almost 160 km off course and miss Oahu entirely. Or flying from Montreal to Melbourne, Australia, you could be almost 320 km off course and miss the continent of Australia.

236

# SO NEAR, AND YET SO FAR

Suppose you were planning a trip to Venus in a spaceship. Venus is 38 640 000 km away. At that distance it would take you almost 5 months to get there. If your flight path were 1° off course, you would miss the planet. Can you imagine by how many kilometres you would miss Venus? Many kilometres

What if you were flying to Pluto and you were 1° off course? Can you imagine by how many kilometres you would miss that planet? Even if you were off course by only 0.5°, you would still miss it by thousands of kilometres. Pluto is 4 266 500 000 km away. It would take you 30 years to travel that distance! Can you imagine travelling for 30 years and then missing the planet? You couldn't let that happen, so you would need equipment that measures angles smaller than 1°.

If you would like to know more about very small angles, you may want to read a book on astronomy or space travel.

237

See activity 8, page 238d.

**goal** Introduction to very small angle measure

**memo** This page is used strictly as an extension

**page 237** A discussion page. The concept of very large distances and very slight changes of direction which result in huge errors or miscalculations may stimulate the imagination of some pupils and lead them to research in the field of space travel. Smaller units of angle measure, minutes and seconds, are not introduced here, but if pupils come upon them in their reading and mention them in discussion, this will extend the scope of their concept of measurement. Very precise, fine measurements are vital to navigation and space travel.

**lesson** Page 238

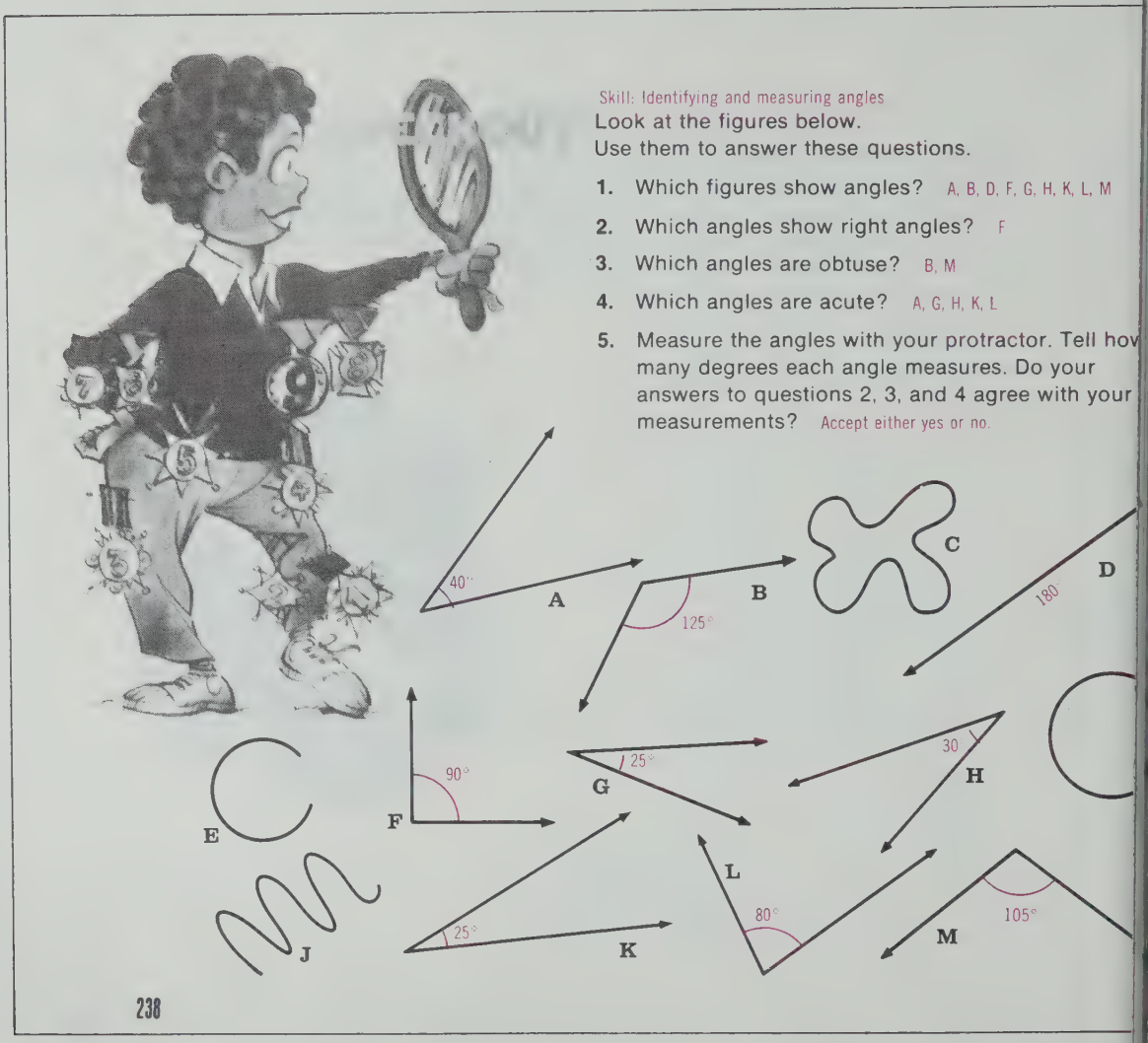
**goal** **Checkout**—identifying and measuring angles

**things** protractors

**page 238** Angle measurements either 1 degree greater or less than the answer given are satisfactory. Youngsters at this level are not expected to have completely mastered the skill of reading the measure of an angle with a protractor. If pupil-made protractors are used, allow a 2-degree error in either direction.

Watch for youngsters who confuse the labels **obtuse** and **acute**. They will have grouped the angles correctly—those greater than 90°, but less than 180°, those less than 90°—and then written the name for the wrong problem. They have the concept but not the vocabulary.

The pupil who can answer problems 1 and 2 correctly will have met the mastery objectives for this level.



See activity 9, page 238d.

See activity 10, page 238d.

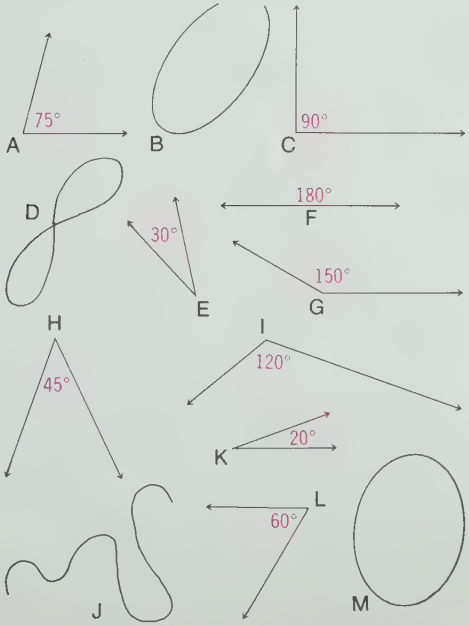
# RESOURCES

## another form of evaluation

**for checkout** – page 238

Look at the figures below. Use them to answer these questions.

1. Which figures show angles?
   A, C, E, F, G, H, I, K, L
2. Which angles are right angles? C
3. Which angles are obtuse? G, I
4. Which angles are acute? A, E, H, K, L
5. Measure the angles with your protractors. Tell how many degrees each angle measures. Do your answers to questions 2, 3, and 4 agree with your measurements?




## activities

**1. things** straws; pipe cleaners; wood rods

With pupils who are not fully convinced that the size of an angle does not depend on the length of its sides, construct additional models (some congruent, some not) and compare them.

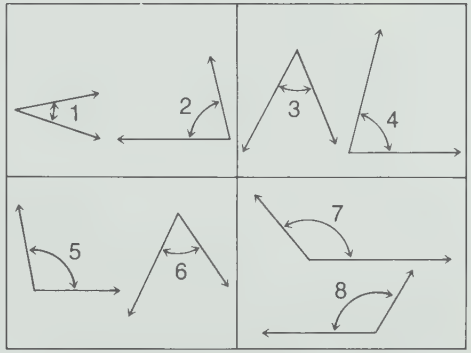
These models can be constructed of straws of various lengths connected with bent pipe cleaners or wood rods from a child's building set.

**2. things** spirit master as shown

WHICH IS LARGER?

Part I

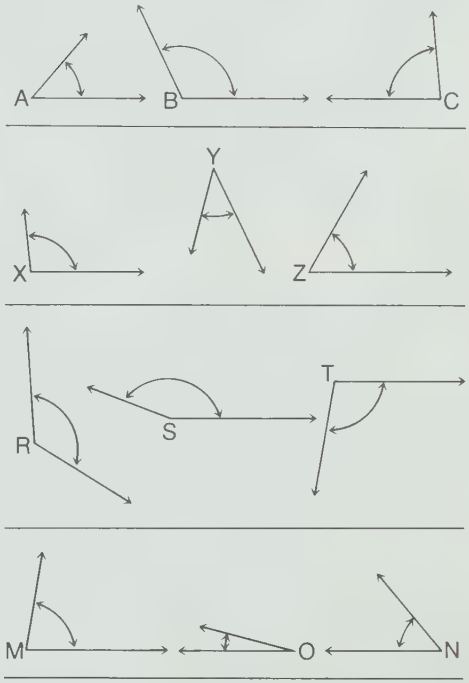
Without measuring, which angle in each pair seems to have the greater amount of opening between the sides? (The double-headed arrows indicate the angles to look at.)




Answers: ∠2, ∠4, ∠5, ∠7

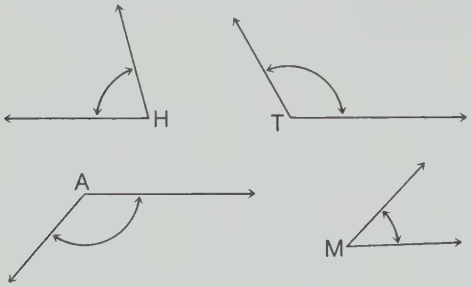
Part II

Arrange each set of angles in order from smallest to largest.




Answers: ∠A, ∠C, ∠B
∠Y, ∠Z, ∠X
∠T, ∠R, ∠S
∠O, ∠N, ∠M

Part III
For each angle pictured, draw one angle that is smaller and one angle that is larger.

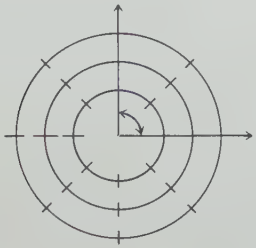


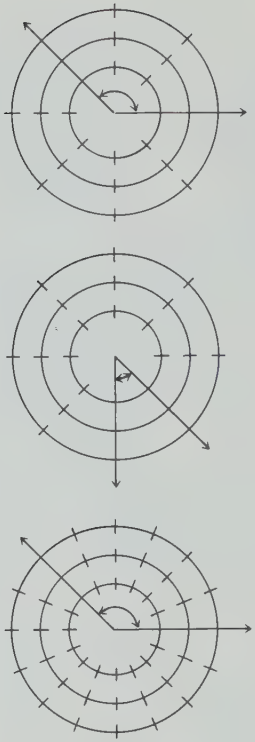

**3. things** spirit master as shown

BUT THE CIRCLES
ARE DIFFERENT SIZES!

Each drawing shows 3 circles and an angle whose vertex is at the center of each circle. Use these drawings to help you answer the following questions.

1. Each circle is divided into how many equal parts?
2. Using the smaller circle, the measure of the angle is what part of the circle?
3. Using the larger circle, the measure of the angle is what part of the circle?





Something to think about.

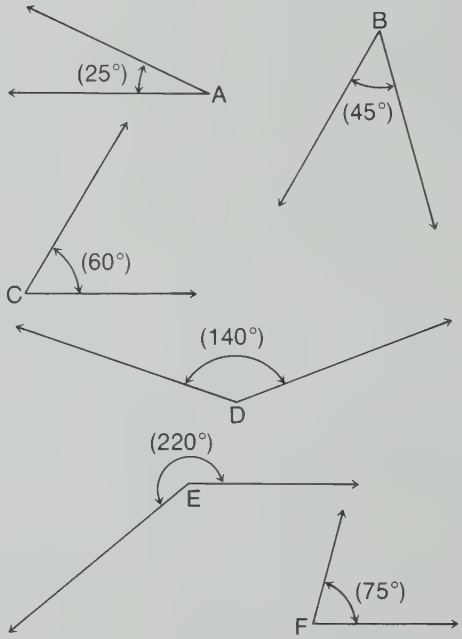
1. For each angle, compare the measurements obtained by using the smaller, middle-size, and larger circle. Were they the same or different? (Same) Were you surprised?
2. Does the size of the circle used to measure an angle have any effect upon the measure of the angle? Why, or why not? (The size of an angle depends on the amount of opening between its sides.)

**4. things** protractors; spirit master as shown

BUT THE SIDES ARE LONGER!

Follow these instructions for the angles pictured below.

1. Use your protractor to measure each angle in degrees. Record your measurements.
2. Make the sides of each angle longer.
3. Measure each angle again. Record your measurements.



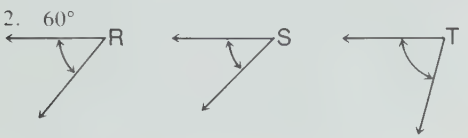

For each angle, compare your measurements—obtained before and after you extended the sides. Were they the same or different? Were you surprised? Does the length of the sides of an angle have any effect upon the size of the angle? Why or why not? (Extending the sides does not change the size of the opening.)

**5. things** protractors; spirit master as shown

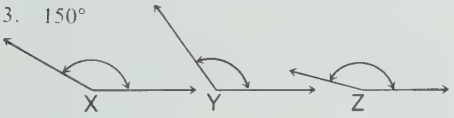
Each problem shows three angles. Without measuring, estimate which angle is closest to the measure given.

1. 25°

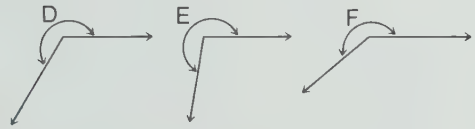
2. 60°



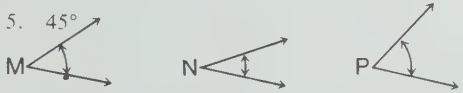

3. 150°




4. 240°




5. 45°

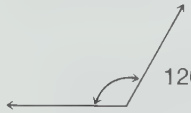


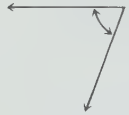

Now check the accuracy of your guesses by measuring each angle in degrees with your protractor. Were you surprised at any of the results?

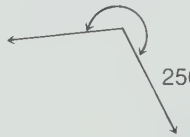
Note: The first angle in each set is closest to the given measure. Angle order should be randomized when you prepare the spirit master.

**6.** For each problem, select the best measure for the angle that is pictured. Do not use your protractor.

1. 40° 30° 20° 50°

2.  120° 100° 140° 110°

3.  70° 80° 60° 50°

4.  250° 110° 230° 260°

5. 160° 170° 150° 140°

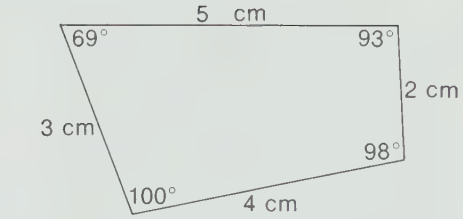
Check the accuracy of your estimates with a protractor. Are you getting better at estimating the size of an angle?

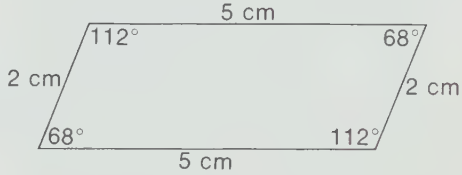
Note: The first measure in each set of possible answers is the best measure. You will want to change the order of the choices for each set when you prepare the spirit master.

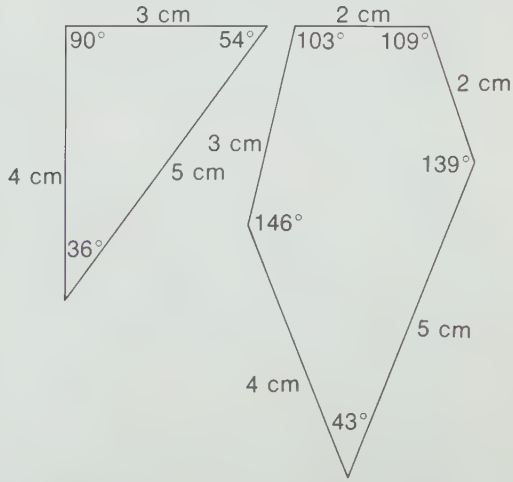
**7. things** centimetre rulers; protractors; spirit master as shown. (Do not include the measurements for the pupil.)

MAKE A COPY

Using a ruler and protractor to measure the sides and angles, make drawings that have the same size and shape as the ones below. Measure angles in degrees and lengths in centimetres.




How can you test whether your drawings have the same size and shape as the original figures? Test them.

**8.** Youngsters who are interested in space and astronomy may enjoy these books if they are in your school or city library.


Asimov, Isaac. *Environments Out There*. New York: Abelard-Schuman, 1967.

Freeman, Mae and Ira. *Fun with Astronomy*. New York: Random House, 1953.

Haber, Heinz. *Stars, Men, and Atoms*. Rev. ed. Racine, Wis.: Western.

Pickering, James S. *Windows to Space*. Boston: Little, Brown, 1967.

Posin, Dan Q. *Exploring and Understanding Our Solar System*. Westchester, Ill.: Benefic Press, 1968.

Reed, W. Maxwell. *Patterns in the Sky: The Story of the Constellations*. New York: Morrow, 1951.


**9. things** ruler; protractor

Using only a straightedge or ruler to draw straight lines, draw an angle that you believe has a measure of:

| | | | |
|---|---|---|---|
| 1. | 30° | 5. | 235° |
| 2. | 60° | 6. | 45° |
| 3. | 90° | 7. | 10° |
| 4. | 150° | 8. | 120° |

Now check the closeness of each of your guesses by measuring each angle drawn. Find the difference between the measure of each angle you drew and the measure that it was supposed to be. (If you came within 10°, you did a very good job.)

1. How many angles did you draw within 10° of the size they were supposed to be?
2. Which angles were the hardest for you to draw? the easiest?
3. What method did you use to draw angles of the specified size?

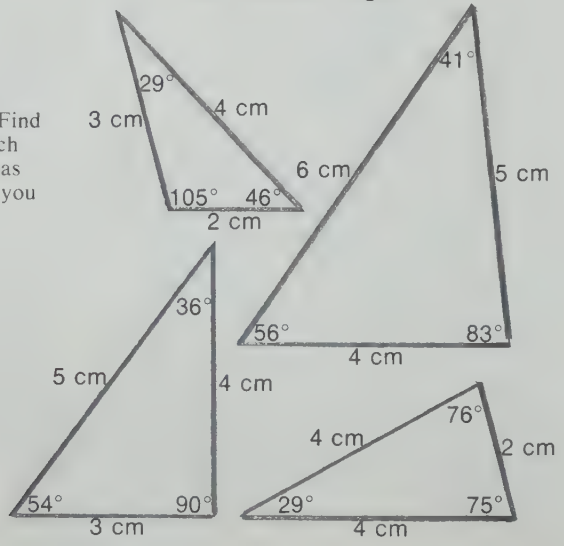
Note: No estimate is incorrect. More practice may be needed before the pupil can come within 10° of accurate measure.

**10. things** centimetre rulers; protractors; spirit master as shown (Do not include the measurements for the pupil.)

ANY HUNCHES?

For each triangle, do the following:

1. Measure each angle to the nearest degree.
2. Measure each side to the nearest centimetre.
3. List the angle measures in order of size from smallest to largest.
4. List the lengths of the sides in order of size from smallest to largest.




Something to think about.

- What seems to be the relationship between the measure of each angle and the length of the side opposite the angle?
- Write any other relationships that seem to be true on the basis of your measurements.

Note: No attempt should be made to teach these relationships now. This is a time to explore. Possible answers:

- The angle opposite the longest side of a triangle has a larger measure than either of the other two angles.
- The shortest side is opposite the smallest angle.
- The sum of the angles of a triangle is 180°.

## additional learning aids

**geometry**—chapter objectives 1, 2

**SRA products**

*Skill through Patterns, level 6*, SRA (1974)
Spirit master: 63
*Visual Approach to Mathematics, Geometry*, SRA (1967)
Visuals: 5–9

**other learning aids** (described on page 288d)—Geoboard Activity Cards (intermediate set), Geoboard Kit, Good Time Mathematics

**measurement**—chapter objectives 3, 4

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit master: M-12
*Math Applications Kit*, SRA (1971)
Science cards: 44, 50
Sports and Games cards: 9, 30, 31, 42
Social Studies cards: 8, 9, 11
Occupations card: 10
*Mathematics Involvement Program*, SRA (1971)
Cards: 245, 315, 346
*Skill through Patterns, level 6*, SRA (1974)
Spirit master: 64

**other learning aids**—Mira, Mira Math for Elementary School, Paper and Pencil Geometry

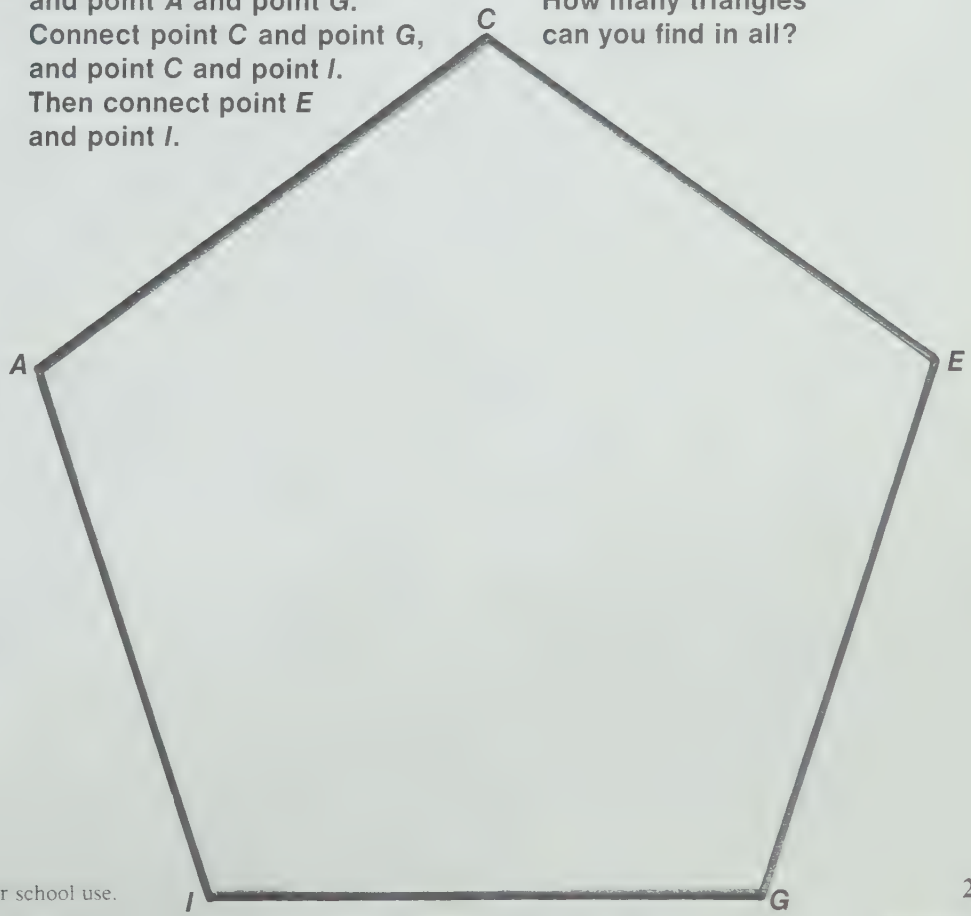
Consider an activity such as this—

**Use a protractor.**
What is the measure of each of the angles inside the pentagon?

What do you see?
Label the new angles that have been formed.
Which angles are the same size?

**Use a ruler.**
Connect point *A* and point *E*, and point *A* and point *G*. Connect point *C* and point *G*, and point *C* and point *I*. Then connect point *E* and point *I*.

How many same-size triangles can you find?
How many triangles can you find in all?

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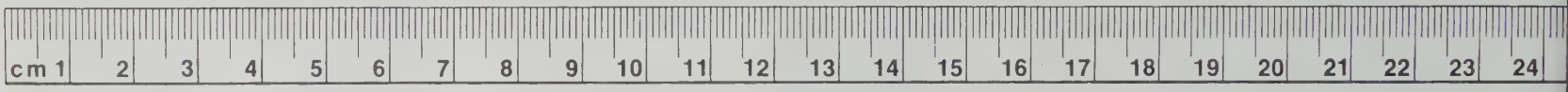


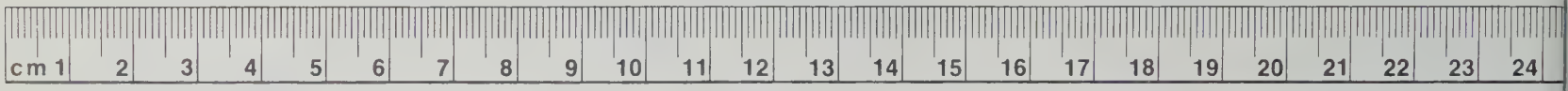


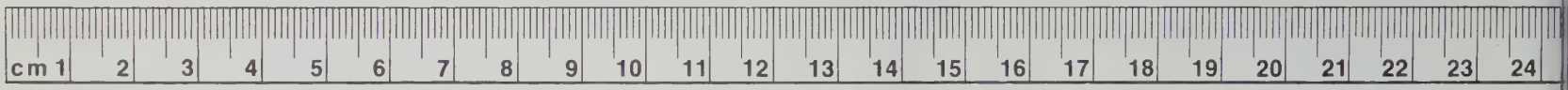


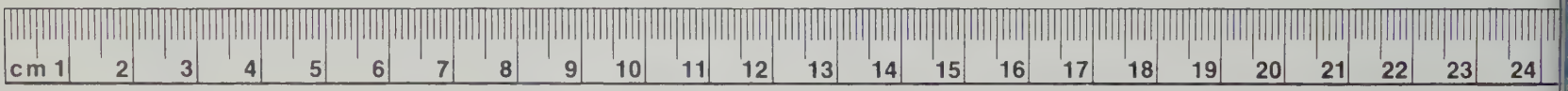



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# 10 FRACTIONS +, −, AND ÷

**before this chapter, the learner has –**

1. Compared two whole numbers or two fractions and identified the greater
2. Renamed whole numbers, mixed numbers, and fractions
3. Identified whether a fraction is in simplest form and, if not, renamed that fraction in simplest form
4. Found the sum or difference for two fractions having like or unlike denominators

**in chapter 10 the learner is –**

1. Adding or subtracting any two fractions or mixed numbers
2. Introduced to division with fractions
3. Estimating quotients to check the reasonableness of the computed quotient

**in later levels the learner will –**

1. Find the quotient for any two fractions or mixed numbers
2. Check the reasonableness of a quotient by estimation

# Notes & Things

This chapter on division of fractions begins with a quick review of adding and subtracting fractions and mixed numbers. The review is good for skill maintenance, but its real purpose is to get the learner to think about common denominators. There is good reason for this.

Rather than the use of an approach that eventually yields the briefly remembered "invert and multiply" rule, you'll find a wonderfully simple common-denominator approach to division of fractions. The attempt has been made to relate this operation with fractions to the concept of division rather than to an inverse relation of multiplication. The common denominator is used as the thrust. Hopefully this will allow the pupil to relate the operation to his past experiences with number operations and to use his past knowledge of equivalent fractions.

By this time the youngsters should begin to understand that mathematics was created by people. Numbers arose out of a need to keep track of things and a need to communicate ideas. It is an oversimplification to consider whole numbers as a way of communicating "how many" and fractions a way of telling "how much," but at this point the distinction serves the purpose of developing concepts and the ability to operate with fractions.

The emphasis on the operations with fractions will probably be decreased sometime in the future as the metric system becomes more popular. Addition, subtraction, and multiplication are operations with fractions that we do sometimes use in our everyday life, but almost always when some sort of measurement is involved. It is very hard to find a use for division of fractions in today's world. Without becoming a student of the history of mathematics, it's also hard to find a use for the operation in the past either. It's fun to think about the possibility of someone, sometime in the past, thinking, "We have addition, subtraction, and multiplication of whole numbers. We have addition, subtraction, and multiplication of fractions. Isn't it a shame we don't have the operation of division?" And then, since most of us like things to be neat and tidy and complete, this person decided to figure out a way to do division—no matter whether it was needed or not.

That whole story probably didn't happen, but it does set the stage for the action of this chapter. You'll find the operation of division treated as gently as possible. It might even be fun for you as well as the students.

For the extra activities you will want to have these things available:

for each pupil: 9 tongue depressors

**goal** Think about and explore ideas through a picture clue

**page 239** If the pupils have just completed chapter 9, enjoy the luxury of some discussion time to talk about the angles that obviously are formed by the spokes of each wheel—the more spokes, the smaller the angle. The pupils can also see some dramatic proof that the length of the sides of any given angle does not affect the size of the angle.

Any one of the wheels also can be thought of as a fraction model. Are all of the wheel rims divided into the same number of fractional parts? Could any wheel be divided into still more parts? Would the wheel be stronger or weaker with more spokes added? As spokes are added, does the space between them become greater or less? Is there a limit to the number of spokes that could be added to a real wheel? Why?

The wheels in the photograph appear to be very old. It's been a long time since wood wheels were used. Can anyone think what they may have been used on? As soon as everyone is thinking about the old days, the scene is set for the next page.

**lesson** Pages 240, 241

**goal** **Survey** — ability to add, subtract, and divide fractions

**page 240** Comparing his skill with great-great-grandfather's should be a challenge for any learner. Examine the problems. Are they different from those found in math books today? What do questions about land, horses, and crops tell about the way people lived in those days?

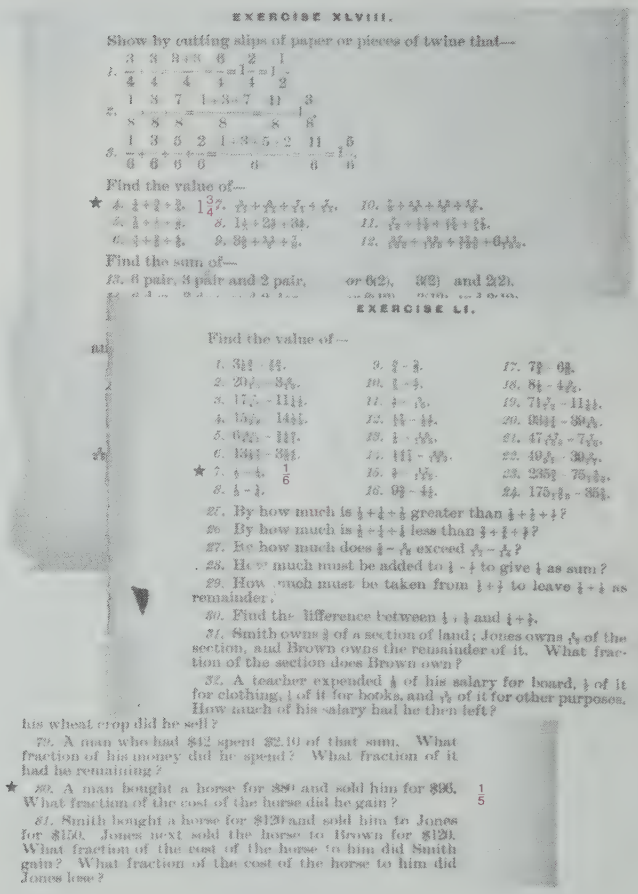
Interested youngsters should be encouraged to do some research work.

Pupils are free to try their skill with any of the problems shown. The starred problems have been selected to help you identify pupils who may be ready to jump right to the Progress Check on page 247.

A gentle reminder—if a pupil can't compute as well now as his great-great-grandparents could, this should be his learning goal for the chapter. That's what it's all about.

These pages come from an old arithmetic book used around 1895. As you can see, your great-great-grandparents had to work very hard. Today we don't have to worry so much about difficult fractions. See if you can do the problems. Your great-great-grandparents could.

Do the problems that are marked with a ★.



EXERCISE XLVIII.

Show by cutting slips of paper or pieces of twine that—

1. $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$.

2. $\frac{1}{8} + \frac{3}{8} + \frac{7}{8} = \frac{1+3+7}{8} = \frac{11}{8} = 1\frac{3}{8}$.

3. $\frac{1}{6} + \frac{3}{6} + \frac{5}{6} + \frac{2}{6} = \frac{1+3+5+2}{6} = \frac{11}{6} = 1\frac{5}{6}$.

Find the value of—

★ 4. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$. $1\frac{3}{4}$ 7. $\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$. 10. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.

5. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$. 8. $1\frac{1}{8} + 2\frac{1}{8} + 3\frac{1}{8}$. 11. $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$.

6. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. 9. $3\frac{1}{8} + 2\frac{1}{8} + \frac{1}{8}$. 12. $\frac{17}{100} + \frac{33}{100} + \frac{13}{100} + 6\frac{11}{100}$.

Find the sum of—

13. 6 pair, 3 pair and 2 pair, or 6(2), 3(2) and 2(2).

EXERCISE LI.

Find the value of—

1. $3\frac{1}{2} - 1\frac{1}{4}$.
2. $20\frac{1}{2} - 3\frac{1}{16}$.
3. $17\frac{1}{3} - 11\frac{1}{12}$.
4. $15\frac{1}{16} - 14\frac{1}{32}$.
5. $6\frac{3}{16} - 1\frac{1}{64}$.
6. $13\frac{1}{12} - 3\frac{1}{24}$.
★ 7. $\frac{1}{2} - \frac{1}{3}$. $\frac{1}{6}$
8. $\frac{1}{2} - \frac{1}{3}$.
9. $\frac{2}{3} - \frac{1}{3}$.
10. $\frac{1}{2} - \frac{1}{3}$.
11. $\frac{1}{2} - \frac{1}{3}$.
12. $1\frac{1}{2} - 1\frac{1}{3}$.
13. $\frac{1}{2} - \frac{1}{3}$.
14. $\frac{11}{12} - \frac{7}{12}$.
15. $\frac{1}{2} - \frac{1}{12}$.
16. $9\frac{2}{3} - 4\frac{1}{2}$.
17. $7\frac{2}{3} - 6\frac{3}{4}$.
18. $8\frac{1}{2} - 4\frac{2}{3}$.
19. $71\frac{7}{8} - 11\frac{1}{2}$.
20. $83\frac{1}{4} - 30\frac{1}{8}$.
21. $47\frac{7}{10} - 7\frac{1}{10}$.
22. $49\frac{1}{8} - 30\frac{1}{4}$.
23. $235\frac{1}{2} - 75\frac{1}{10}$.
24. $175\frac{1}{10} - 35\frac{1}{2}$.

27. By how much is $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ greater than $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$?

28. By how much is $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ less than $\frac{3}{4} + \frac{2}{3} + \frac{1}{2}$?

29. By how much does $\frac{3}{4} - \frac{1}{8}$ exceed $\frac{1}{2} - \frac{1}{4}$?

30. How much must be added to $\frac{1}{2} - \frac{1}{3}$ to give $\frac{1}{2}$ as sum?

31. How much must be taken from $\frac{1}{2} + \frac{1}{3}$ to leave $\frac{1}{2} + \frac{1}{3}$ as remainder?

40. Find the difference between $\frac{1}{2} + \frac{1}{3}$ and $\frac{1}{2} + \frac{1}{3}$.

41. Smith owns $\frac{3}{4}$ of a section of land; Jones owns $\frac{1}{8}$ of the section, and Brown owns the remainder of it. What fraction of the section does Brown own?

42. A teacher expended $\frac{1}{4}$ of his salary for board, $\frac{1}{3}$ of it for clothing, $\frac{1}{6}$ of it for books, and $\frac{1}{12}$ of it for other purposes. How much of his salary had he then left?

his wheat crop did he sell?

79. A man who had $42 spent $2.10 of that sum. What fraction of his money did he spend? What fraction of it had he remaining?

★ 80. A man bought a horse for $80 and sold him for $96. What fraction of the cost of the horse did he gain? $\frac{1}{5}$

81. Smith bought a horse for $120 and sold him to Jones for $150. Jones next sold the horse to Brown for $120. What fraction of the cost of the horse to him did Smith gain? What fraction of the cost of the horse to him did Jones lose?

240

**Your goal is to learn as much about the addition, subtraction, and division of fractions as your great-great-grandfather did.**

Complete these addition problems.

**Set E** (easy)

1. Start with an easy one: $\frac{1}{7} + \frac{5}{7} = ?$ There is no $\frac{6}{7}$ renaming.

2. Here is a rather easy one: $\frac{1}{8} + \frac{3}{8}$ You will have to rename the sum. $\frac{4}{8} = \frac{1}{2}$

**Set A** (average)

3. Try an average type: $\frac{2}{3} + \frac{1}{6}$ You will rename to get a common denominator. $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

4. Here is another average type: $\frac{2}{3} + \frac{5}{6}$ You will rename to get a common denominator. $\frac{4}{6} + \frac{5}{6} = \frac{9}{6}$ And the sum will need renaming too. $\frac{9}{6} = 1\frac{1}{2}$

5. This is average: $\frac{3}{4} + \frac{4}{5}$. Rename both fractions to get a common denominator. And rename the sum. $\frac{15}{20} + \frac{16}{20} = \frac{31}{20}$ $\frac{31}{20} = 1\frac{11}{20}$

**Set H** (hard)

6. This one isn't hard unless you don't know how to do it. Try it.

There is renaming all over the place.

$$\begin{array}{r} 3\frac{2}{3} \\ +\,2\frac{1}{2} \\ \hline ? \\ 6\frac{1}{6} \end{array}$$

Need help? You'll find it on practice pages that follow. But check out subtraction first, please. Complete the following.

1. Easy type: $\frac{5}{9} - \frac{1}{9} = ?$ $\frac{4}{9}$

2. Rather easy type: $\frac{5}{6} - \frac{1}{6} = ?$ $\frac{2}{3}$

3. Average type: $\frac{1}{2} - \frac{1}{4} = ?$ $\frac{1}{4}$

4. Another average type: $\frac{3}{4} - \frac{2}{8} = ?$ $\frac{1}{2}$

There is more renaming involved in these.

5. $\frac{2}{3} - \frac{1}{4}$ $\frac{5}{12}$ 6. $2 - \frac{3}{4}$ $1\frac{1}{4}$

7. And these are your challenges.

**a** $3\frac{1}{2} - 1\frac{1}{8}$ $2\frac{3}{8}$

**b** $4\frac{2}{3} - 2\frac{1}{5}$ $2\frac{7}{15}$

**c** $5\frac{1}{4} - 2\frac{3}{4}$ $2\frac{1}{2}$

**d** $4\frac{1}{3} - 2\frac{1}{2}$ $1\frac{5}{6}$



**goal** **Survey**—ability to add and subtract fractions with unlike denominators

**page 241** Each problem checks a specific skill that has been developed and practiced in earlier chapters. Use this page to help you identify those pupils who may need your personal additional help as they progress through the next pages.

Use the following chart to guide you in making appropriate assignments.

| Errors on problems | Pages to watch performance |
|---|---|
| 1–5 addition | 242 and 243 |
| 6 addition | 244 and 245 |
| 1–5 subtraction | 243 and 244 |
| 6–7 subtraction | 244–246 |

**lesson** Pages 242, 243, 244, 245, 246

**goal** Review of adding fractions with like and unlike denominators and renaming the sum in simplest form

**page 242** This self-explanatory page provides a sequential review of addition with **like** and **unlike** denominators. Check for understanding of **multiple** and **greatest common factor**. In the example

$$\frac{16}{10} = \frac{16}{10} \div \frac{2}{2} = \frac{8}{5}$$

make sure that pupils are able to rename $\frac{8}{5}$ as $1\frac{3}{5}$.

The easy type of problem had a common denominator
(**Set E** on the last page).

$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2}$ $\qquad$ $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{?}{5}$ 4

$\frac{3}{8} + \frac{5}{8} = \frac{8}{8}$ ← You know that's another name for 1.

Pay attention here. $\frac{7}{10} + \frac{9}{10} = \frac{7+9}{10} = \frac{16}{10}$ ← That needs to be renamed.

Divide the numerator and the denominator by the same number.
What is the greatest common factor of 16 and 10? 2? Maybe 4? No! 5? No! 8? No!

$\frac{16}{10} = \frac{16 \div 2}{10 \div 2} = \frac{8}{5}$ ← What's the simplest name for $\frac{8}{5}$? $1\frac{3}{5}$

You probably don't ever make a mistake in addition.
But renaming can cause trouble.

Renaming is needed in the average-type addition problems too.
This might help you with problems in **Set A** on the last page.
Focus on this problem. If you know how to do this one, you know how to do all problems of the average type.

$\frac{2}{3} + \frac{5}{8} = ?$

**THINK**

No common denominator.
Get that job done first.
Thirds can't be renamed as eighths.
You need a denominator that has a factor of 3 and a factor of 8.
So multiply: $3 \times 8$. Use 24 as the denominator.

*rename* $\frac{16}{24} + \frac{15}{24}$. Then add: $\frac{16+15}{24} = \frac{31}{24}$ AND *rename* again! You finish it. $\frac{31}{24} = 1\frac{7}{24}$

242

Here is a QUICK review of renaming if you need it.

**1.** When you need a simplest name, divide the numerator and the denominator by the same number.

Pick the number that is the greatest common factor of the numerator and the denominator. It will save you a lot of time and work.

$\frac{12}{16}$ needs renaming. What is the greatest common factor?

2 is a factor of both 12 and 16. Is 3? No. Is 4? Yes.
But is it the greatest common factor? Yes
Try 6. Try 8. $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

Here's another. No new thinking is needed.

$\frac{15}{12}$ needs renaming. What is the greatest common factor? 2? 3? 4? 5? 6?

$\frac{15}{12} = \frac{15 \div 3}{12 \div 3} = ?$ $\frac{5}{4}$ You finish it. There is still another renaming. $1\frac{1}{4}$

**2.** This is the rest of the renaming story.
When you need to find a common denominator, you rename.

$\frac{3}{8} + \frac{5}{6} = ?$ You can multiply $8 \times 6$ to get a common denominator.
Or you can find a common multiple of the denominators.

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, and 48. Why can you stop at 48? $8 > 6 = 48$

Multiples of 8 are 8, 16, 24. You can stop here, because 24 is a common multiple.

$\frac{3}{8} + \frac{5}{6}$ *Rename* $\frac{?}{24}$ $^{9}$ $+ \frac{?}{24}$ $^{20}$. You take it from there. Complete the addition.
$\frac{9 + 20}{24} = \frac{29}{24} = 1\frac{5}{24}$

243

**goal** Review of renaming a fraction in simplest form and with a common denominator

**page 243** Two types of renaming are reviewed.

- Problem 1—renaming in simplest form
- Problem 2—renaming with a common denominator

Direct each pupil to the specific type of help he needs.

Check for understanding of the terms **greatest common factor** and **common multiple**.

It is sometimes necessary to remind the learners that the first thing they should look at in addition or subtraction of fractions is the denominators. If the denominators are the same, they can launch the operation. If the denominators are not the same, some work must be done before they can operate. There has been an emphasis on renaming for many chapters that came before this one. This page and the next are literally the last chance for an instructional review of common multiples. If any youngster is still in trouble, he will need your personal help.

**goal** Review of subtracting fractions with like and unlike denominators; adding and subtracting mixed numbers with like denominators

**page 244** Check for understanding of **common multiple**.

The review of common multiple is completed on this page and the review of adding and subtracting mixed numbers begins. You'll notice that the vertical notation is stressed to make it easier for a child to view the multiproblems involved. Discourage any youngster who operates on the whole numbers first. If he learns to tackle the fractions first in addition, he establishes a good work pattern for renaming in subtraction.

**1.** The easy type of subtraction had a common denominator.

$\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8}$ ← This is easy to rename. Do it. $\frac{1}{2}$

$\frac{3}{5} - \frac{1}{6}$ ← You must find a common denominator.

What is a common multiple of 5 and 6?
Multiples of 5 are 5, 10, 15, 20, 25, and 30. ← Why can you stop here?
Multiples of 6 are 6, 12, 18, 24, 30. Stop!
30 is the common multiple. $5 \times 6 = 30$

$\frac{3}{5} - \frac{1}{6}$

*Rename* $\frac{18}{30} - \frac{5}{30} = \frac{18-5}{30} = \frac{?}{}$ You finish it. $\frac{13}{30}$

Once again, renaming is the important skill.

**2.** Nothing was done about mixed numbers in addition or subtraction. Take care of that now.

$2\frac{3}{5} + 1\frac{1}{5}$ or $3\frac{6}{7} - 2\frac{3}{7}$ are easier to work if you stack them.

$2\frac{3}{5}$
$+1\frac{1}{5}$ There is a common denominator.
$3\frac{4}{5}$ ← Add the fractions first.
↑ Then add the whole numbers.

$3\frac{6}{7}$
$-2\frac{3}{7}$ There is a common denominator.
$\blacksquare$ ← Subtract fractions first.
$1\frac{3}{7}$ ↑ Then subtract whole numbers.
You finish it.

You know all problems will not be that easy.
Roll up your sleeves and get ready to add and subtract mixed numbers. It won't be too bad. That is a promise.



244

**1.** Jake worked for $2\frac{1}{4}$ hours on Friday. He worked $5\frac{1}{2}$ hours on Saturday. He had to know how much time he worked before he could get paid.

$$\begin{array}{r} 2\frac{1}{4} \\ +\,5\frac{1}{2} \\ \hline \end{array}$$ You need a common denominator.

$$\begin{array}{r} 2\frac{1}{4} \\ +\,5\frac{2}{4} \\ \hline 7\frac{3}{4} \end{array}$$ ← Where did $\frac{2}{4}$ come from? $\frac{1\times 2}{2\times 2}=\frac{2}{4}$

Now you can add. Fractions first.
Then add the whole numbers.
Do you have to rename the fraction? No
This time you're lucky.

There are no new rules for subtraction.

**2.** Jake worked $10\frac{1}{2}$ hours one week. He worked $7\frac{1}{4}$ hours the second week. How many more hours did he work in the first week?

$$\begin{array}{r} 10\frac{1}{2} \\ -\,7\frac{1}{4} \\ \hline \end{array}$$ *Rename*

$$\begin{array}{r} 10\frac{2}{4} \\ -\,7\frac{1}{4} \\ \hline 3\frac{1}{4} \end{array}$$ ← Now you can subtract.

← Do you have to *rename*? No

Practice on these. Watch the operation symbols.

**LOOK OUT**

Get your whole numbers all together.

| | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|
| **3.** | $2\frac{1}{2}$ | $9\frac{5}{6}$ | $5\frac{5}{6}$ | $6\frac{1}{5}$ | $6\frac{7}{8}$ | $8\frac{4}{5}$ | $2\frac{1}{3}$ | $3\frac{5}{6}$ |
| | $+\,5\frac{1}{4}$ | $-\,2\frac{1}{3}$ | $-\,2\frac{1}{2}$ | $+\,2\frac{2}{3}$ | $-\,2\frac{1}{3}$ | $-\,2\frac{1}{2}$ | $+\,5\frac{3}{4}$ | $+\,8\frac{5}{8}$ |
| | $7\frac{3}{4}$ | $7\frac{1}{2}$ | $3\frac{1}{3}$ | $8\frac{13}{15}$ | $4\frac{13}{24}$ | $6\frac{3}{10}$ | $8\frac{1}{12}$ | $12\frac{11}{24}$ |

245

**goal** Review of adding and subtracting mixed numbers with unlike denominators

**page 245** These problems require a lot of computation—first rename with a common denominator, then add or subtract. Finally rename the answer. Note that three types of denominators are included:

- One denominator a multiple of the other denominator
- Two denominators without a common factor
- Two denominators having a common factor

When selecting problems make sure to include all three types.

**goal** Review of renaming a whole as fractional parts to make subtraction possible

**page 246** Renaming with a common denominator isn't always good enough in subtraction. Sometimes a whole must be renamed as fractional parts to make subtraction possible. That's the focus of the page. This step is comparable to renaming a ten as ones to make subtraction possible with whole numbers. You may want to be selective in assigning specific problems. Watch for types of denominators and this extra renaming step. These problems involve many steps and require fortitude on the part of less capable students. Problem **4a** is a special type. You may want to talk about this one before making an assignment.

**1** Subtraction of mixed numbers does have one curve ball.

$5\frac{1}{4}$ / $-2\frac{1}{2}$ / ? — *Rename* → $5\frac{1}{4}$ / $-2\frac{2}{4}$ / ?

*Rename* again. $4\frac{4}{4}+\frac{1}{4}$ or $4\frac{5}{4}$

This is subtraction. $-2\frac{2}{4}$

$2\frac{3}{4}$

**2** Look at one more example before you practice.

$7\frac{1}{3}$ / $-2\frac{5}{8}$ / ? — *Rename* → $7\frac{8}{24}$ / $-2\frac{15}{24}$ / ?

*Rename* again. $6\frac{24}{24}+\frac{8}{24}$ or $6\frac{32}{24}$

$-2\frac{15}{24}$

You subtract this. $4\frac{17}{24}$

**3** Take your time on this practice. Think carefully.

| a | b | c | d | e | f |
|---|---|---|---|---|---|
| $3\frac{2}{5}$ | $8\frac{1}{2}$ | $4\frac{1}{6}$ | $7\frac{2}{3}$ | $2\frac{3}{4}$ | $3\frac{1}{3}$ |
| $-1\frac{2}{3}$ | $-2\frac{7}{8}$ | $-2\frac{4}{5}$ | $-2\frac{7}{8}$ | $-1\frac{9}{10}$ | $-2\frac{5}{6}$ |
| $1\frac{11}{15}$ | $5\frac{5}{8}$ | $1\frac{11}{30}$ | $4\frac{19}{24}$ | $\frac{17}{20}$ | $\frac{1}{2}$ |

**4**

| | | | | | |
|---|---|---|---|---|---|
| 4 | 5 | $6\frac{1}{4}$ | $8\frac{1}{6}$ | 3 | $5\frac{1}{8}$ |
| $-2\frac{1}{8}$ | $-2\frac{3}{5}$ | $-2\frac{1}{8}$ | $-2\frac{1}{8}$ | $-1\frac{3}{10}$ | $-2\frac{3}{4}$ |
| $1\frac{7}{8}$ | $2\frac{2}{5}$ | $4\frac{1}{8}$ | $6\frac{1}{24}$ | $1\frac{7}{10}$ | $2\frac{3}{8}$ |

Yes, you have to rename 4 as $3\frac{8}{8}$.

If you got through these in good shape, you are an expert.
Don't feel bad if you need more practice.
They aren't the easiest problems in the world.

246

# PROGRESS CHECK

Skills: Adding and subtracting fractions – like denominators

Compute. Watch the operation symbols.

(1.) $\frac{2}{7}+\frac{3}{7}$ $\frac{5}{7}$ (2.) $\frac{5}{9}-\frac{4}{9}$ $\frac{1}{9}$ (3.) $\frac{3}{8}+\frac{3}{8}$ $\frac{3}{4}$ (4.) $\frac{9}{10}-\frac{3}{10}$ $\frac{3}{5}$

Skills (5, 6): Adding and subtracting fractions – unlike denominators

(5.) $\frac{1}{2}+\frac{3}{4}$ $1\frac{1}{4}$ (6.) $\frac{5}{6}-\frac{3}{4}$ $\frac{1}{12}$ (7.) $1\frac{1}{8}+2\frac{3}{8}$ $3\frac{1}{2}$ (8.) $4\frac{3}{5}-1\frac{1}{5}$ $3\frac{2}{5}$

Skills (7 to 12): Adding and subtracting mixed numbers

(9.) $2\frac{2}{3}+1\frac{5}{6}$ $4\frac{1}{2}$ (10.) $5\frac{1}{4}-2\frac{3}{4}$ $2\frac{1}{2}$ (11.) $4-\frac{1}{6}$ $3\frac{5}{6}$ (12.) $1\frac{5}{6}+2\frac{1}{6}$ 4

Skill: Word problems with fractions

(13.) Joan worked $1\frac{3}{4}$ hours in her garden Saturday morning. She worked the same amount of time in the afternoon. How much time has she spent in her garden so far? $3\frac{1}{2}$ hours

On Sunday she worked for another $2\frac{1}{2}$ hours. How many hours did she spend in her garden during the weekend? 6

(14.) In one way Pauline is fortunate. She has homework only 3 days a week. But on those days she works very hard. On Monday and Tuesday she works $1\frac{3}{4}$ hours each day. How many hours has she worked so far? $3\frac{1}{2}$ Wednesday is free, but her homework on Thursday totals $2\frac{1}{2}$ hours. How many hours of homework a week does Pauline have? 6

247

See activity 1, page 268a.

See activity 2, page 268a.

**lesson** Page 247

**goal** **Progress Check** – adding and subtracting fractions and mixed numbers; applying fraction skills to solving word problems

**page 247** Problems 1 through 12 parallel the problems on survey page 241. Use this chart to guide each pupil to the specific help he needs. This is a good time for peer tutors.

| Page 247 Problems | Page 241 Problems | For help see page |
|---|---|---|
| 1, 3 | 1, 2 addition | 242, top half |
| 2, 4 | 1, 2 subtraction | 244, problem 1 |
| 5, 7 | 3–5 addition | 242, bottom half<br>243, problem 2 |
| 6, 8 | 3–6 subtraction | 244, problem 2 |
| 9, 12 | 6 addition | 244, problem 2 |
| 10, 11 | 7 subtraction | 245, 246 |

The word problems should be solved independently. Do help with reading if necessary.

**lesson** Pages 248, 249, 250

**goal** Introduction to the idea of dividing with fractions

**memo** The explanation to the pupil of why he should learn to divide fractions is honest and straightforward. Finding real-world problems that would convince the learner that division with fractions is a necessary skill, rather than an academic exercise, is virtually impossible. So when all else fails, perhaps the youngsters will appreciate this simple introduction. Don't let reading stand in the way.

**page 248** It is difficult to imagine living in 2400 B.C. Why would anyone ever have decided that he needed fractions? Do fractions allow us to think about and answer questions that could not be answered with whole numbers?

Can you guess why someone would have needed to add fractions? to subtract them? to multiply? More and more educators are agreeing that there is little reason to teach division of fractions other than to complete the pattern of operations. Most real-world problems requiring division of fractions could be solved with multiplication instead.

Learners should understand that mathematics is a creative product of people's necessity and curiosity.

*It's hard to find situations in everyday life to show that you need to know how to divide fractions.*

# THEN WHY LEARN HOW TO DO IT?

There are lots of reasons, people might say. But think about this reason:

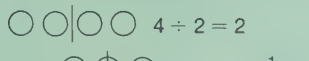
History books about mathematics don't tell much about when operations on fractions were invented. But they do give us an idea of when people needed these numbers.

Clay tablets dating back to 2400 B.C. show that the Babylonians had a way to represent fractions. They probably wouldn't have thought of it if they hadn't needed the idea of a fraction. The Egyptians got into the act hundreds of years later and carried the fraction idea further. The Romans gave the things a name, but they didn't like to use them unless they ha to. Finally, the Hindus figured out an easy way to write them.

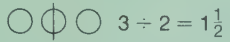
From 2400 B.C. to this year is a long time. It's hard to tell when people started to compute with fractions. But you can guess why they needed to add, subtract, and multiply.

Maybe somebody decided that since they knew how to add, subtract, multiply, and divide whole numbers, it would be great to do all those operations with fractions, too. You know the operations of addition, subtraction, and multiplication with fractions. Why not join the rest of the world and take a look at division of fractions? It won't hurt at all.

1. Look at some patterns in the answers to division problems.

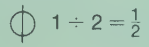


$4 \div 2 = 2$



$3 \div 2 = 1\frac{1}{2}$

$2 \div 2 = 1$

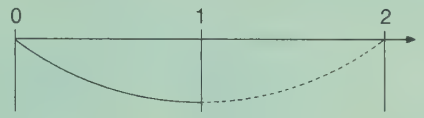


$1 \div 2 = \frac{1}{2}$

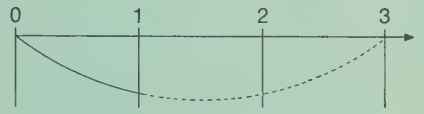
How many 2s in 1? $\frac{1}{2}$

2. A number line would be helpful to see the action of division.

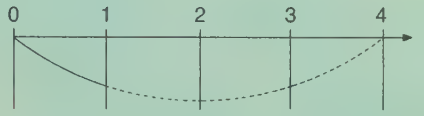
The loop shows 2.
Is there a two in 1? No
How much of a two is in 1? $\frac{1}{2}$
$1 \div 2 = \frac{1}{2}$




3. The loop shows 3.
Is there a three in 1?
How much of a three is in 1? $\frac{1}{3}$
$1 \div 3 = \underline{?}$ $\frac{1}{3}$




4. The loop shows 4.
Is there a four in 1? No
How much of a four is in 1? $\frac{1}{4}$
$1 \div 4 = \underline{?}$ $\frac{1}{4}$




249

**goal** Exploration of dividing a whole-number dividend by a whole-number divisor greater than the dividend

**memo** The ability to predict whether the quotient will be greater than 1 or less than 1 is most important later in the chapter. Readiness for this later work is built by examining division with whole numbers first. One of the most difficult things for the learner to accept is the difference between the size of the answer when he divides fractions rather than whole numbers.

**page 249** A gab session is needed for this page. Focus on the pattern formed by the quotients in problem 1. Examine the relationship between the size of the divisor and the size of the quotient. When is the quotient equal to 1? When is it greater than 1? less than 1?

Make a big deal out of the loops on the number lines. The pupil must see that part of the loop is solid and part is a broken line. **How much** is shown by the solid part of the loop?

Emphasize the divisor–denominator relationship.

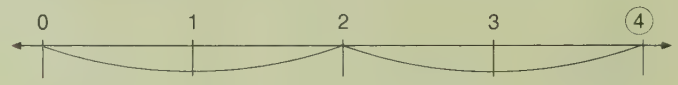
**goal** Dividing whole numbers that yield fraction quotients

**page 250** Take time to talk together some more. The examples are sequenced, leading to a fraction quotient. Point out the size of the dividend and the size of the divisor in each example. How does this relationship affect the size of the quotient? Emphasize the divisor–denominator relationship in the last two examples.

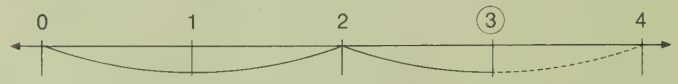
Note that for problems 1 through 8, the dividend is consistently less than the divisor. Set 1 on page 267 includes problems of this form together with problems where the dividend is greater than the divisor. Use this set as a base for tutorial help rather than as an across-the-board assignment.

Make sure you understand. Backtrack. Start with what you know about division of whole numbers.

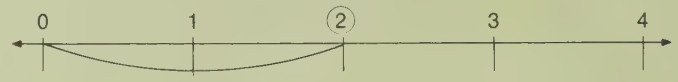
How many 2s in 4? 2
$4 \div 2 = \underline{?}$ 2



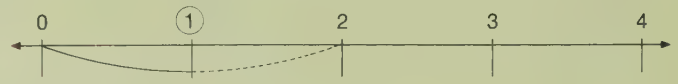

How many 2s in 3? $1\frac{1}{2}$
$3 \div 2 = \underline{?}$ $1\frac{1}{2}$




How many 2s in 2? 1
$2 \div 2 = \underline{?}$ 1

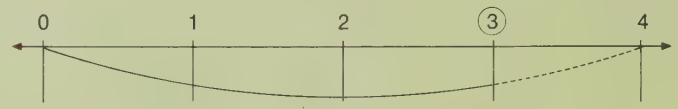



How many 2s in 1? $\frac{1}{2}$
$1 \div 2 = \underline{?}$ $\frac{1}{2}$




Does the number line show $\frac{1}{2}$ of two in 1? Yes

Look at one more example.
How many 4s in 3? $\frac{3}{4}$
$3 \div 4 = \frac{?}{4}$ 3




Complete these sentences. Each quotient will be a fraction.

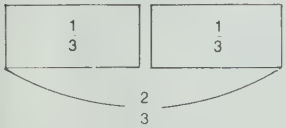
**1.** $2 \div 3 = \underline{?}$ $\frac{2}{3}$ **2.** $4 \div 5 = \underline{?}$ $\frac{4}{5}$ **3.** $3 \div 5 = \underline{?}$ $\frac{3}{5}$ **4.** $8 \div 9 = \underline{?}$ $\frac{8}{9}$

**5.** $6 \div 11 = \underline{?}$ $\frac{6}{11}$ **6.** $4 \div 7 = \underline{?}$ $\frac{4}{7}$ **7.** $5 \div 7 = \underline{?}$ $\frac{5}{7}$ **8.** $4 \div 9 = \underline{?}$ $\frac{4}{9}$

*Extra practice on page 267 – set 1*

250

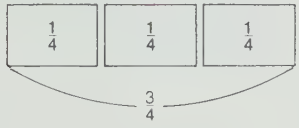
. After dinner, he had $\frac{2}{3}$ of a meat loaf left. How many $\frac{1}{3}$-pieces did he have? 2




How many $\frac{1}{3}$s in $\frac{2}{3}$? 2

$\frac{2}{3} \div \frac{1}{3} = 2$

2. She had $\frac{3}{4}$ of a chocolate bar. How many $\frac{1}{4}$-pieces did she have? 3

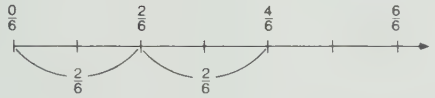



How many $\frac{1}{4}$s in $\frac{3}{4}$? 3

$\frac{3}{4} \div \frac{1}{4} = 3$

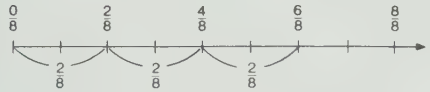
3. How many $\frac{2}{6}$s in $\frac{4}{6}$? 2

$\frac{4}{6} \div \frac{2}{6} = \underline{?}$ 2



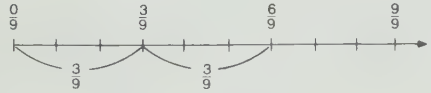

4. How many $\frac{2}{8}$s in $\frac{6}{8}$? 3

$\frac{6}{8} \div \frac{2}{8} = \underline{?}$ 3




5. How many $\frac{3}{9}$s in $\frac{6}{9}$? 2

$\frac{6}{9} \div \frac{3}{9} = ?$ 2




251

**lesson** Pages 251, 252

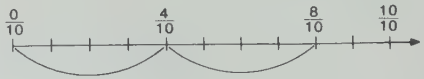
**goal** Exploration of dividing a fraction by a fraction with a like denominator

**page 251** The "how many" question will encourage the pupils to find the answer intuitively, using the number-line diagrams. It is premature to point out the division operation itself.

**goal** Practice in dividing a fraction by a fraction with a like denominator

**page 252** The emphasis remains on the "How many" question. The question at the bottom of the page is the **first** step in acquiring understanding, then skill, with the division algorithm. Do not examine the relationship of the numerators yet. That comes on the next page. For now the important thing is to see that the denominators are the same.

How many $\frac{4}{10}$s in $\frac{8}{10}$? 2




$\frac{8}{10} \div \frac{4}{10} = \underline{?}$ 2

As you work, pay special attention to the denominators.

How many $\frac{2}{9}$s in $\frac{6}{9}$? 3

$\frac{6}{9} \div \frac{2}{9} = \underline{?}$ 3

Use a number line to help when necessary.

1. How many $\frac{3}{8}$s in $\frac{9}{8}$? 3 $\frac{9}{8} \div \frac{3}{8} = \underline{?}$ 3
2. How many $\frac{2}{5}$s in $\frac{4}{5}$? 2 $\frac{4}{5} \div \frac{2}{5} = \underline{?}$ 2
3. How many $\frac{3}{9}$s in $\frac{9}{9}$? 3 $\frac{9}{9} \div \frac{3}{9} = \underline{?}$ 3
4. How many $\frac{2}{3}$s in $\frac{4}{3}$? 2 $\frac{4}{3} \div \frac{2}{3} = \underline{?}$ 2
5. How many $\frac{3}{4}$s in $\frac{9}{4}$? 3 $\frac{9}{4} \div \frac{3}{4} = \underline{?}$ 3
6. How many $\frac{2}{5}$s in $\frac{6}{5}$? 3 $\frac{6}{5} \div \frac{2}{5} = \underline{?}$ 3
7. How many $\frac{1}{10}$s in $\frac{7}{10}$? 7 $\frac{7}{10} \div \frac{1}{10} = \underline{?}$ 7
8. How many $\frac{3}{12}$s in $\frac{9}{12}$? 3 $\frac{9}{12} \div \frac{3}{12} = \underline{?}$ 3

What did you notice about the denominators in every problem? They are the same.

252

Some people think of estimating as a guessing game. The notion of estimation is so important that you should at least be able to make an "educated guess." (That means you have a *reason* for your guess.)

Can you know whether an answer will be greater than 1 or less than 1? Here's one way.

**1.** $\frac{5}{3} \div \frac{2}{3} \,?\, 1$

Is there one $\frac{1}{3}$ in $\frac{5}{3}$? Yes

Is there one $\frac{2}{3}$ in $\frac{5}{3}$? Yes

More than one $\frac{2}{3}$ in $\frac{5}{3}$? Yes

$\frac{5}{3} \div \frac{2}{3} > 1$

**2.** $\frac{5}{7} \div \frac{3}{7} \,?\, 1$

Is there one $\frac{1}{7}$ in $\frac{5}{7}$? Yes

Is there one $\frac{2}{7}$ in $\frac{5}{7}$? Yes

Is there one $\frac{3}{7}$ in $\frac{5}{7}$? Yes

More than one $\frac{3}{7}$ in $\frac{5}{7}$? Yes

$\frac{5}{7} \div \frac{3}{7} > 1$

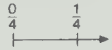
**3.** Here is another way.

$\frac{7}{6} \div \frac{4}{6} \,?\, 1$

How many $\frac{4}{6}$s in $\frac{7}{6}$? $1\frac{3}{4}$ $\frac{7}{6} \div \frac{4}{6} > 1$

**4.** This is a special problem.

$\frac{1}{4} \div \frac{3}{4} \,?\, 1$




How many $\frac{3}{4}$s in $\frac{1}{4}$? $\frac{1}{3}$ $\frac{1}{4} \div \frac{3}{4} < 1$



**lesson** Pages 253, 254

**goal** Introduction to estimating the quotient when dividing two fractions

**page 253** You'll want to talk about the examples together. Look at the denominators. They are the same in every problem. Then focus on the numerators. How are the numerators related? This is the **second** step in developing understanding. When will the quotient be less than 1? (When the numerator of the divisor is greater than the numerator of the dividend) Use the number line to reinforce this generalization.

Pupils who think, "How many three-fourths are there in one-fourth?" will probably have a far easier time estimating quotients than those who think, "One-fourth divided by three-fourths."

Go right on to page 254.

**goal** Practice in estimating quotients when dividing two fractions

**page 254** Talking about the words on this page is very important, so continue the discussion from page 253 by using the examples at the top of the page. By this time the pupils should be looking at the numerators and their size relationship. Everyone is on his own for problems 1 through 6, but the page summary is worth talking about to make sure everyone is on the right track.

Use whatever way you want to determine whether the answer is greater than or less than 1.

$\frac{3}{8} \div \frac{2}{8} \,?\, 1$

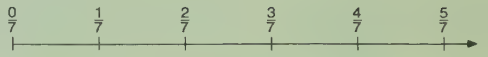
Is there one $\frac{1}{8}$ in $\frac{3}{8}$? Yes

Is there one $\frac{2}{8}$ in $\frac{3}{8}$? Yes

Is there more than one $\frac{2}{8}$ in $\frac{3}{8}$? Yes

$\frac{3}{8} \div \frac{2}{8} > 1$

$\frac{4}{7} \div \frac{5}{7} \,?\, 1$




How many $\frac{5}{7}$s in $\frac{4}{7}$? $\frac{4}{5}$

$\frac{4}{7} \div \frac{5}{7} < 1$

Replace the ? with < or >.

1. $\frac{1}{2} \div \frac{3}{2} < 1$
2. $\frac{5}{6} \div \frac{4}{6} > 1$
3. $\frac{7}{3} \div \frac{9}{3} < 1$
4. $\frac{7}{8} \div \frac{2}{8} > 1$
5. $\frac{4}{5} \div \frac{2}{5} > 1$
6. $\frac{8}{4} \div \frac{9}{4} < 1$

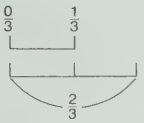
7. Compare the denominators in each problem. They are the same.
8. Compare the numerators in each problem. They differ.
9. Look at the problems in which you used >.
   Compare the fractions.
   Is the first fraction less than the second fraction? No
10. Look at the problems in which you used <.
    Compare the fractions. What is the pattern here?
    The first fraction is less than the second fraction.

254

**1.** $\frac{1}{3} \div \frac{2}{3} < 1$

How many $\frac{2}{3}$s in $\frac{1}{3}$? $\frac{1}{2}$

What part of $\frac{2}{3}$ is in $\frac{1}{3}$? $\frac{1}{2}$



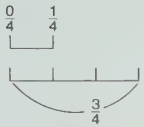

Only half of $\frac{2}{3}$ is in $\frac{1}{3}$.

$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$

**2.** $\frac{1}{4} \div \frac{3}{4} < 1$

How many $\frac{3}{4}$s in $\frac{1}{4}$? $\frac{1}{3}$

What part of $\frac{3}{4}$ is in $\frac{1}{4}$? $\frac{1}{3}$

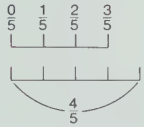



Only $\frac{1}{3}$ of $\frac{3}{4}$ is in $\frac{1}{4}$.

$\frac{1}{4} \div \frac{3}{4} = \frac{1}{3}$

**3.** $\frac{3}{5} \div \frac{4}{5} < 1$

What part of $\frac{4}{5}$ is in $\frac{3}{5}$? $\frac{3}{4}$




Only $\frac{3}{4}$ of $\frac{4}{5}$ is in $\frac{3}{5}$.

$\frac{3}{5} \div \frac{4}{5} = \frac{3}{4}$

Compare the answers with the numerators of the fractions in the problem. There's a pattern. Do you see it?

The numerator of the fraction divided becomes the numerator of the quotient. The numerator of the divisor becomes the denominator of the quotient.

Ask yourself, will the answers be less than 1? Yes

**4.** $\frac{1}{2} \div \frac{3}{2} = \underline{?}$ $\frac{1}{3}$

**5.** $\frac{1}{4} \div \frac{2}{4} = \underline{?}$ $\frac{1}{2}$

**6.** $\frac{2}{5} \div \frac{3}{5} = \underline{?}$ $\frac{2}{3}$

**7.** $\frac{2}{6} \div \frac{4}{6} = \underline{?}$ $\frac{1}{2}$

**8.** $\frac{1}{3} \div \frac{4}{3} = \underline{?}$ $\frac{1}{4}$

**9.** $\frac{3}{8} \div \frac{6}{8} = \underline{?}$ $\frac{1}{2}$

**10.** $\frac{3}{10} \div \frac{4}{10} = \underline{?}$ $\frac{3}{4}$

**11.** $\frac{3}{12} \div \frac{5}{12} = \underline{?}$ $\frac{3}{5}$



**lesson** Page 255

**goal** Exploration of dividing fractions with quotients less than 1

**page 255** First examine the denominators of the problems. Are they alike? Now examine the numerators to determine whether the quotient will be less than 1 or greater than 1.

Some of your learners may already be thinking of a good shortcut.
$\frac{1}{3} \div \frac{2}{3}$ The denominators are the same.
So the problem is really $1 \div 2$.
Think division: dividend ÷ divisor
This sentence may also be written

$$\frac{1}{2} \frac{\text{dividend}}{\text{divisor}}$$

The continued emphasis on the size of the answers may require another review of the last page.

$\frac{①}{2} \div \frac{③}{2}$ - less than 1 $(1 \div 3)$ $\frac{⑤}{6} \div \frac{④}{6}$ - more than 1 $(5 \div 4)$

$\frac{⑦}{3} \div \frac{⑨}{3}$ - less than 1 $(7 \div 9)$ $\frac{⑦}{8} \div \frac{②}{8}$ - more than 1 $(7 \div 2)$

Watch for any youngster who seems confused when the denominator of the quotient is different from the denominator of the problem.

**lesson** Pages 256, 257

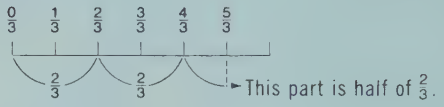
**goal** Exploration of dividing fractions with quotients greater than 1

**page 256** Examine the numerators for each example to determine whether the quotient will be less than 1 or greater than 1. The focus is on the "How many" question. Continue to avoid reading the division sentence as five-thirds divided by two-thirds. The emphasis is still on building the **concept** of division rather than on the operation itself; therefore, the sentence should be read, "How many two-thirds in five-thirds?"

Examine each example with your pupils. If the development on page 255 is understood, there should be no problem.

**1.** $\frac{5}{3} \div \frac{2}{3} > 1$ How many $\frac{2}{3}$s in $\frac{5}{3}$?

$\frac{5}{3} \div \frac{2}{3} = \underline{?}$



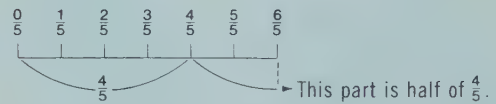

From $\frac{0}{3}$ to $\frac{2}{3}$ is one $\frac{2}{3}$.

From $\frac{2}{3}$ to $\frac{4}{3}$ is one more $\frac{2}{3}$.

From $\frac{4}{3}$ to $\frac{5}{3}$ is $\frac{1}{2}$ of a $\frac{2}{3}$. So $\frac{5}{3} \div \frac{2}{3} = 2\frac{1}{2}$

**2.** $\frac{6}{5} \div \frac{4}{5} > 1$ How many $\frac{4}{5}$s in $\frac{6}{5}$?

$\frac{6}{5} \div \frac{4}{5} = \underline{?}$

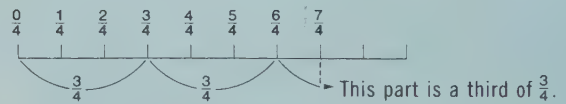



From $\frac{0}{5}$ to $\frac{4}{5}$ is one $\frac{4}{5}$.

From $\frac{4}{5}$ to $\frac{6}{5}$ is $\frac{1}{2}$ of a $\frac{4}{5}$. So $\frac{6}{5} \div \frac{4}{5} = \underline{?}$ $1\frac{1}{2}$

**3.** $\frac{7}{4} \div \frac{3}{4} > 1$ How many $\frac{3}{4}$s in $\frac{7}{4}$?

$\frac{7}{4} \div \frac{3}{4} = \underline{?}$

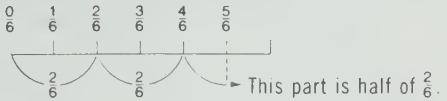



So $\frac{7}{4} \div \frac{3}{4} = \underline{?}$ $2\frac{1}{3}$

256

Draw a number line for each problem. Then divide.

Example: $\frac{5}{6} \div \frac{2}{6} = ?$




$\frac{5}{6} \div \frac{2}{6} = 2\frac{1}{2}$

## YOUR TURN

1. $\frac{6}{8} \div \frac{4}{8} = \underline{?}$ $1\frac{1}{2}$ How many $\frac{4}{8}$s in $\frac{6}{8}$? $1\frac{1}{2}$

2. $\frac{5}{4} \div \frac{2}{4} = \underline{?}$ $2\frac{1}{2}$ How many $\frac{2}{4}$s in $\frac{5}{4}$? $2\frac{1}{2}$

3. $\frac{9}{10} \div \frac{4}{10} = \underline{?}$ $2\frac{1}{4}$ How many $\frac{4}{10}$s in $\frac{9}{10}$? $2\frac{1}{4}$

4. $\frac{7}{3} \div \frac{2}{3} = \underline{?}$ $3\frac{1}{2}$ How many $\frac{2}{3}$s in $\frac{7}{3}$? $3\frac{1}{2}$

You can also use this idea on problems written as $1\frac{2}{3} \div \frac{2}{3}$.
All you have to do is rename $1\frac{2}{3}$ as $\frac{5}{3}$ and then—

5. $\frac{5}{3} \div \frac{2}{3} = \underline{?}$ $2\frac{1}{2}$ How many $\frac{2}{3}$s in $\frac{5}{3}$? $2\frac{1}{2}$

6. $1\frac{1}{4} \div \frac{3}{4} = \underline{?} = \frac{5}{4} \div \frac{3}{4} = \underline{?}$ $1\frac{2}{3}$ How many $\frac{3}{4}$s in $\frac{5}{4}$? $1\frac{2}{3}$

Look back at the example at the top of the page.
The answer is $2\frac{1}{2}$. Think of $2\frac{1}{2}$ expressed as the fraction $\frac{5}{2}$.

Do you see a relationship between the answer $\frac{5}{2}$ and
the two numerators of the fractions in the problem?

The numerator of $\frac{5}{6}$ is the numerator of the quotient. The numerator of $\frac{2}{6}$ is the denominator of the quotient.

Can you find this pattern in the other problems?

Yes – all of them



**goal** Practice in dividing fractions with quotients greater than 1

**page 257** Appreciate the fact that the kids have had experience with only whole-number division. When dividing with whole numbers, the answer that results is "smaller." Now suddenly when dividing with fractions, the answer is "bigger." It's no wonder this type of division is a mystery to so many people.

Encourage your pupils to continue using the number line if it helps them visualize the problem. A model is given at the top of the page. You will find some youngsters who can read a completed number line but cannot make one independently. Get a peer tutor to help.

Discuss the questions at the bottom of the page with everyone. There should be some pupils who are ready to generalize and share their insight with the others.

**lesson** Pages 258, 259, 260

**goal** Exploration of dividing a whole number by a fraction

**page 258** Making a number line to find each answer becomes tedious. There must be a way to compute the answer with numbers. The action of dividing each hour into thirds on the number line parallels renaming 2 as $\frac{6}{3}$ in written form. How many $\frac{1}{3}$s in $\frac{6}{3}$ ? can then be easily answered. This is the pupil's first encounter in division without common denominators. Can any whole number be named as a fraction? What denominator should be used for this renaming step? (Denominator of the divisor) Once this renaming step is completed, there should be no problem in completing the division.

**1.** June was helping her father, a fruit farmer, pick cherries. She worked for 2 hours. It took her $\frac{1}{3}$ of an hour to fill a basket. How many baskets of cherries did she pick? 6

The math sentence for this story is $2 \div \frac{1}{3} = \underline{?}$
You think, How many $\frac{1}{3}$s in 2? You can picture it on a number line.

Each arrow on the number line shows $\frac{1}{3}$ of an hour.
The arrowhead shows the end of a period worth $\frac{1}{3}$ h .
Count to find how many periods of $\frac{1}{3}$ h there are in 2 h .

Does $2 \div \frac{1}{3} = 6$? Yes

You could have computed rather than counted.

You are dividing by $\frac{1}{3}$.
Can 2 also be named as some number of thirds? Yes

$2 \div \frac{1}{3} = \frac{?}{3} \div \frac{1}{3}$ Is $\frac{6}{3}$ another name for 2? Yes

$\frac{6}{3} \div \frac{1}{3}$ How many $\frac{1}{3}$s in $\frac{6}{3}$? 6

$\frac{6}{3} \div \frac{1}{3} = 6$

**2.** What if June had worked for 3 h? How many baskets would she have filled? 9

$3 \div \frac{1}{3}$ How many $\frac{1}{3}$s in 3? 9

**3.** Can 3 be renamed as some number of thirds? Yes

$\frac{?}{3} \div \frac{1}{3}$ Is $\frac{9}{3}$ another name for 3? Yes

$\frac{9}{3} \div \frac{1}{3}$ How many $\frac{1}{3}$s in $\frac{9}{3}$? 9

258

**goal** Practice in dividing a whole number by a fraction

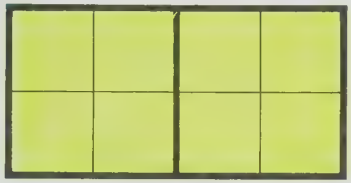
**page 259** Another model is introduced—the region model. Pupils will now have two ways to visualize dividing a whole number by a fraction.

The prerequisite step to actually dividing is renaming the whole number as a fraction. You may want to check this skill orally before making an assignment. Make and use models for any tutorial work that is needed.

## The Small Deal Land Co.

is dividing 2 acres of land into $\frac{1}{4}$-acre lots. How many lots will there be?

The math sentence for this story is $2 \div \frac{1}{4} = \underline{?}$
You think, How many $\frac{1}{4}$s in 2? You can picture it.



There are $\frac{4}{4}$ in 1 acre.
And another $\frac{4}{4}$ in the other acre.
Now you can count to find how many $\frac{1}{4}$s in 2.

But see if you know how to compute the answer.

$2 \div \frac{1}{4}$ You're dividing by fourths, so rename 2 as fourths. $\frac{8}{4} \div \frac{1}{4} = 8$

Renaming whole numbers is the first step to computing problems like this. Show that you know how.

**1.** $1 = \frac{?}{2}$ 2 **2.** $1 = \frac{?}{3}$ 3 **3.** $1 = \frac{?}{4}$ 4 **4.** $1 = \frac{?}{5}$ 5

**5.** $2 = \frac{?}{2}$ 4 **6.** $3 = \frac{?}{2}$ 6 **7.** $4 = \frac{?}{2}$ 8 **8.** $5 = \frac{?}{2}$ 10

**9.** $4 = \frac{?}{3}$ 12 If you're having trouble, draw a picture.

**10.** $2 = \frac{?}{5}$ 10 Still in trouble? Then try a number line.

**11.** $5 = \frac{?}{4}$ 20 **12.** $3 = \frac{?}{6}$ 18 **13.** $2 = \frac{?}{8}$ 16 **14.** $6 = \frac{?}{2}$ 12

**15.** $2 = \frac{?}{3}$ 6 **16.** $4 = \frac{?}{4}$ 16 **17.** $3 = \frac{?}{3}$ 9 **18.** $10 = \frac{?}{10}$ 100

**goal** Practice in dividing a whole number by a fraction

**page 260** Problems 1 through 5 are rigged—the numerator of each divisor is 1. The number lines should help the pupil see that when the numerator of the divisor is 1, the quotient is the numerator of the dividend—$\frac{5}{1}$ is another name for 5.

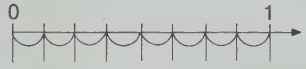
Everyone should be able to handle the computations on the page independently. Emphasize being able to **read** a number line, but don't expect everyone to be able to make the number-line model independently. The focus continues to be on renaming the whole number first.

Use set II on page 267 for tutorial purposes only. Pupils who are successful on this page should continue on to page 261.

Complete each sentence to describe the division shown on the number line.

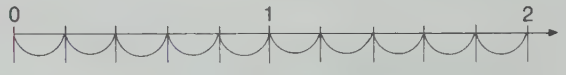
**1.** $1 \div \frac{1}{5} = \frac{?}{5} \div \frac{1}{5} = \underline{?}$ 5

**2.** $1 \div \frac{1}{8} = \frac{?}{8} \div \frac{1}{8} = \underline{?}$ 8




**3.** $1 \div \frac{1}{4} = \frac{?}{4} \div \frac{1}{4} = \underline{?}$ 4

**4.** $2 \div \frac{1}{5} = \frac{?}{5} \div \frac{1}{5} = \underline{?}$ 10




**5.** $3 \div \frac{1}{2} = \frac{?}{2} \div \frac{1}{2} = \underline{?}$ 6

**6.** You draw a number line to show $2 \div \frac{1}{2}$. Then compute the answer.

You don't need the number line for these.
Rename each whole number as a fraction and compute.

**7.** $4 \div \frac{1}{3} = \underline{?}$ $\frac{12}{3} \div \frac{1}{3} = \frac{12}{1}$ or 12

**8.** $3 \div \frac{1}{6} = \underline{?}$ $\frac{18}{6} \div \frac{1}{6} = \frac{18}{1}$ or 18

**9.** $1 \div \frac{1}{7} = \underline{?}$ $\frac{7}{7} \div \frac{1}{7} = 7$

**10.** $2 \div \frac{1}{8} = \underline{?}$ $\frac{16}{8} \div \frac{1}{8} = 16$

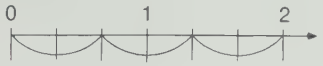
**11.** $5 \div \frac{1}{5} = \underline{?}$ $\frac{25}{5} \div \frac{1}{5} = 25$

**12.** $1 \div \frac{1}{10} = \underline{?}$ $\frac{10}{10} \div \frac{1}{10}$

*Extra practice on page 267— set II*

See activity 3, page 268a.

$2 \div \frac{2}{3} = \underline{?}$ How many $\frac{2}{3}$s in 2? Rename 2 as thirds and compute.

$2 \div \frac{2}{3} = \frac{?^{6}}{3} \div \frac{2}{3} = \underline{?}\ ^{3}$ 


**1.** Complete each sentence to describe the division shown on the number line.

**a** $3 \div \frac{3}{4} = \frac{?^{12}}{4} \div \frac{3}{4}\ ^{4}$


**b** $2 \div \frac{2}{5} = \frac{?^{10}}{5} \div \frac{2}{5}\ ^{5}$

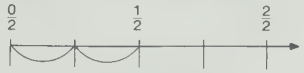

Renaming the whole number so that it has the same denominator as the fraction divisor is a very important first step.

**2.** And guess what.
Sometimes denominators have to be renamed too.

**a** $\frac{1}{2} \div \frac{1}{4} = \underline{?}$

Each loop shows $\frac{1}{4}$.

How many $\frac{1}{4}$s in $\frac{1}{2}$?




$\frac{1}{2}$ has to be renamed as fourths. A common denominator is needed.

$\frac{1}{2} \div \frac{1}{4}$

*Rename* $\frac{2}{4}$

*Rewrite* $\frac{2}{4} \div \frac{1}{4} = \underline{?}\ ^{2}$ Now you can divide without any trouble.



**lesson** Pages 261, 262, 263

**goal** Introduction to dividing fractions with unlike denominators and whole-number quotients

**page 261** The focus is on renaming—renaming so that division is possible. Dividing fractions by the common-denominator method requires writing the problem so that the dividend and the divisor have the same denominator. If a whole number is involved, it is renamed as an appropriate fraction. If the problem consists of two fractions with unlike denominators, then renaming with a common denominator is the first step.

Compare the size of each divisor to each dividend. Is the quotient less than 1 or greater than 1?

**goal** Dividing fractions with unlike denominators and mixed number quotients

**page 262** On the preceding page, the quotient is always a whole number. Here the quotient may be a mixed number. Problem 4 introduces the possibility of a fraction quotient as well.

Praise the pupil who sees these patterns:
- When the numerator of the dividend is a multiple of the numerator of the divisor, the quotient is a whole number.
- When the numerator of the dividend is not a multiple of the numerator of the divisor, the quotient is a mixed number.

Don't expect anyone to use this rigorous language. The pupil's pointing to the numerator and talking about it in his own language does a far better job of indicating understanding.

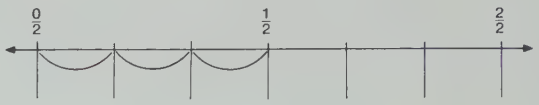
Be alert for signs of two types of difficulty that may arise:
- Uneven division errors
- Naming the denominator of the fraction in a mixed-number quotient

How many $\frac{2}{4}$s in $\frac{3}{4}$? One $\frac{2}{4}$ unit and $\frac{1}{2}$ of another $\frac{2}{4}$ unit. Once the fractions have been changed to a common denominator, we work only with the numerators.

**1** Look at another one.
How many $\frac{1}{6}$s in $\frac{1}{2}$?

$\frac{1}{2} \div \frac{1}{6}$

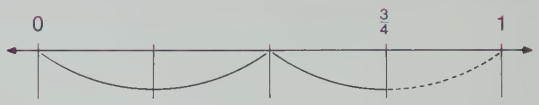
*Rename* $\frac{3}{6} \div \frac{1}{6} = \underline{?}$ 3




**2** How many $\frac{1}{2}$s in $\frac{3}{4}$?
Find a common denominator.

$\frac{3}{4} \div \frac{1}{2}$

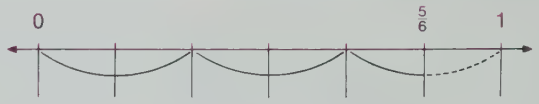
$\frac{3}{4} \div \frac{2}{4} = \underline{?}$ $1\frac{1}{2}$



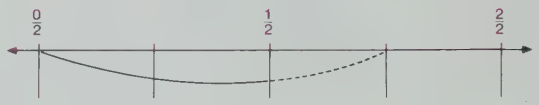

**3** How many $\frac{1}{3}$s in $\frac{5}{6}$?
Find a common denominator.

$\frac{5}{6} \div \frac{1}{3}$

$\frac{5}{6} \div \frac{2}{6} = \underline{?}$ $2\frac{1}{2}$




**4** $\frac{1}{2} \div \frac{3}{4} = \underline{?}$ The loop shows $\frac{3}{4}$.
How many $\frac{3}{4}$s in $\frac{1}{2}$?
How much of a $\frac{3}{4}$ in $\frac{1}{2}$?




Once again, $\frac{1}{2}$ has to be renamed as fourths.

Find a common denominator. $\frac{1}{2} \div \frac{3}{4}$

*Rename* $\frac{2}{4}$

*Rewrite* $\frac{2}{4} \div \frac{3}{4} = \underline{?}$ 2/3

262

Watch the denominators!
Make sure you have a common denominator.
Then all you have to do is divide the numerators.

**Can I write the problem like this: $\frac{5}{6} \div \frac{1}{3} = 5 \div 2 = 2\frac{1}{2}$?**

Can you answer Dan's question? What steps did he leave out? Where did the second step, $5 \div 2$, come from? Look back at the problems you have done on other pages. After you have a common denominator, can you just divide the numerators and find the quotient? Yes
Think carefully!

In this example the number line shows the need for a common denominator.

How many $\frac{1}{2}$s in $\frac{2}{3}$?

$\frac{2}{3} \div \frac{1}{2}$

$\frac{4}{6} \div \frac{3}{6} = 4 \div 3 = 1\frac{1}{3}$

└You can write this step or just think it. It's up to you.

You complete these problems.

**1.** $\frac{4}{5} \div \frac{1}{2}$

$\frac{8}{10} \div \frac{5}{10} = 8 \div 5 = \underline{?}$ $1\frac{3}{5}$

**2.** $\frac{7}{10} \div \frac{1}{5}$

$\frac{7}{10} \div \frac{2}{10} = 7 \div 2 = \underline{?}$ $3\frac{1}{2}$

**3.** $\frac{8}{9} \div \frac{1}{3}$

$\frac{?}{9} \div \frac{?}{9} = \underline{?}$ $2\frac{2}{3}$

**4.** $\frac{3}{4} \div \frac{1}{3}$

$\frac{?}{12} \div \frac{?}{12} = \underline{?}$ $2\frac{1}{4}$

**5.** $\frac{7}{6} \div \frac{1}{3}$

$\frac{?}{6} \div \frac{?}{6} = \underline{?}$ $3\frac{1}{2}$

**6.** $\frac{9}{10} \div \frac{2}{5}$

$\frac{?}{10} \div \frac{?}{10} = \underline{?}$ $2\frac{1}{4}$

263

**goal** Practice in dividing fractions

**page 263** Dan's question has been included for a reason—to check whether the algorithm is understood. Dan mentally changed to a common denominator, then divided numerators to find the quotient. If your pupils don't realize this, encourage them to work out the example, then compare their work with Dan's. Remember, lead—don't tell. Those who are capable may use this mental shortcut for problems 1 through 6.

**lesson** Pages 264, 265

**goal** Estimating quotients

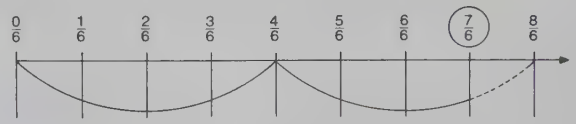
**page 264** The page reviews each type of division problem developed in the chapter. For each example, compare the size of the divisor to the size of the dividend and the type of quotient that results. Summarize the findings in a chart and look for a pattern.

- Dividend > divisor? Then the quotient > 1.
- Dividend < divisor? Then the quotient < 1.

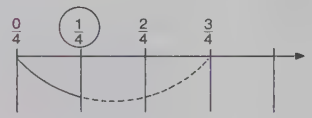
Look, but don't tell. If your pupils don't find them, these patterns will be discussed at the top of page 265.

**1.** Here's a special review.

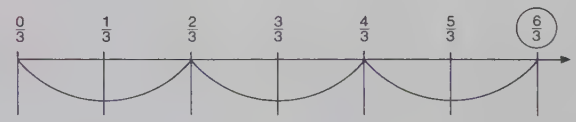
**a** How many $\frac{4}{6}$s in $\frac{7}{6}$?
More than 1? Yes
More than 2? No
$\frac{7}{6} \div \frac{4}{6} = \underline{?}$ $1\frac{3}{4}$



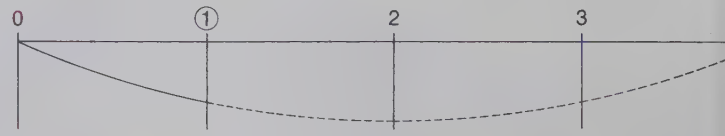

**b** How many $\frac{3}{4}$s in $\frac{1}{4}$?
More than 1? No
Less than 1? Yes
$\frac{1}{4} \div \frac{3}{4} = \underline{?}$ $\frac{1}{3}$



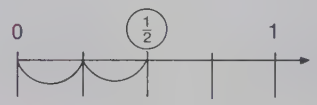

**c** How many $\frac{2}{3}$s in 2?
More than 1? Yes
More than 2? Yes
$2 \div \frac{2}{3} = \underline{?}$ 3




**d** How many 4s in 1?
More than 1? No
Less than 1? Yes
$1 \div 4 = \underline{?}$ $\frac{1}{4}$




**e** How many $\frac{1}{4}$s in $\frac{1}{2}$?
More than 1? Yes
Less than 1? No
$\frac{1}{2} \div \frac{1}{4} = \underline{?}$ 2




*Extra practice on page 267—set III*

264

Estimation can help you decide whether your answer is reasonable.

**1.** Is $\frac{1}{2}$ greater than or less than $\frac{1}{3}$?

Is $\frac{1}{2} \div \frac{1}{3}$ greater than or less than 1?

Is $\frac{1}{3} \div \frac{1}{2}$ greater than or less than 1?

**2.** Is $\frac{1}{5}$ greater than or less than $\frac{1}{3}$?

Is $\frac{1}{5} \div \frac{1}{3}$ greater than or less than 1?

Is $\frac{1}{3} \div \frac{1}{5}$ greater than or less than 1?

**3.** Complete each sentence with > or <.

**a** $\frac{3}{5} \div \frac{1}{2} \; ? \; 1$ (>)  **b** $\frac{1}{3} \div \frac{1}{4} \; ? \; 1$ (>)  **c** $\frac{5}{6} \div \frac{1}{4} \; ? \; 1$ (>)

**d** $\frac{2}{3} \div \frac{5}{6} \; ? \; 1$ (<)  **e** $\frac{1}{5} \div \frac{1}{2} \; ? \; 1$ (<)  **f** $\frac{5}{8} \div \frac{1}{3} \; ? \; 1$ (>)

**g** $\frac{2}{3} \div \frac{1}{6} \; ? \; 1$ (>)  **h** $\frac{1}{2} \div \frac{5}{6} \; ? \; 1$ (<)  **i** $\frac{5}{12} \div \frac{1}{2} \; ? \; 1$ (<)

## PROGRESS CHECK

**DIVIDE**

Skill: Dividing a whole number by a fraction

(1.) $1 \div \frac{1}{8} = \; ?$ 8  (2.) $1 \div \frac{3}{5} = \; ?$ $1\frac{2}{3}$  (3.) $2 \div \frac{2}{3} = \; ?$ 3

Skill: Dividing a fraction by a fraction

(4.) $3 \div \frac{4}{5} = \; ?$ $3\frac{3}{4}$  (5.) $\frac{3}{4} \div \frac{1}{3} = \; ?$ $2\frac{1}{4}$  (6.) $\frac{4}{5} \div \frac{1}{2} = \; ?$ $1\frac{3}{5}$

265

STOP See activity 4, page 268b.

See activity 5, page 268b.

**goal** Checking the reasonableness of a quotient by estimation; **Progress Check**—dividing fractions

**page 265** Models are not provided here, and the examples are not in common-denominator form. Comparison, not computation, is the emphasis. Encourage your pupils to change to common-denominator form before they compare. Discourage guessing—this helps no one. Some pupils will be capable of sophisticated thinking at this point. For example:

$\frac{5}{6} \div \frac{1}{4}$ Think: $\frac{5}{6}$ is almost 1
$\frac{1}{4}$ is less than $\frac{1}{2}$
Therefore $\frac{5}{6} > \frac{1}{4}$

Learners capable of this level of thinking should not be required to change to common-denominator form. You will probably want to do problems 1 through 3 as a group.

Before assigning the Progress Check, you may want to have your pupils estimate orally whether the quotient for each problem is greater than 1 or less than 1. You may also want to determine the common denominator for each problem as a group—especially problems 5 and 6.

Look for problems in renaming with a common denominator. Practice only the specific skill that is causing trouble.

Check for errors with uneven division.
$3 \div \frac{4}{5} = \frac{15}{5} \div \frac{4}{5} = (15 \div 4)$
$15 \div 4$ can be thought of as
$\frac{15}{4} = 3\frac{3}{4}$ or as $4\overline{)15}$ $3\frac{3}{4}$

The additional practice sets on page 267 can be used if problems are needed as a basis for tutorial work.

**lesson** Page 266

**goal** Introduction to division with mixed numbers

**page 266** One last step in completing the division picture—division with mixed numbers. Renaming each mixed number as a fraction is a prerequisite skill. Watch for those pupils who need practice in this type of renaming. Assign peer tutors if necessary. This skill must be mastered before a pupil can do the problems on this page. Continue to emphasize estimating the size of the quotient.

Division involving two mixed numbers introduces no new rules. But it does involve more renaming. Mastery is not expected at this time; yet, pupils should not be left in a state of confusion.

Set IV on page 267 is included for tutorial help.

$4 \div 1\frac{1}{3}$ should be thought of as $4 \div \frac{4}{3}$.
Where did the $\frac{4}{3}$ come from? $1\frac{1}{3} = \frac{4}{3}$
The whole number must be renamed as thirds.

$$4 \div \frac{4}{3} = \frac{12}{3} \div \frac{4}{3} = 12 \div 4 = \underline{?} \quad 3$$

$3 \div 1\frac{1}{2}$ should be thought of as $3 \div \frac{3}{2}$.
Then get ready to compute. Rename the whole number.

$$3 \div \frac{3}{2} = \frac{?}{2}^{6} \div \frac{3}{2} = \underline{?} \quad 2$$

Practice on these. Find the quotients.

**1.** $5 \div 1\frac{1}{4} = \underline{?}$ 4 **2.** $5 \div 1\frac{2}{3} = \underline{?}$ 3 **3.** $8 \div 1\frac{1}{3} = \underline{?}$ 6

**4.** $7 \div 1\frac{1}{6} = \underline{?}$ 6 **5.** $4 \div 2\frac{1}{2} = \underline{?}$ $1\frac{3}{5}$ **6.** $6 \div 1\frac{3}{8} = \underline{?}$ $4\frac{4}{11}$

**7.** $8 \div 2\frac{1}{2} = \underline{?}$ $3\frac{1}{5}$ **8.** $2 \div 1\frac{3}{4} = \underline{?}$ $1\frac{1}{7}$ **9.** $3 \div 1\frac{1}{2} = \underline{?}$ 2

What would you do with a problem like $2\frac{1}{2} \div 1\frac{1}{3}$?
Think of it as $\frac{5}{2}_{15} \div \frac{4}{3}_{8}$. Then find a common denominator.

$$\frac{5}{2} \div \frac{4}{3} = \frac{?}{6} \div \frac{?}{6} = \underline{?} \quad 1\frac{7}{8}$$ There are really no new rules.

Find these quotients.

**10.** $3\frac{1}{2} \div 1\frac{1}{5} = \underline{?}$ $2\frac{11}{12}$ **11.** $3\frac{2}{3} \div 1\frac{1}{8} = \underline{?}$ $3\frac{7}{27}$ **12.** $1\frac{3}{4} \div 2\frac{1}{3} = \underline{?}$ $\frac{3}{4}$

**13.** $1\frac{4}{5} \div 4\frac{1}{2} = \underline{?}$ $\frac{2}{5}$ **14.** $1\frac{1}{2} \div 2\frac{1}{2} = \underline{?}$ $\frac{3}{5}$ **15.** $2\frac{1}{2} \div 1\frac{1}{4} = \underline{?}$ 2

*Extra practice on page 267—set IV*

# Extra Practice and Review

## Set I

Compute these sentences.

| a | b | c |
|---|---|---|
| $7 \div 10 = ?$ $\frac{7}{10}$ | $10 \div 7 = ?$ $1\frac{3}{7}$ | $8 \div 6 = ?$ $1\frac{1}{3}$ |
| $6 \div 8 = ?$ $\frac{3}{4}$ | $6 \div 4 = ?$ $1\frac{1}{2}$ | $4 \div 6 = ?$ $\frac{2}{3}$ |
| $3 \div 7 = ?$ $\frac{3}{7}$ | $8 \div 7 = ?$ $1\frac{1}{7}$ | $1 \div 10 = ?$ $\frac{1}{10}$ |

## Set II

Rename each whole number as a fraction and compute.

| a | b | c |
|---|---|---|
| $8 \div \frac{1}{2}$ 16 | $4 \div \frac{1}{4}$ 16 | $3 \div \frac{1}{8}$ 24 |
| $8 \div \frac{1}{3}$ 24 | $10 \div \frac{1}{5}$ 50 | $5 \div \frac{1}{10}$ 50 |
| $4 \div \frac{1}{6}$ 24 | $10 \div \frac{1}{10}$ 100 | $1 \div \frac{1}{10}$ 10 |

## Set III

Compute.

Watch the denominators!

| a | b | c |
|---|---|---|
| $\frac{3}{5} \div \frac{1}{2}$ $1\frac{1}{5}$ | $\frac{3}{5} \div \frac{1}{10}$ 6 | $\frac{1}{3} \div \frac{5}{6}$ $\frac{2}{5}$ |
| $\frac{1}{2} \div \frac{1}{5}$ $2\frac{1}{2}$ | $\frac{5}{6} \div \frac{2}{3}$ $1\frac{1}{4}$ | $\frac{1}{8} \div \frac{1}{2}$ $\frac{1}{4}$ |
| $\frac{1}{2} \div \frac{1}{8}$ 4 | $\frac{1}{2} \div \frac{3}{5}$ $\frac{5}{6}$ | $\frac{1}{2} \div \frac{4}{5}$ $\frac{5}{8}$ |

## Set IV

Find the quotient.

| a | b | c |
|---|---|---|
| $2\frac{1}{3} \div 1\frac{1}{2}$ $1\frac{5}{9}$ | $6 \div 2\frac{2}{3}$ $2\frac{1}{4}$ | $2\frac{1}{4} \div 3\frac{3}{8}$ $\frac{2}{3}$ |
| $3\frac{3}{4} \div 1\frac{1}{2}$ $2\frac{1}{2}$ | $2\frac{1}{6} \div 1\frac{1}{4}$ $1\frac{11}{15}$ | $3 \div 3\frac{3}{4}$ $\frac{4}{5}$ |
| $5 \div 2\frac{2}{5}$ $2\frac{1}{12}$ | $4 \div 3\frac{1}{3}$ $1\frac{1}{5}$ | $3\frac{1}{3} \div 4$ $\frac{5}{6}$ |



**lesson** Pages 267, 268

**goal** Providing additional practice with division skills developed in the chapter

**page 267** The problem sets are provided to serve as a basis for tutorial work.

Skill: Computing with fractions

Add.

| | a | b | c | d |
|---|---|---|---|---|
| 1. | $1\frac{1}{2} + 2\frac{1}{3}$ $3\frac{5}{6}$ (Set A) | $2\frac{1}{8} + 1\frac{3}{4}$ (Set H) | $\frac{1}{3} + \frac{2}{5}$ $\frac{11}{15}$ (Set A) | $\frac{3}{8} +$ (Set |

Subtract.

| | a | b | c | d |
|---|---|---|---|---|
| 2. | $2\frac{7}{8} - 1\frac{5}{8}$ $1\frac{1}{4}$ (Set H) | $2 - \frac{1}{2}$ (Set H) | $3\frac{3}{8} - 1\frac{3}{4}$ $1\frac{5}{8}$ (Set H) | $\frac{7}{10} -$ (S |

Divide.

| | a | b | c | d |
|---|---|---|---|---|
| 3. | $4 \div \frac{1}{2}$ 8 (Set II) | $3 \div \frac{1}{2}$ 6 | $3 \div \frac{3}{4}$ 4 | $2 \div$ |
| 4. | $3 \div 1$ 3 (Set I) | $2 \div 4$ $\frac{1}{2}$ (Set I) | $\frac{1}{2} \div \frac{1}{6}$ 3 (Set III) | $\frac{1}{4} \div$ (S |
| 5. | $\frac{1}{4} \div \frac{1}{3}$ $\frac{3}{4}$ (Set III) | $\frac{2}{3} \div \frac{1}{2}$ $1\frac{1}{3}$ | $\frac{5}{8} \div \frac{1}{4}$ $2\frac{1}{2}$ | $\frac{3}{10} \div$ |
| 6. | $4 \div 1\frac{1}{3}$ 3 (Set IV) | $5 \div 1\frac{1}{4}$ 4 | $6 \div 1\frac{1}{2}$ 4 | $2\frac{1}{2}$ |

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See activity 6, page 268b.

See activity 7, page 268b.

# RESOURCES

## another form of evaluation

**for progress check** – page 247

Compute. Watch the operation symbols.

1. $\frac{5}{9} + \frac{2}{9}$ $\frac{7}{9}$ 2. $\frac{6}{7} - \frac{4}{7}$ $\frac{2}{7}$ 3. $\frac{1}{8} + \frac{5}{8}$ $\frac{3}{4}$
4. $\frac{11}{12} - \frac{5}{12}$ $\frac{1}{2}$ 5. $\frac{1}{3} + \frac{7}{9}$ $1\frac{1}{9}$ 6. $\frac{9}{10} - \frac{2}{5}$ $\frac{1}{2}$
7. $2\frac{5}{8} + 3\frac{1}{8}$ $5\frac{3}{4}$ 8. $5\frac{5}{6} - 3\frac{1}{6}$ $2\frac{2}{3}$ 9. $1\frac{4}{5} + 3\frac{3}{10}$ $5\frac{1}{10}$
10. $4\frac{1}{3} - 1\frac{2}{3}$ $2\frac{2}{3}$ 11. $3 - \frac{1}{4}$ $2\frac{3}{4}$ 12. $2\frac{7}{9} + 3\frac{2}{9}$ 6
13. Tuesday was a busy day for Mark. After school, he helped in the library for $\frac{3}{4}$ hour. Then he attended band practice for $1\frac{1}{2}$ hours. How many extra hours had he spent at school? $2\frac{1}{4}$
    Mark's homework took $1\frac{5}{6}$ hours. How many hours in all had he spent on these activities? $4\frac{1}{12}$

Here's an 1895 kind of problem to try.

14. Ann bought a bicycle for $90. Then she spent $20 on equipment. What fraction of the cost of the bicycle did the equipment cost? $\frac{2}{9}$

**for progress check** – page 265

Divide.

1. $1 \div \frac{1}{6} = \underline{?}$ 6 2. $1 \div \frac{2}{3} = \underline{?}$ $1\frac{1}{2}$ 3. $3 \div \frac{3}{8} = \underline{?}$ 8
4. $2 \div \frac{4}{5} = \underline{?}$ $2\frac{1}{2}$ 5. $\frac{2}{3} \div \frac{3}{5} = \underline{?}$ $1\frac{1}{9}$ 6. $\frac{5}{6} \div \frac{1}{4} = \underline{?}$ $3\frac{1}{3}$

**for checkout** – page 268

Add.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 1. | $\frac{3}{4} + \frac{3}{4}$ $1\frac{7}{20}$ | $2\frac{1}{6} + 3\frac{2}{3}$ $5\frac{5}{6}$ | $3\frac{3}{8} + \frac{1}{3}$ $\frac{17}{24}$ | $\frac{7}{9} + \frac{5}{9}$ $1\frac{1}{3}$ |

Subtract.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 2. | $3\frac{5}{6} - 1\frac{1}{6}$ $2\frac{2}{3}$ | $3 - \frac{1}{4}$ $2\frac{3}{4}$ | $4\frac{4}{9} - 2\frac{2}{3}$ $1\frac{7}{9}$ | $\frac{9}{10} - \frac{7}{10}$ $\frac{1}{5}$ |

Divide.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 3. | $2 \div \frac{1}{3}$ 6 | $3 \div \frac{1}{3}$ 9 | $5 \div \frac{5}{6}$ 6 | $4 \div \frac{4}{5}$ 5 |
| 4. | $4 \div 1$ 4 | $2 \div 6\frac{1}{3}$ | $\frac{1}{3} \div \frac{1}{9}$ 3 | $\frac{1}{5} \div \frac{1}{10}$ 2 |
| 5. | $\frac{1}{6} \div \frac{1}{5}$ $\frac{5}{6}$ | $\frac{3}{4} \div \frac{1}{3}$ $2\frac{1}{4}$ | $\frac{5}{12} \div \frac{1}{6}$ $2\frac{1}{2}$ | $\frac{7}{8} \div \frac{1}{2}$ $1\frac{3}{4}$ |
| 6. | $5 \div 1\frac{2}{3}$ 3 | $7 \div 1\frac{1}{6}$ 6 | $6 \div 1\frac{1}{5}$ 5 | $3\frac{1}{3} \div 1\frac{1}{3}$ $2\frac{1}{2}$ |

7. Try some problems like those from the 1895 book.
   a) Three families share a large lot for their vegetable gardens. The Smiths have $\frac{1}{2}$, the Thompsons $\frac{1}{3}$, and the Browns have the rest of it. What fraction of the lot do the Browns have? $\frac{1}{6}$
   b) One week Tim's father worked $45\frac{1}{4}$ hours. His usual week at work was $37\frac{1}{2}$ hours. How many hours had he worked overtime? $7\frac{3}{4}$
   c) $62\frac{1}{2} \div 6\frac{1}{4}$ 10

## activities

**1. things** spirit master or laminated cards

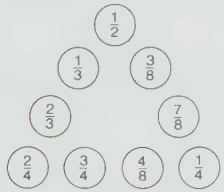
Prepare a spirit master as shown or use laminated cards. Pupils write on the cards with washable crayons or watercolor pens so that they can be reused.

| | − | | + | | − | | + | |
|---|---|---|---|---|---|---|---|---|
| 3 | $\frac{1}{2}$ | $(2\frac{1}{2})$ | $\frac{1}{4}$ | $(2\frac{3}{4})$ | $1\frac{3}{4}$ | (1) | $1\frac{1}{2}$ | $(2\frac{1}{2})$ |
| $\frac{4}{8}$ | $\frac{1}{8}$ | | $\frac{3}{4}$ | | $\frac{3}{8}$ | | $\frac{1}{2}$ | |
| $\frac{5}{6}$ | $\frac{1}{2}$ | | $\frac{5}{12}$ | | $\frac{1}{2}$ | | $\frac{3}{4}$ | |
| $\frac{8}{12}$ | $\frac{2}{3}$ | | $\frac{4}{5}$ | | $\frac{1}{10}$ | | 5 | |
| 8 | $3\frac{1}{4}$ | | $\frac{5}{8}$ | | $2\frac{1}{8}$ | | $\frac{3}{4}$ | |
| 10 | $5\frac{1}{3}$ | | $4\frac{1}{2}$ | | $6\frac{2}{3}$ | | $\frac{1}{6}$ | |

**2.** Use only these fractions:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{7}{8}$

Write one fraction in each circle so that the sum of the numbers on each side of the triangle is the same. (One solution is shown.)




**3.** Both multiplication and division of fractions can throw a youngster if he has never taken time to think about the size of the answer in relation to the size of the multiplier or divisor. If a pupil insists that the answers don't make sense, have him arrange the following numbers in order from greatest to least:

3, $\frac{2}{3}$, 2, $\frac{1}{2}$, 1, $\frac{1}{3}$

Next present these patterns:

| | |
|---|---|
| $3 \times 3 = 9$ | $6 \div 3 = 2$ |
| $3 \times 2 = 6$ | $6 \div 2 = 3$ |
| $3 \times 1 = 3$ | $6 \div 1 = 6$ |
| $3 \times \frac{2}{3} = \frac{6}{3}$ or 2 | $6 \div \frac{2}{3} = \frac{18}{2}$ or 9 |
| $3 \times \frac{1}{2} = \frac{3}{2}$ or $1\frac{1}{2}$ | $6 \div \frac{1}{2} = \frac{12}{1}$ or 12 |
| $3 \times \frac{1}{3} = \frac{3}{3}$ or 1 | $6 \div \frac{1}{3} = \frac{18}{1}$ or 18 |

By examining the size of the multiplier or divisor and of the resulting answer, perhaps the youngsters can see that the answers make sense after all.

**4.** Use the answers to these problems and the decoder to find a coded message.

| | | |
|---|---|---|
| 1. $1 \div \frac{1}{4}$ | 2. $3 \div \frac{2}{3}$ | 3. $8 \div \frac{1}{4}$ |
| 4. $\frac{1}{3} \div \frac{1}{2}$ | 5. $10 \div \frac{1}{5}$ | 6. $\frac{2}{3} \div \frac{1}{3}$ |
| 7. $4 \div \frac{1}{3}$ | 8. $\frac{2}{3} \div \frac{5}{6}$ | 9. $\frac{3}{4} \div \frac{3}{4}$ |
| 10. $\frac{3}{4} \div \frac{3}{8}$ | 11. $2 \div \frac{1}{6}$ | 12. $\frac{7}{10} \div \frac{3}{5}$ |
| 13. $\frac{1}{4} \div \frac{1}{18}$ | 14. $16 \div \frac{1}{2}$ | 15. $\frac{5}{8} \div \frac{3}{16}$ |

| Answers | 2 | 4 | $\frac{2}{3}$ | $1\frac{1}{6}$ | 1 | 32 | $4\frac{1}{2}$ | 50 | $3\frac{1}{3}$ | 12 | $\frac{4}{5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Code letter | A | C | G | I | L | N | O | R | S | T | U |

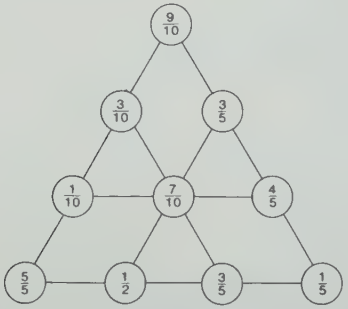
Solution: CONGRATULATIONS

**5.** Provide the pupil with the following figure and directions.

Here are some fractions:

$\frac{1}{2}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}$

Write these fractions in the circles so that all the lines with four circles have the same sum **and** that all the lines with three circles have the same sum. (The answer is shown.)




**6. things** for each pupil: 9 tongue depressors

Multiple sticks make renaming with a common denominator easy. To make multiple sticks, divide each tongue depressor into 10 same-size sections. Write the first ten nonzero multiples of 1 in the sections on one stick, the first ten nonzero multiples of 2 on another. Continue until a stick has been made for the multiples of 1 through 9.

To add $\frac{2}{3} + \frac{5}{8}$ place the 2-stick above the 3-stick and the 5-stick above the 8-stick.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |

Look only at the 3-stick and the 8-stick to find the first (least) common multiple of both 3 and 8—and therefore the common denominator for these two fractions. On the $\frac{2\text{-stick}}{3\text{-stick}}$, 16 is above 24. So, $\frac{2}{3} = \frac{16}{24}$.
On the $\frac{5\text{-stick}}{8\text{-stick}}$, 15 is above 24. So, $\frac{5}{8} = \frac{15}{24}$.

The denominators are alike. Now simply add the numerators to find the sum. The sticks can be used for addition, subtraction, and division.

**7.** Challenge: Can you write a name for number 1, using exactly 4 fours?

Rules:
- Use at least one fraction.
- Choose any number operation.

A name can be written for every number from 1 through 10, with exactly 4 fours. Try it.

$1 = \frac{4+4}{4+4}$ $\quad 2 = \frac{4 \times 4}{4+4}$

$3 = \frac{4+4+4}{4}$ $\quad 4 = \frac{4-4}{4} + 4$

$5 = \frac{4 \times 4 + 4}{4}$ $\quad 6 = \frac{4+4}{4} + 4$

$7 = \frac{44}{4} - 4$ $\quad 8 = \frac{4 \times (4+4)}{4}$

$9 = 4 + 4 + \frac{4}{4}$ $\quad 10 = \frac{44-4}{4}$

# additional learning aids

**operation**—chapter objectives 1, 2, 3

### SRA products

*Mathematics Learning System,*
*Activity Masters, level B*, SRA (1974)
Spirit masters: W-31, 33, 34; F-15, 33, 34
*Computapes*, SRA (1972)
Module 5, Lessons: FR 22–24
*Computational Skills Development Kit*, SRA (1965)
Addition of Fractions cards: 1–16
Subtraction of Fractions cards: 1–9
Division of Fractions cards: 1–6
*Cross-Number Puzzles* (*Fractions*), SRA (1966)
Addition cards: 1–15
Subtraction cards: 1–15
Division cards: 1–6
*Cross-Number Puzzles* (*Story Problems*), SRA (1966)
Fraction cards: 1–5
Mixed Fraction cards: 1–5
*Diagnosis: an instructional aid—Mathematics Level B*, SRA (1972)
Probes: M-5, 6
*Math Applications Kit*, SRA (1971)
Appeteasers card: 9
Sports and Games: 49
Occupations cards: 26, 44
Social Studies cards: 19, 25, 26
Everyday Things cards: 13, 21, 41, 42
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 29, 30, 31, 32, 33, 34, 38, 39, 54, 62

**other learning aids** (described on page 288d)—Experiments in Fractions, Fraction Bars Student Activity Book (levels I and II), Fraction Tally, Tripletts

# 11 GEOMETRY

**before this chapter the learner has –**

1. Identified polygons by the number of sides
2. Determined by tracing and testing whether two polygons were congruent
3. Found the line(s) of symmetry of a plane figure by folding
4. Found the area and perimeter of a rectangle on geopaper

**in chapter 11 the learner is –**

1. Examining figures that have been flipped or turned and determining by tracing and matching whether they are congruent
2. Finding reflected images of a figure
3. Finding the number of axes of symmetry for a polygon
4. Rotating figures a given number of degrees and identifying the degree of rotation for a figure that has been turned

**in later levels the learner will –**

1. Identify whether two congruent figures differ by a rotation
2. Identify the direction of a rotation and the amount of rotation

# Notes & Things

This chapter provides a review of several major concepts of geometry previously explored – congruence, symmetry, and circular (degree) measurement. These ideas are synthesized in the study of motion geometry. The importance of congruence is shown when reflections and rotations are examined. Reflections extend the learners' knowledge of symmetry. Studying an object as it turns relates circular motion to degree measurement. All this sounds very technical, but not when these ideas are thought about in terms of **flips** and **turns**.

An activity approach is featured throughout the chapter. Pupils explore to find how area and perimeter relate to congruence. Scale drawings are introduced as possible furniture placements are examined. Do-it-yourselfers have an opportunity to demonstrate their ability by rearranging tile patterns. Messages written in reverse need to be decoded. Mirror reflections are examined. Axes of symmetry are found by folding tracings of figures. The turning of figures is related to movements of clock hands and degree measure – this means more practice with a protractor.

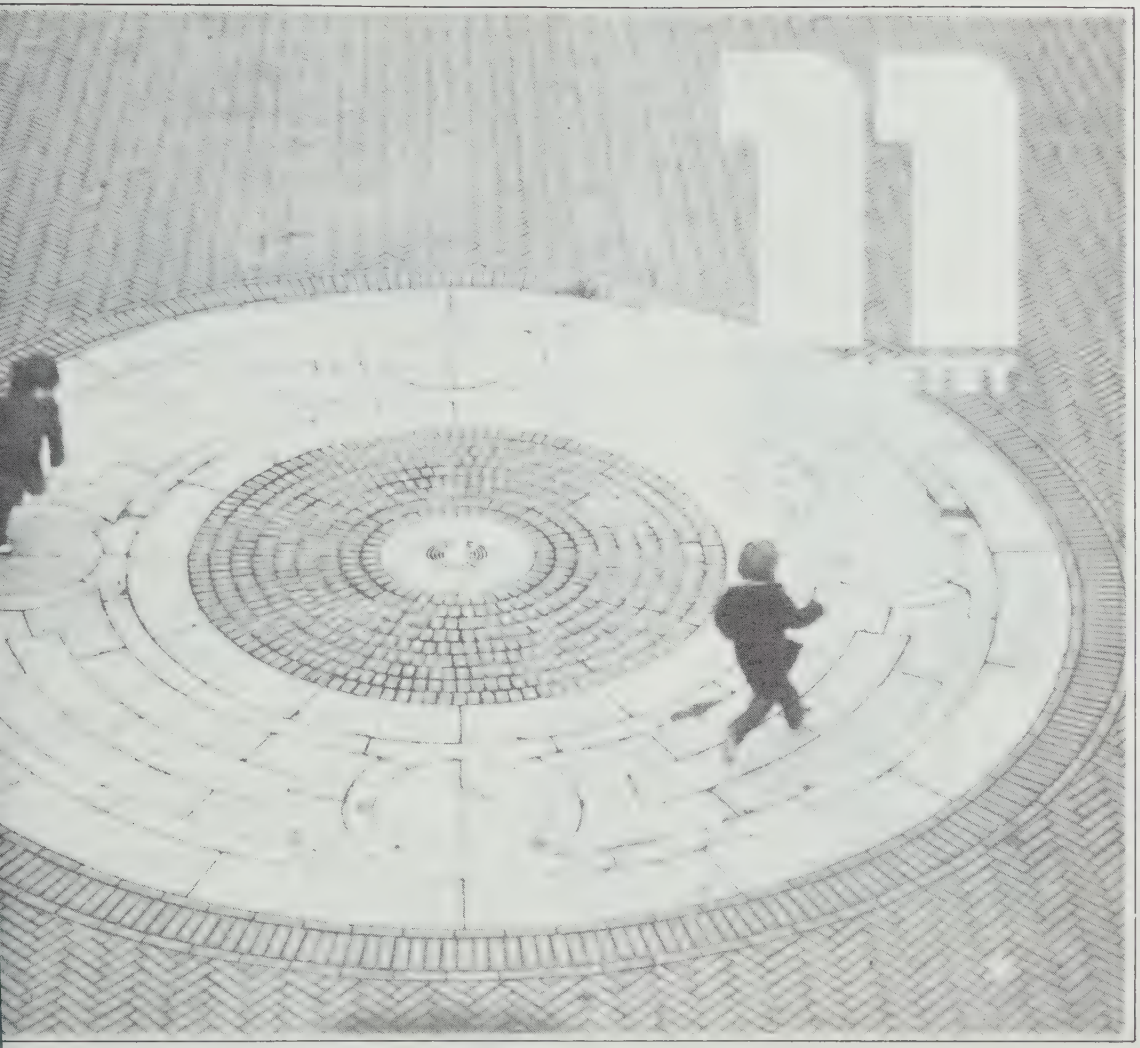
The study of rotations and reflections will continue in the next levels. This chapter is simply meant to explore these ideas and get the youngsters to observe the things that happen in their world that are related to geometry.

## things

rulers (optional)
paper for tracing
straightedge
graph paper
scissors
construction paper
paper clips
mirrors
clock with movable hands
protractor

For the extra activities you will want to have these things available:

- for each pupil: circle protractor or index card
- colored pencils or crayons
- paper punch
- equilateral-triangle grid paper



**goal** Think about and explore ideas through a picture clue

**page 269** This plaza might be found in many countries of the world. The pupil's little brothers and sisters would probably play the same running game if a plaza like this were close to their home. Encourage your pupils to analyze the circular shape. How many parts of the circular design are very nearly the same? If you counted the number of different shapes needed to do $\frac{1}{4}$ of the design, would you have to count to find out how many different shapes you would need for the entire design? Which shape is used most frequently?

Would a plaza like this be practical to build in the area in which you live? Why not? Is there a design such as this one somewhere in your community? All pupils should keep their eyes open. Sometimes a bathroom floor or the wall of an older building will contain an equally interesting tile design.

**lesson** Pages 270, 271

**goal** Exploring whether moving an object changes its size and shape

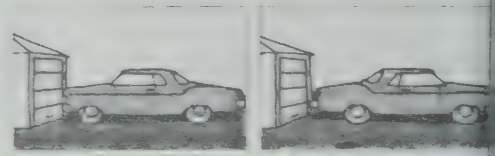
**page 270** These discussion questions are designed to explore ideas, to open doors to inquiry, and to encourage considering many possibilities. Distance can cause an optical illusion. A car, a boat, or a plane, for example, certainly looks different when it's far away. Does distance affect size?

Someone is sure to think that the car in problem 3 may have been in an accident during the two-day period. Will this change any answers?

What are the characteristics of an object that changes shape and size? After the discussion, try to generalize. Is there any one word that describes all objects that do not change when moved? (*Rigid*) What about those that do change? (*Not rigid*)

1. John stood against the right wall of the room and Bill measured his height. He was 160 cm tall. If John walked over to the other side of the room, how tall would he be there? 160 cm Would he measure the same around the waist? Would his arms be the same length? Yes Yes What about his legs? What about his feet? Yes Yes
2. If John walked two blocks away from Bill, he would look smaller. Did John's size change? No

3. One day John's mother parked the family car in the driveway facing the garage. The next day she parked the car by backing into the driveway.



Is the car still the same car? Is its location the same? Is its position the same? Is Yes No anything else about the car different? Is it the same size? Is it the same shape? Yes Yes

4. When an object is moved, its location changes. It may also be in a different position and direction. But the size and shape of most objects don't change.
   a. Name some objects in the room that do not change in size or shape when they are moved. Answers will vary. Exam
   b Can you think of any objects whose size or shape does change when you move them? Answers will vary. Examples: rag; beanbag chair

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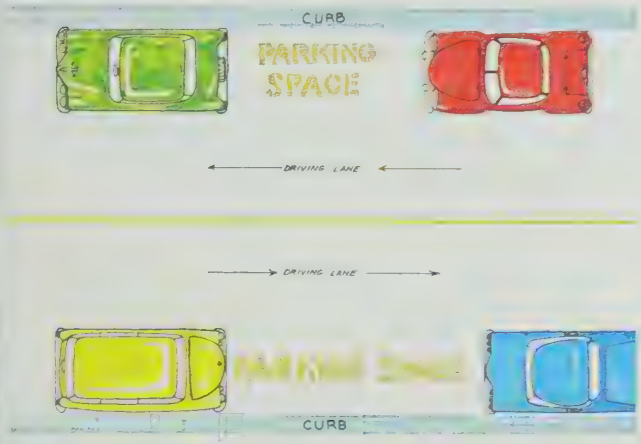
Do you think most people know that solid shapes do not change size or shape when they are moved? Yes
If so, why do you think there are "fender bender" accidents when people try to park cars? Carelessness of driver




Cut out a piece of paper 1.5 cm wide and 4 cm long. Pretend it is a model of a car.

There are two typical parking situations shown. Will your car fit into both spaces? Try to park your car so that you don't hit any of the other cars. Don't forget you can't go over the curb. No

What kind of motions do you have to think about as you try to get your car into position?

Forward, backward, turning

Believe it or not, geometry can help you solve problems like parallel parking. Your goal is to explore the motion of shapes.

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**goal** Exploring motions of solid shapes

**page 271** Here's everyone's golden opportunity to show how much he knows about parking a car. Explore the kinds of motions possible in driving a car. Remind the youngsters that sliding sideways is not usually possible with a car without a slippery road surface. Emphasize the goal of the chapter—to learn about motions of shapes.

**lesson** Pages 272, 273

**goal** Review of the characteristics of polygons

**things** rulers (optional)

**page 272** It's time to review the minimum vocabulary and previously examined concepts. Careful of problem **1a**. Some youngster may just draw a second figure congruent to the first. This is not likely, but it is possible. If in doubt, have the pupil use tracing as a check.

In math, **why not** is as important as **why is**. After naming the polygons in problem 4, zero in on figure **e** and why the figure **isn't** a polygon.

Using rulers or marking segments on the edge of a piece of paper will verify any doubtful answers for problem 5.

1. Draw a closed figure with three straight sides. How many angles? 3

   **a** Draw another one. Is it the same size? Not necessarily, but it could be

   **b** What is the name for all closed figures with three straight sides and three angles? Triangle

2. Draw a closed figure with four straight sides. ▭ or ▱ or ▭ How many angles? 4

   **a** Draw another one with opposite sides the same length and one with each side the same length. ▭ or ▭

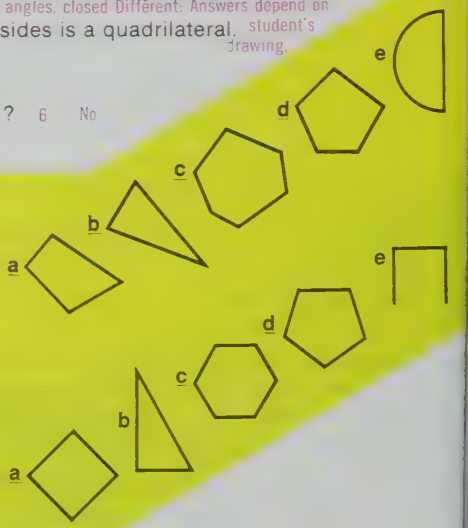
   **b** How are these two figures alike? How are they different? Alike: Opposite sides same length, 4 straight sides, 4 angles, closed Different: Answers depend on

   **c** Any closed figure with four straight sides is a quadrilateral. student's drawing.

3. Draw a closed figure with six straight sides. How many angles? Is this figure a quadrilateral? 6 No Why? Does this figure have a name? Hexagon It has 6 sides.

4. Wouldn't it be nice if all straight-sided closed figures had just one name? Many people thought so. They started to use the name *polygon* (*poly-* comes from a Greek word meaning "many" and *-gon* from a Greek word meaning "angles"). Which of these closed figures are polygons?

5. If every side of a polygon is the same length AND if every angle is the same size, the polygon is called a *regular* polygon. Which of these closed figures are regular polygons?

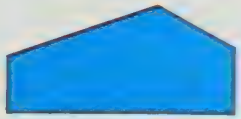



272

1. This is a polygon.

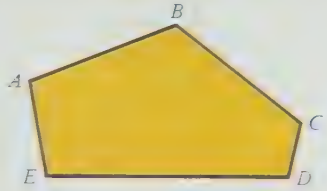
   **a** How many sides does the polygon have? 5

   **b** Could the front of a house have this shape? Yes

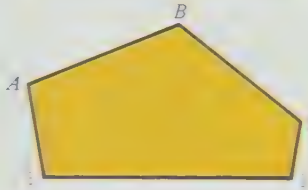


2. You can label each line segment that makes the side of a polygon.

   **a** The sides of this polygon are $AB$, $BC$, ?, ?, and ?. CD, DE and EA, or DC, ED and AE

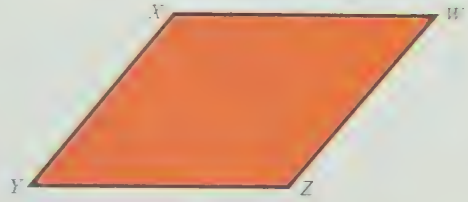



   **b** This polygon is a tracing of the one above. Tell how you would label the corners of the tracing.




3. Trace this polygon on a separate sheet of paper. Label the corners of your tracing so that they match the original figure.

   Is the figure you drew the same size and shape as the original one? Yes




*Figures that are the same size and shape are congruent.*

273

**goal** Review of the concept of congruent figures

**things** paper for tracing

**page 273** Here's some more review. This time it's a review of naming the parts of a polygon and of actually tracing a congruent model. The technique is a familiar one.

Few exact definitions are presented on pupil pages in this program. Much of the vocabulary is left up to you for development, and its use by pupils is then optional. When vocabulary does appear on the page, the pupil is expected to know and use it. Find the vocabulary on pages 272 and 273 that everyone needs.

**lesson** Pages 274, 275

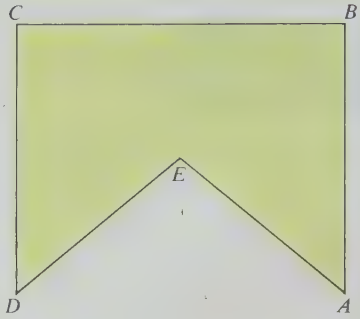
**goal** Testing two figures to determine whether they are congruent

**things** paper for tracing
straightedge

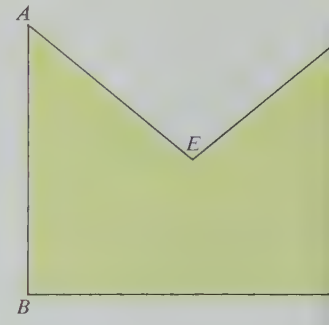
**page 274** Two methods have been developed for testing congruency in this program:

- The basic method of making a tracing or movable model
- Verifying the arrangement and number of square units of area measure for two figures

Have the pupils use both methods for problems 3 and 4.

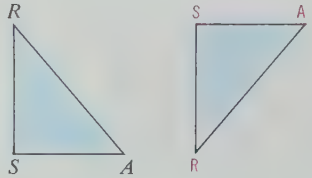



How many sides has the figure on the left? How many angles? 5; 5
Make a tracing of the polygon on the left. Is your tracing Should be congruent to the original figure? Is your tracing congruent to the figure on the right? Yes

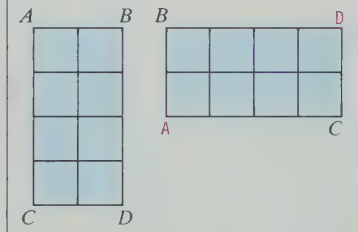



**1.**
Did you need to turn the tracing of the figure to make sure the tracing was congruent to the figure on the right? Yes

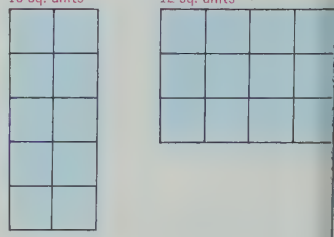
**2.**
Are the two triangles below congruent? Trace to make sure. Yes
Tell how you would label the second triangle. Label angles that are the same size with the same label.




**3.**
Are the two rectangles below congruent? How could you Yes
make sure? How would you Trace
finish labeling the second rectangle? Do both rectangles cover the same number of square units? Yes



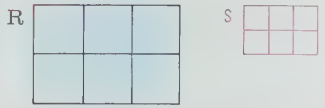

**4.**
What is the area of each rectangle below? Do they have the same area? Do No
they have the same shape? No
Are they congruent? No




274

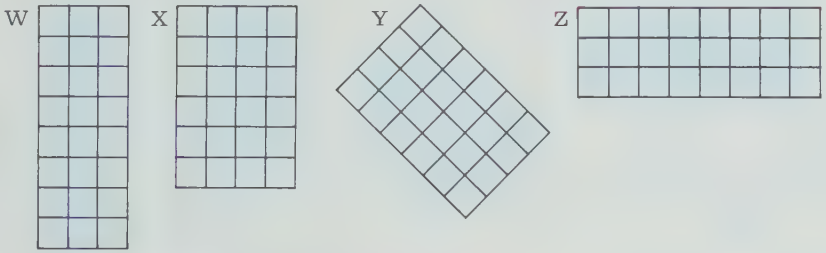
1. Two figures must have the same size and the same shape before they are congruent. Which of the following statements is true?
   - **a** If two polygons are congruent, they have the same area. True
   - **b** If two polygons have the same area, they are congruent. Not true

2. The area of this polygon is 6 square units. Draw a figure that is congruent to it. Call it figure S. Position of figure may vary.

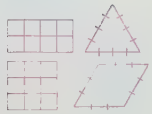



   - **a** Do both figures have the same area? Yes
   - **b** Draw a figure that has the same area but is not congruent to figure R or figure S. Label your noncongruent figure T. Drawings may vary. Example: T
   - **c** What is the perimeter (distance around) of figure R? 10
   - **d** What is the perimeter of figure S? 10
   - **e** What is the perimeter of figure T? Answers vary according to figure drawn in b.

3. Which of the rectangles below have the same area? All do.




   - **a** Which have the same perimeter? W and Z, X and Y
   - **b** Which are congruent? W and Z, X and Y
   - **c** Do figures that are congruent have the same perimeter? Yes
   - **d** Can figures that are *not* congruent have the same area? Yes
   - **e** Can figures that are *not* congruent have the same perimeter? Yes. Examples:





**goal** Exploring the relationship of area and perimeter to congruent and noncongruent figures

**things** graph paper
straightedge

**page 275** Anyone who doubts that the statement in problem **1b** is false need only take a piece of graph paper and make as many quadrilaterals as he can, each containing 24 square units.

Graph paper and a straightedge will simplify the work for problem 2.

After completing problem 3, challenge your pupils to think about two figures that have exactly the same area **and** the same perimeter. Ask what these facts tell about the two figures. (They are congruent.)

**lesson** Page 276

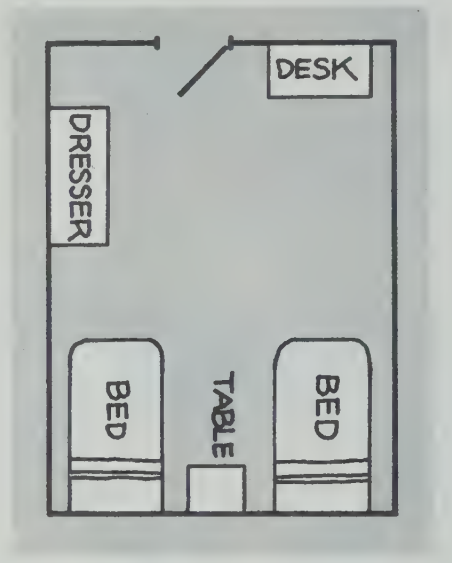
**goal** Introduction to scale drawings

**things** straightedge
scissors (optional)

**page 276** The pupils have had some experience using a scale on a map to measure distance, so the idea of using a smaller unit to represent a larger one will not be new to them.

Graph paper and movable cutout paper models for furniture are often used by interior decorators for this type of work. Graph paper may prove to be a hindrance though, for some students will be reluctant to place furniture over the lines. The cutout models of furniture will help in deciding possible furniture arrangements. Room-planning kits are available at some furniture stores. If one is available, you will have the raw material to extend the learning on this page.

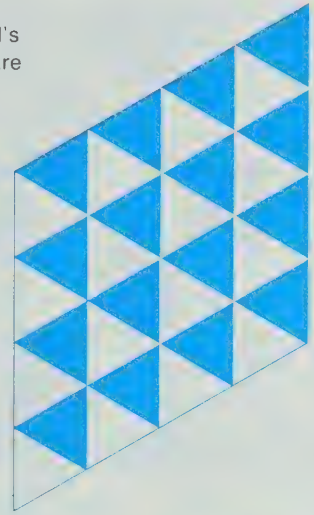
Bette and Jean are planning how to rearrange the furniture in their room. This is the way the room is arranged now:



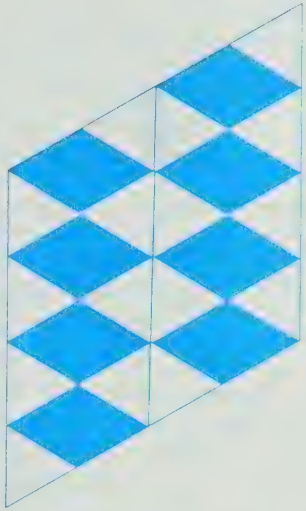

1. Bette wants one of the beds where the dresser is now. Jean says it won't fit.
   - **a** Will moving the bed change its size? No
   - **b** Will the space by the door change size?
   - **c** How can they check to see whether the bed will fit? Answers will vary. Examples: Measure bed and space by door; make scale drawings of room and bed, and then try; move the bed and try it.
2. They can also use a drawing. The drawing above is much smaller than the room and furniture. Each inch of the drawing represents 4 feet of the room. Each $\frac{1}{8}$ inch of the drawing represents $\frac{1}{2}$ foot of the room. Such a drawing is called a *scale drawing*.
   - **a** Trace one of the beds in the drawing above. Use your tracing to see whether the bed will fit by the door where Bette wants it. Will it fit? Yes—if a sh is against th
   - **b** Will the two beds fit end to end? Yes
   - **c** What pieces of furniture can fit between the beds if the beds are not moved? Desk (or table)
3. Make at least two sketches showing how the room could be rearranged. How would you arrange the room?
   Sketches will vary.

276

**1.**
The tiles on Fred's bathroom floor are arranged in a pattern like this:

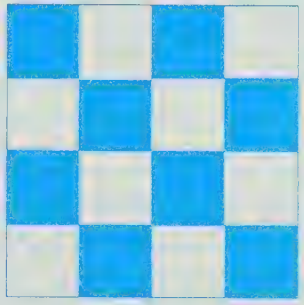


He imagines another design using the same size tiles:



Is the number of blue tiles in each design the same? Yes
the number of white tiles? Is there the Yes
same number of blue tiles as white? Yes

**2.**
Find ways to rearrange each of these patterns.
Patterns will vary.



277

Have everyone hunt for actual tile patterns—at home, in school, in a public building. A sketch of the pattern can be made, indicating the different colors. Problem: Try to rearrange the pattern, using the same number of tiles of each color as in the original pattern.

**lesson** Pages 277, 278

**goal** Exploration of rearranging patterns

**things** construction paper
scissors

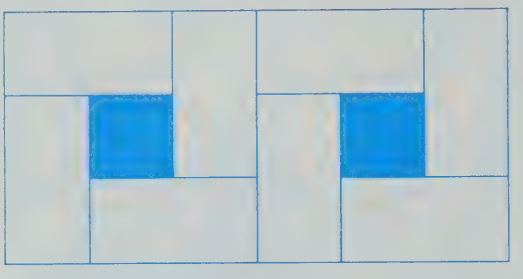
**page 277** The illustrations in problem 1 do not show the entire bathroom floor, for goodness sake, but you'll have someone who will think that is the case. Have your pupils verify whether the two regions are congruent. When it comes to a floor, changing the pattern is of little value if the same area isn't covered. Challenge the youngsters to figure out how the diamonds were formed.

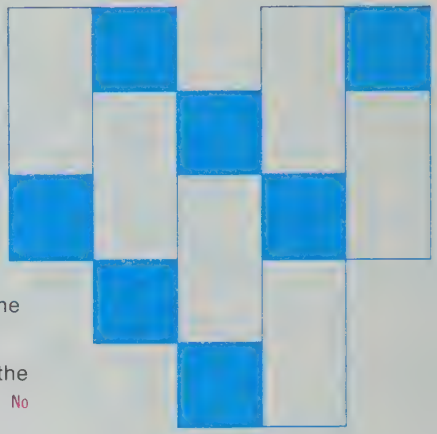
The new figures in problem 2 need not be congruent to those on the page. The emphasis is on changing the pattern. The pupil is free to do his thing as long as he maintains the same number of tiles of each color. Your pupils might enjoy finding their answers with tiles cut from construction paper. (The tiles used need not be congruent to those on the page.)

**goal** Continuing to explore the rearrangement of patterns; examining right and left hands for congruency

**page 278** The procedure for problem 2 on the preceding page applies to problem 3.

Generally, if an individual is right-handed, his right hand is a little larger; and if he is left-handed, his left hand is a little larger. Check the tracings for problem 4. Is there sufficient evidence to think that this may be true? If it is true, what might be the reason? (One hand is used more, so muscles develop more.)





**3.** These two patterns are made from tiles of the same shape.

**a** Is the number of blue tiles the same as the number of white ones in both designs? No

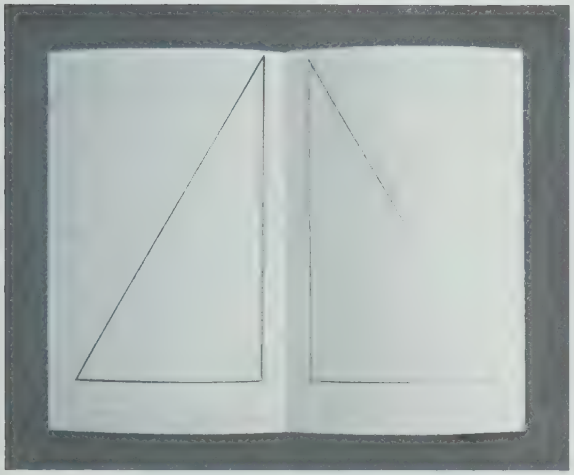
**b** Use tiles of the same shape as shown here and make up a pattern of your own. See how many ways you can rearrange the shapes. Patterns will vary.

**4.** Look at your right hand and your left hand. How are the two hands alike? How are they different?
Same shape; they're reverses of each other.

**a** Put your left hand palm down on a piece of paper. Trace the outline of your hand.

Make the lines dark enough to see from the other side of the paper. Now turn the paper over and put your right hand on the outline. Does it fit? Not exactly (The hand used most often will be slightly larger.)
There are small differences between the right and the left hand for most people. Except for these differences, the hands are the same.

278



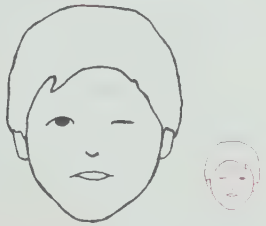
1. Bill drew a triangle on a piece of paper. Then he folded the paper and traced the outline of the triangle on the other side of the fold. The two triangles look very much alike, but the tracing is reversed.

   Are the two triangles the same size and shape? Make a tracing of them, Yes
   together with the fold line. Fold your tracing along the line so that the two triangles lie on top of each other. Do they match? Are they congruent? Yes
   Yes

2. Put a mirror on the fold line facing to the left. Do you see the image that looks like the triangle on the right of the fold line? Yes

3. Sue parts her hair on the right side. On which side does the part appear when she looks in a mirror? Trace this Left side
   sketch and draw her image in the mirror.

Things on the right look as if they are on the left in a mirror image. Things on the left look as if they are on the right.



4. Fred is winking his left eye. Trace the sketch and draw his image in the mirror. Which eye is the image winking?
   The right eye

279

**lesson** Pages 279, 280, 281

**goal** Investigating reflections of figures

**things** paper for tracing
paper clips

**page 279** Clipping the tracing paper to the page may help keep the paper from slipping. Stress tracing the fold line too. This will help the pupils line up the figures. They've had much experience tracing, so there shouldn't be any problems.

**goal** Examining reflections of figures

**things** mirrors

**page 280** Seeing is believing. If mirrors are available—perhaps from a science kit—use them to verify the messages in problems 1 and 2. Let the youngsters experiment to find where to place the mirror so that the message can be read. This will be approximately the same distance from the message as the fold lines are from the figures on page 279.

Before attempting problem 3, discuss what must be done with each letter in a word and with each word on the line to make the message come out reversed. Writing these messages may not be easy for some students. This may have to be a homework assignment since mirrors are necessary to check the results.

So far only a vertical line of reflection has been examined. A horizontal line of reflection is also possible. Careful of the letter *M* when discussing problem 4. The vertical reflection of *M* is identical, while the horizontal reflection looks like a *W*. Now look at **MOM** in problem 5. This is a parallel situation. A horizontal line of reflection need not be pointed out, but some sharpie may make this observation. Note that problems 4 and 5 call for reflections that **look alike**.

1. Tom and Mike wanted to write messages to each other. But they didn't want anyone else to read what they wrote.

   Tom planned to send this message:

   MIKE,
   I WILL MEET
   YOU AT THE
   CORNER AFTER
   SCHOOL.
   TOM

   But he wrote it so that it looked like this:

   MIKE,
   I WILL MEET
   YOU AT THE
   CORNER AFTER
   SCHOOL.
   TOM

   Can you read it?

2. Here is another mirror image. Write the original message. If you can't figure it out, hold the message up to a mirror.

   THIS SECRET
   MESSAGE IS
   A REFLECTION.

   THIS SECRET MESSAGE IS A REFLECTION.

3. Now try writing a message of your own as it would look in a mirror. MESSAGES WILL VARY

4. What does the mirror image of the capital letter M look like? What does O look like? How many other capital letters look the same as their mirror image? List them. Same Same A, H, I, T, U, V, W, X, Y

5. There are some whole words that look the same as their image. MOM is one example. Find as many words as you can think of that look the same as their mirror image.

   Answers will vary
   Examples: TOT, WOW, HUH, TAT, and so on.

6. A pool of water can act like a mirror. But when you see the image of the bridge, the image is reversed *and* upside down.

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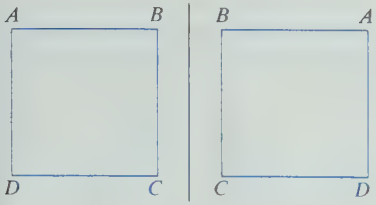
**things** spirit master; mirrors

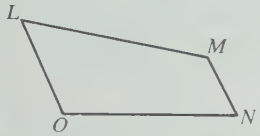
Preparing a spirit master with the letters and art printed in reverse is not difficult. Place the spirit master facedown with the protective tissue removed. Now write on the back of the carbon sheet. Have fun!



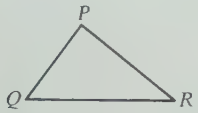

Look at the figures on the left.
The second rectangle is a mirror image of the first.

Trace the following figures and then draw each shape as it would look in a mirror. Be sure to label the three polygons in your tracing and on your drawing.

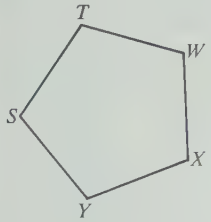
1.

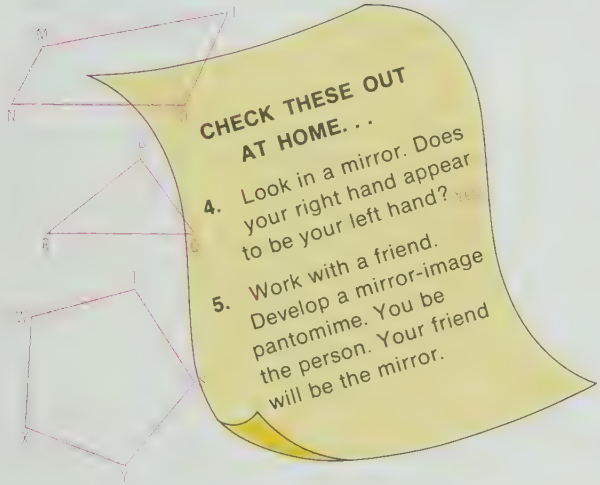



2.




3.



**CHECK THESE OUT AT HOME. . .**

4. Look in a mirror. Does your right hand appear to be your left hand?

5. Work with a friend. Develop a mirror-image pantomime. You be the person. Your friend will be the mirror.

281

**goal** Making reflections of a figure

**things** paper for tracing

**page 281** Check carefully that the letters used to name the sides of each figure are in the correct position on the reflection. Students need not reflect the letter itself, only the figures. For example, *P* need not be written ꟼ. Any youngster who is having a great deal of visual difficulty may need to use a mirror first in order to get a visual image of what he is to draw. Stress tracing the vertical line to help in estimating the location of the reflection.

**lesson** Pages 282, 283

**goal** Review of finding lines of symmetry

**things** paper for tracing
straightedge

**page 282** Pupils found it impossible to test real-world objects for congruency because the objects could not be folded or traced. These same problems exist in trying to test objects for symmetry.

Has anyone forgotten what makes a polygon regular (problem 2)? Send that youngster back to page 272 to look for the definition. Earlier in the program the pupils used tracing and folding to find lines of symmetry. The formal name AXES OF SYMMETRY is introduced here. (Point out the singular and the plural forms of the word **axis**.) Using a straightedge and making dark lines when tracing will simplify the matching procedure when folding.

**1** A plane figure is symmetrical if it can be folded to form two halves that match. The idea of symmetry can be seen if you look at a human being, for example. The eye on the right matches the eye on the left. The arm on the right matches the arm on the left. Does a human being have a nose on the right that matches a nose on the left? But does the part of the nose on the left match the part on the right? We can't test a human being to see if the body is symmetrical, because we can't fold a human being. We could fold a picture of a human being, of course. But let's start with plane figures.

No

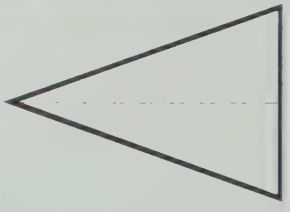
Yes

**2** All regular polygons are symmetrical. Trace the regular polygon below. How many different ways can you fold your tracing so that two symmetrical parts are formed?

Mark each fold that divides the figure. These fold-mark lines are called the *axes of symmetry*. If a figure can be folded only one way, it will have only one line or *axis* of symmetry.

**3** Is the figure below a regular polygon? No Trace it. How many different ways can you fold it so that one half matches the other half? Is the figure symmetrical? Does it have more than one axis of symmetry? No

Yes

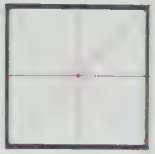


**4** Is the figure below a regular polygon? No Trace it. Is it symmetrical? Each of the polygons on this page is a triangle. Think about how the lengths of the sides relate to the number of axes of symmetry.

No

282

**1** The polygon on the right is a square. What has to be true about the length of each side? Trace it. On the tracing draw its axes of symmetry.



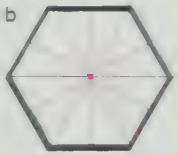
Must be equal

**2** Does a rectangle have four axes of symmetry?
Trace the rectangle below.
On the tracing draw its axes of symmetry.

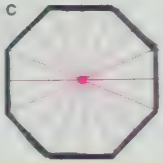
**3** Trace each of the regular polygons below. On each tracing draw the axes of symmetry for each polygon.

a

b



c



d

**4** Review the readings you have made for problem 3.

a How many sides does each polygon have?

b Is each of the sides the same length?

c How many angles does each polygon have?

d Is each angle the same size?

e How many axes of symmetry does each have?

f Suppose a regular polygon has eleven sides. How many axes of symmetry do you think it has?

g How many axes of symmetry would a 50-sided regular polygon have? a 100-sided one?

**5** The more sides a regular polygon has, the more it looks like a circle. How many axes of symmetry do you think a circle has?

Infinitely many



See activity 1, page 288a.

goal Finding axes of symmetry for regular polygons

things paper for tracing
straightedge

page 283 By the time they complete problem 4, the youngsters will generalize that for a regular polygon the number of axes of symmetry is equal to the number of sides. Careful—so far nothing has been done to prove that the reverse is also true. If a figure has 5 axes of symmetry, it could be a 5-sided regular polygon. But could it have some other shape, perhaps? The youngsters haven't tested enough yet to know. Proving or disproving this idea might make a good research project.

**lesson** Pages 284, 285

**goal** Introduction to rotations of a figure

**things** clock with movable hands
protractor

**warmup** Use a clock to review angle measurement before using these ideas to introduce rotations.

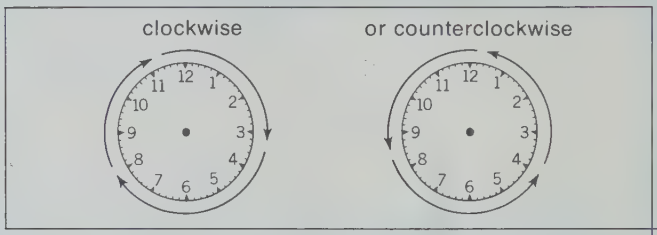
Use a clockface made from a paper plate. Fasten the hands with a wire paper fastener. Use the hands on the clock to define CLOCKWISE and COUNTERCLOCKWISE motions.

Review angle measurements as follows:

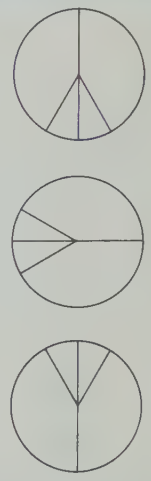
- Set both of the hands at 12. Keep the hour hand on 12 and rotate the minute hand to 3. What kind of angle is shown? How many degrees in this angle? Verify the answer with a protractor. How many degrees did the minute hand move going from 12 to 3? Return the minute hand to 12. Use a counterclockwise motion to 3 and repeat the questions.
- Set both hands at 12. Repeat the procedure. Rotate the minute hand to 6 first with a clockwise motion, then with a counterclockwise motion. Compare the results of the two motions.
- Set both hands at 12. Repeat. Rotate the minute hand to 9—both clockwise and counterclockwise.

**page 284** Have any pupil who has difficulty with problem 2 make a tracing of the first figure and verify the degree of rotation with movements of clock hands.

**1.** Many things turn or rotate. You rotate a radio or TV dial; you see wheels rotate. What other things rotate?
Answers will vary. Examples: clock hands, phone dial, earth, and so on
Two-dimensional figures can turn or rotate in either of two directions,




**2.** The figures at the right are congruent.



**a** The first symbol, when rotated 90° in a clockwise direction, is in the same position as the second symbol. How far must the second symbol be rotated to be in the same position as the third symbol? 90° clockwise

**b** How far must the first symbol be rotated to be in the same position as the third figure?
180° clockwise

**c** The first symbol can also be rotated 270° in a counterclockwise direction to be in the same position as the second symbol. How far must it be turned in a counterclockwise direction to be in the same position as the third figure? 180°

284

**1.** Now draw a stickman in each final position of the stickmen in this chart.

| Original position | Degrees clockwise | *OR* | Degrees counter-clockwise | Final position |
|---|---|---|---|---|
| | 90° | | 270° | ? |
| | 180° | | 180° | ? |
| | 270° | | 90° | ? |
| | 360° | | 0° | ? |

**2.** Look at this chart. The first chair has to be turned 90° clockwise *or* 270° counterclockwise to get into the final position. How many degrees must each be turned?

| Original position | Degrees clockwise | *OR* | Degrees counter-clockwise | Final position |
|---|---|---|---|---|
| | 90° | | 270° | |
| | ? 180 | | ? 180° | |
| | ? 270 | | ? 90 | |
| | ? 360 or 0 | | ? 0 or 360° | |

285

See activity 2, page 288a.

**goal** Finding the number of degrees of rotation

**things** clockface with movable hands
protractor

**page 285** Anyone who needs to make a tracing to rotate should be free to do so. The degree of rotation can be checked with a protractor.

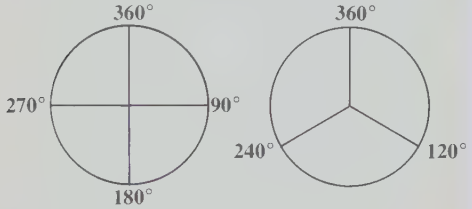
**lesson** Pages 286, 287

**goal** Exploring more rotations

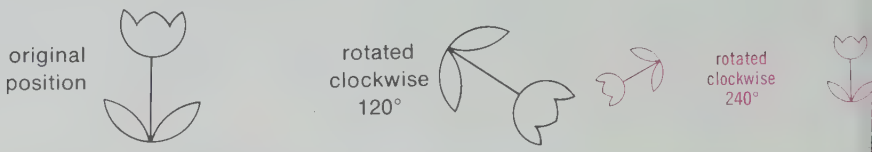
**memo** Pages 286 and 287 are optional. You may not want to use them with all your pupils, or you may choose to skip them entirely.

**things** paper for tracing
straightedge

**page 286** Determining the degree of rotation will be the greatest difficulty your pupils encounter. A protractor could be used. Two circles, marked as follows, make a far simpler aid.




**1.** The following illustration has been rotated clockwise 120°. Trace the figure and show how it would look after it has been rotated 240° and 360°.

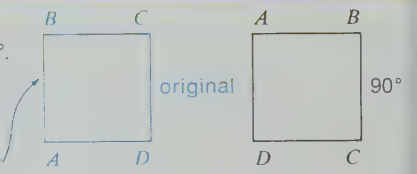



How many degrees must you rotate the figure until it is back in its original position? 360°

**2.** This illustration is being rotated in a clockwise direction.

When a figure is rotated 360°, is it always back in its original position? Are there any other rotations that return the snowflake to its original position? Name some of them. Yes
2 turns (720°), 3 turns (1080°), . . .
Yes

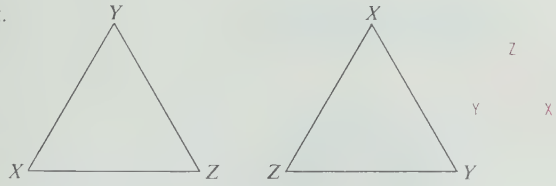
**3.** Draw a square. Use congruent squares to show its position after it has been rotated clockwise 90°, 180°, 270°, and 360°.




**a** Can you tell that each square shows a rotation?
Not until you label it

**b** Label your original square.
Then label the others to show the rotation. For example

1. The second triangle below is a tracing of the first. How far has the tracing been rotated?
120°

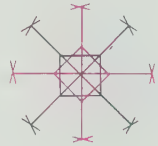



Rotate the tracing the same number of degrees again. Does it still look like the original? Yes, except that the letters have changed positions.

2. Will this figure look the same after it has been rotated 120°? Trace and rotate to make sure of your answer. Yes

3. Draw at least two other shapes that will not be changed by a 120° rotation.
Shapes will vary. Examples:

4. **a** Will this figure look the same after it has been rotated 120°? Trace and rotate to make sure. No



**b** Will it look the same after it has been rotated 90°? Trace and make sure. Yes

**c** Draw at least two other shapes that will not be changed by a 90° rotation.
Examples: Shapes will vary.

5. Will this figure look the same after it has been rotated 60°? Trace and rotate to make sure.
Yes

287

**goal** Examining figures that appear the same after a rotation

**things** paper for tracing
straightedge

**page 287** Making a tracing and rotating the tracing continue to be the method for proving an answer. Some youngsters will begin to see a connection between axes of symmetry, degree of rotation, and figures that appear the same after being rotated. Great! They are intuitively on their way to the concepts established in the next level.

**lesson** Page 288

**goal** **Checkout**—understanding of concepts related to size and shape, congruency, reflection, and rotation

**things** paper for tracing

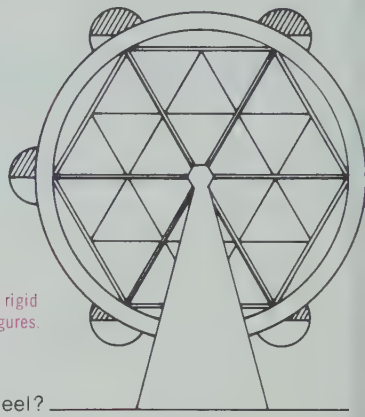
**page 288** The Checkout pages for all exploratory chapters in this program are different from checkout pages for other chapters. The questions are exploratory, and many of them are quite a challenge too. The emphasis is on **thinking**. Relax and have fun with this page. Question 2 should lead to a good discussion. You will probably be delighted with some of the children's original thinking.

Additional practice or tutoring is not appropriate at this time. Once again, one of the major goals of this chapter was to get the kids to observe the things that happen in their world that are related to geometry. Encourage the sharing of these ideas whenever possible.

# CHECKOUT

Skill: Telling whether geometric figures differ with rotation

1. A Ferris wheel is a beautiful thing to see and fun to ride on, too. If you got on a seat at the bottom and rode to the top, would you still be the same size and shape? Would you look the same size and shape to someone on the ground? Yes / No



   a The Ferris wheel is constructed with triangular shapes to give strength to the huge wheel. Why are triangles used? They are rigid figures. How many triangles can you count in the picture of this Ferris wheel? 38 (Accept other answers.)

   b How many different sizes and shapes are there among the triangles you counted? 3 sizes of 1 shape (triangle)

   Skill: Identifying congruent geometric shapes

   c Are the smaller triangles congruent? Are the larger ones congruent? Make tracings to check your answers. Yes / Yes

   d What figures do the triangles together form? Make sure you have the right names. How many sides does it have? Triangles (larger)—3 sides; parallelograms—4 sides; hexagons—6 sides

2. An amusement park or a carnival may be a good place to see congruent shapes. What congruent things can you think of that you can be sure to find there? Might you see any examples of reflections or rotations? Surely you can think of one thing, or two things, or . . . Answers will vary.

   Examples: Congruent things: ducks in shooting gallery, bars on birdcages, and so on
   Reflections: reflections in mirror mazes
   Rotations: seats on Ferris wheel, car tires, and so on

288

See activity 3, page 288a.

See activity 4, page 288b.

# RESOURCES

## another form of evaluation

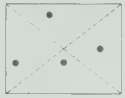
**for checkout**—page 288

1. Think about the largest building you have ever seen.
   a) Draw a sketch of the building.
   b) If you stood on the top floor of the building, would you still be the same size and shape? Would you look the same size and shape to someone on the ground? Yes No
   c) What geometric shapes were used in the construction of the building? Why do you think these shapes were used? Answers may vary.
   d) Were there different sizes of the same shape in the building? Answers may vary.
   e) Name some congruent shapes you saw in the building. Answers may vary.
2. A city street is a good place to see congruent shapes. What congruent things can you think of that you might see? Do you think you would see any examples of reflections or rotations? See if you can think of at least one thing.
   Answers may vary. Examples:
   Congruent: meters
   Reflections: reflections in pools
   Rotation: turnstyle

## activities

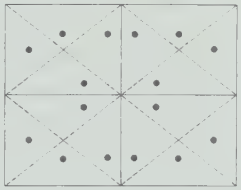
**1. things** plain paper; colored pencils or crayons; paper punch

Fold your paper in half. Fold it in half again so that the fold lies on top of itself. Leave your paper folded and fold diagonally. You want both diagonals to show. Punch a hole in each triangle so that the congruent triangles do not look alike. Your paper should look like this.



Before unfolding your paper, guess—
- how many holes there will be.
- how many triangles there will be.
- how many sets of triangles will be congruent, including the hole.

Unfold your paper. Shade each triangle in one-fourth of the sheet a different color. Now shade all the other congruent triangles with the same color. Remember—the hole is important too! How many axes of symmetry does your completed figure have? (2)



**2. things** for each pupil: circle protractor or index card

Each pupil will need his circle protractor from page 227 or the figure shown below reproduced on an index card. The pupil is to hold the card directly before him with the 180° mark pointed straight ahead. He begins walking forward. Give him instructions to direct him around obstacles, away from walls, and towards a preselected location. Instructions should be given as follows: Walk straight ahead. Stop. Turn 90° clockwise. Walk straight ahead. Stop. Turn 180° counterclockwise.

The gym, cafeteria, and playground are excellent places to do this activity. Group the youngsters with a leader who makes up the directions for his group.

**3. things** copy of figure shown; tracing paper

How good are you at finding different shapes?
1. How many different-size triangles can you find in the figure below? Trace each size just once.
2. Can you find two triangles with different shapes?
3. Hunt for different-size squares. Trace each size only once.
4. This time you're looking for different-size parallelograms. Trace each size once.
5. Can you find any **other** quadrilaterals? Trace each one.
6. Find a 6-sided polygon in the grid. Trace it.
7. Can you find two 6-sided polygons that are not congruent? Trace them.

**4. things** equilateral-triangle grid paper →

Here are some real challenges for you to try. Show your answers by shading in triangles.

- Make two parallelograms that do not have the same shape (one long and narrow, the other with all sides the same length).
- Can you make two hexagons that do not have the same shape?
- Can you make two triangles that do not have the same shape?
- Can you make a square, a rectangle, a pentagon, or an octagon?

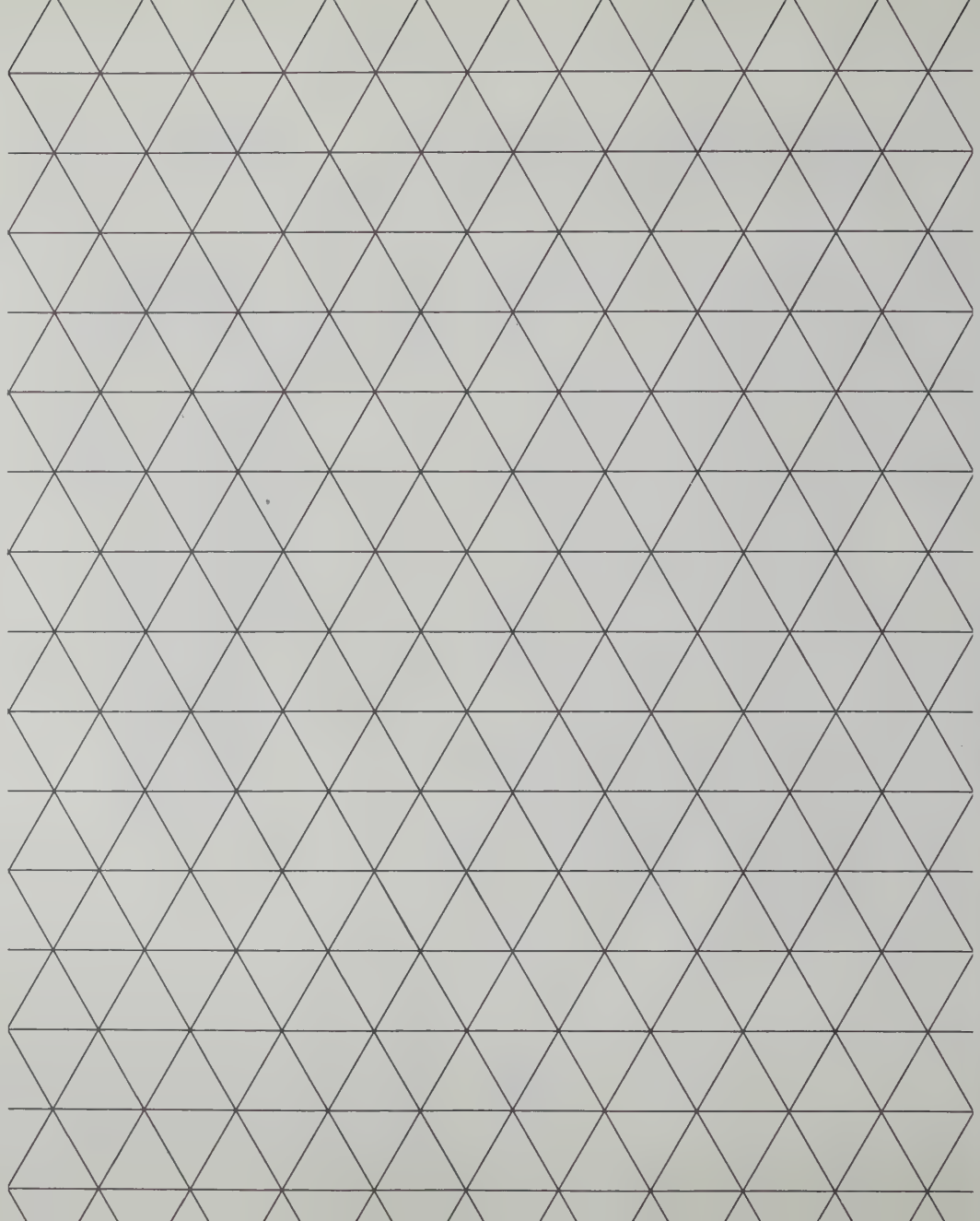
## additional learning aids

**geometry**—chapter objectives 1, 2, 3, 4

**SRA products**

*Mathematics Learning System*,
*Activity Masters*, *level B*, SRA (1974)
Spirit masters: G–4, 5, 6, 7
*Math Applications Kit*, SRA (1971)
Everyday Things card: 37
Occupations cards: 3, 36
Science card: 29
*Mathematics Involvement Program*, SRA (1971)
Cards: 65, 75, 47, 277, 297
*Skill through Patterns*, *level 6*, SRA (1974)
Spirit masters: 35, 36, 42, 43
*Visual Approach to Mathematics*, *Rational Numbers*, SRA (1967)
Visuals: 5–9

**other learning aids** (described on page 288d)—Geoboard Activity Cards (intermediate set), Geoboard Kit, Geoboards and Motion Geometry, Learn to Fold—Fold to Learn, Mira, Mira Math for Elementary School, Mirror Magic, Paper and Pencil Geometry



# 12 INTEGERS

**before this chapter the learner has –**

1. Shown addition and subtraction on a number line
2. Illustrated data on a bar graph
3. Compared and ordered whole numbers

**in chapter 12 the learner is –**

1. Showing positive and negative integers on a number line
2. Comparing and ordering a set of integers
3. Writing integers to describe gains and losses
4. Exploring the addition of positive and negative integers

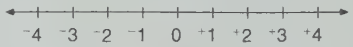
**in later levels the learner will –**

1. Identify the integers in a set of rational numbers
2. Order a set of integers
3. Name the opposite of an integer
4. Master finding the sum or difference of two integers

# Notes & Things

This exploratory chapter about integers will be easy for nearly all pupils. They already know about above- and below-zero temperatures, about being in debt, and about geographic locations that are above or below sea level. They have heard the countdown before the blast-off of a rocket, and a few of the sophisticated ones probably have seen negative numbers on the stock-market pages of the newspaper. The idea isn't new, but the notation may be new and even confusing. The symbols that have been used for addition and subtraction change size and location, and all of a sudden they become symbols that tell whether a number has a positive or negative value—whether it is to the right or left of zero on the number line. One way to minimize the possible notation confusion is to make very sure those symbols are always called by their correct names. The names used for these numbers can really get troublesome when the integers are used in a math sentence. $^{+}2 + {}^{-}1 = {}^{+}1$, for example, doesn't make an ounce of sense unless the correct words are used. "Positive two plus negative one equals positive one" can be figured out. "Plus two plus minus one equals plus one" is nonsense!

Knowing how to say things correctly is one problem and knowing what things not to say is still another. The integers are introduced on the number line. Both positive and negative numbers belong to the set of integers.



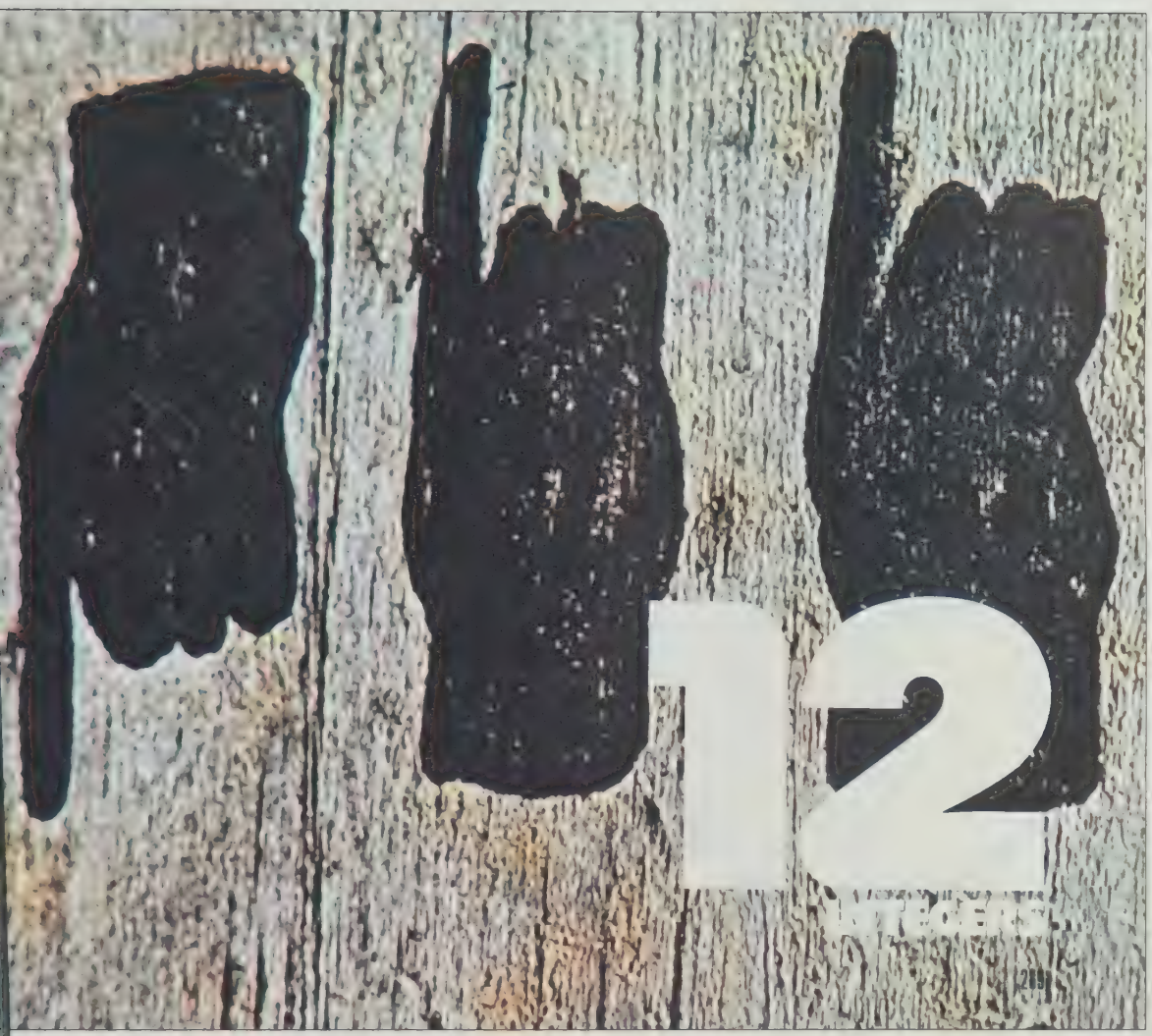

Zero is neither positive or negative, but it also belongs in the set of integers.

Some children think that $^{+}1, {}^{+}2, {}^{+}3, \ldots$ aren't the same whole numbers they have been using for years. Assure them that they are. The whole numbers are a subset of the integers.

The operation of addition is explored thoroughly. Please don't expect anyone to show mastery. There is plenty of time for that later. For now the job is simply to understand that at long last $2 + 2 = 4$ if we are working with whole numbers but that $^{-}2 + {}^{+}2 = 0$ if we are operating with the set of integers! The pupils should be delighted to find what a difference those little positive or negative symbols can make. It should also be a source of pleasure to see how integers are used in everyday situations. They should be feeling pretty smart at the end of the chapter. Goodness, those integers are exactly the same thing that big brother or sister is studying in high school.

## things

rulers
graph paper



**goal** Think about and explore ideas through a picture clue

**page 289** Finger symbols were used by the Greeks and Romans. We can find them printed in a 1532 German arithmetic book, and they are used today also. The members of the Board of Trade use complex finger-hand symbols to buy and sell wheat, corn, beans, and so on in the commodities market. (They shout a lot too but the noise doesn't count.) Today's hitchhiker uses a simple finger symbol.

The photograph reminds us that pictures of hands are also used to give simple directions. Use the up and down positions to investigate associated words and ideas.

increase prices—decrease prices
rising temperature—falling temperature
above freezing—below freezing
gaining weight—losing weight

If the notion of thinking about opposites is not enjoyable, please don't force the discussion. The chapter investigates many ideas that can be associated with positive and negative numbers. The more complete story situation may be far more profitable for your group.

**lesson** Page 290, 291, 292, 293

**goal** Examining zero as a whole number and in place value

**page 290** Zero is one of our greatest inventions. What would we do without it? Two functions of zero reviewed on the page are—

- Zero as the number that answers "How many?" when there are none
- Zero as a placeholder in our system of numeration

Without zero or a similar symbol, place value does not exist. The lack of this number was a limitation in such early systems of numeration as the Roman and Babylonian.

GOAL is to get acquainted with integers.
Integers isn't the name of the new family that moved in. It isn't a new disease. It's another set of numbers.

Every day we use numbers to count.

In the Lane family there are Mr. and Mrs. Lane, Lillian, Leonard, Louis, Lance, and Louise. How many children are there in the Lane family? How many girls? How many boys? How many people are there in the Lane family? 7

2 (3 if you count Mrs. Lane)

5

3 (4 if you count Mr. Lane)

People invented counting numbers long ago to answer the question "How many are there?"

Centuries later, people invented a new number. It was a number that indicated that there are *no* objects in a set. What was this number? 0

The Jones family lives next door to the Lanes. The Jones children are named Jane and Janice. How many boys in the family? 0

Tim had a bag with 5 marbles in it. One day they all fell out through a hole in the bag. Tim lost all 5 marbles. How many did he have then? Which number answers this question? 0

None

Zero is especially useful in a numeration system that uses place value. Without it we couldn't tell the difference between 1 and 100, or 101 and 11.

290

A great deal has been written concerning zero. Use zero as a research topic for your inquisitive students.

One reference is Marnie Luce, *Zero Is Something* (Minneapolis: Lerner, 1969).

Sometimes there is a need for numbers less than 0. One of these times is when cold weather is being discussed.

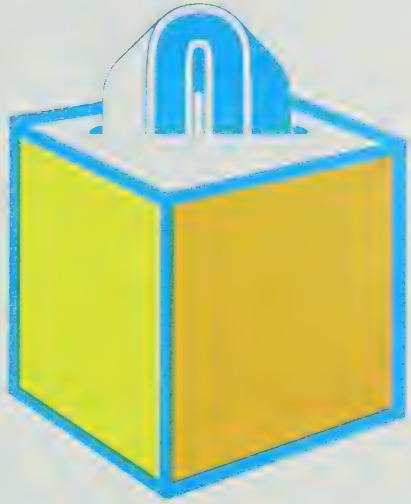
On Friday evening the local TV weatherman gave the weekend weather forecast:

> Today's high was 5 degrees, and tonight's low will be 2 degrees.
> Saturday the high will be 1 degree and the low will be 7 degrees below zero.
> On Sunday the high will be 2 degrees below zero and the low will be 10 degrees below zero.

He wrote the temperatures on a chart. He used a small raised circle as a symbol for the word *degrees*. He used another symbol too. Instead of writing "7° below zero," he wrote "$^{-}7°$."

| | Friday | Saturday | Sunday |
|---|---|---|---|
| High | 5° | 1° | $^{-}2°$ |
| Low | 2° | $^{-}7°$ | $^{-}10°$ |

1. Does the temperature 5° mean 5 degrees above zero or 5 degrees below zero?
2. Is 5° warmer than 2°? Yes
3. Is $^{-}7°$ warmer than $^{-}10°$? Yes
4. Which is the lower temperature?
   **a** 2° or $^{-}2°$ **b** 1° or $^{-}7°$ **c** $^{-}10°$ or $^{-}2°$



291

**goal** Exploring the need for numbers less than zero

**memo** Zero as an origin point for numbering serves to separate positive numbers from negative numbers. This idea will be developed over the next few pages.

**page 291** As civilization advanced, people found that the counting numbers were not enough. They needed a system that contained numbers less than zero. This idea is introduced to the pupil by examining the need for numbers less than zero when temperature is recorded.

Watch out for problems 3 and 4. The youngsters are accustomed to thinking with whole numbers, where the smallest number is the one nearest zero. The opposite is true with negative numbers. None of the temperatures given are very warm, but **think**—Is it coldest at 7° below zero, or at 10° below zero, or at 12° below zero? Why not have someone look up the coldest temperature ever recorded on earth, and the hottest too. It is quite a range!

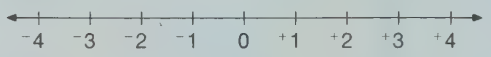
**goal** Introduction to positive and negative numbers

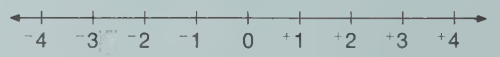
**page 292** The number-line model is really important. What number separates positive numbers from negative numbers? Be consistent in naming $^{-}1$ as negative one, **not** minus one. The latter can cause confusion when subtraction of integers is introduced and the pupil is confronted with "minus minus one," rather than "minus negative one." The same thing holds true for positive numbers. The symbol does look like a plus sign, but it's not in the same position. The positive and negative symbols indicate the position of a number on the number line—nothing more and nothing less.

**1.** The next weekend was even colder. Friday's high was 0°, and the low was 3° below zero. Saturday's high was 1° below zero, and the low was 6° below zero. Sunday's high was 5° below zero, and the low was 12° below zero. Make a chart to show these temperatures.

| | Fri. | Sat. | Sun. |
|---|---|---|---|
| High | 0° | $^{-}1°$ | $^{-}5°$ |
| Low | $^{-}3°$ | $^{-}6°$ | $^{-}12°$ |

Think of the thermometer as a number line.




One less than zero is a negative number.
It is written $^{-}1$ and read as "negative one."
It is to the left of zero on the number line.
One more than zero is a positive number.
It can be written $^{+}1$ and read as "positive one."

Two less than zero is read "negative two" and is written $^{-}2$. Two more than zero is read "positive two" and can be written $^{+}2$.

The negative symbol is *never* left off any number to the left of zero, but sometimes the positive symbol is not written with the numbers to the right of zero.

All numbers to the left of zero are negative numbers.
All numbers to the right of zero are positive numbers.

**2.** Tell whether each of these numbers is to the left or to the right of zero.

**a** $^{-}5$ Left **b** $^{+}3$ Right **c** $^{-}3$ Left

**3.** THINK Then describe each of these situations with a positive or negative number.

**a** You lost 35¢ $^{-}35¢$ **b** You found 50¢ $^{+}50¢$

292

A coral reef lies near a small island. Several ships hit the reef. So divers were sent down to measure the depth. Here is the chart they drew.

1. What is the most dangerous point on the reef for a boat to cross? C
2. How far below sea level is the reef at that point? 2 m
3. What number is used to show sea level? 0
4. The negative sign can be used to show metres below sea level. Using that sign, answer these questions. What is—
   - **a** the lowest charted point on the reef? A, -8 m
   - **b** the highest charted point on the reef? C, -2 m
5. Copy and complete this table, giving the depth or height of each point.

| Point | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Level | ? | ? | ? | ? | ? | ? | ? | ? | ? | ? | ? |

-8 m -4 m -2 m -3.5 m -2.5 m -7 m 0 m +4 m +8 m +6 m 0 m

293

**goal** Examining an application of positive and negative numbers

**page 293** Positive and negative numbers are shown on a vertical number line The illustration should help pupils see why the negative numbers are frequently thought of as **below** zero and the positive numbers as **above**. They can also see that the lowest charted point on the reef is the farthest from sea level. There can be no doubt that -8 m is farther from sea level than -5 m, for example.

The thickness of the lines and the size of the dots on the chart make very exact measurement difficult. Therefore a range of a few points either more or less should be accepted for the measurements. This variation can lead to discussion of the difficulty or impossibility of making completely exact measurements with the tools and equipment so far devised. What is complete precision in measurement?

**lesson** Pages 294, 295

**goal** Showing positive and negative numbers on a number line

**memo** There is no absolute rule for placing numbers in a given direction on a number line. It is merely more convenient if convention is followed – less confusion results. Conventional vertical and horizontal directions are used in this program.

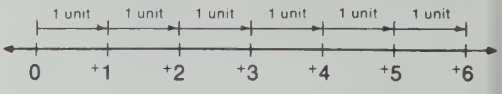
**things** rulers

**page 294** Preparing a number line will be a review for most of your pupils. As they make their number lines, emphasize the following:

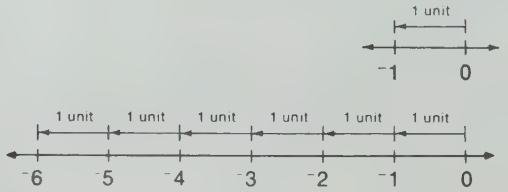
- Locate a beginning point first. This point is labeled "0."
- All units must be the same length. Any length may be chosen as the unit.
- The line, therefore the units and the numbers, could go on forever.
- On a horizontal line, larger positive numbers go to the right; smaller numbers go to the left. The system is reversed for negative numbers. Larger negative numbers go to the left; smaller numbers go to the right.

Here's an extra question. Do you rotate the number line 90 degrees clockwise or counterclockwise to use it to show sea level or temperature?

You know how to make a number line.
You start from a point marked zero.
You choose a unit of length, measure 1 unit to the right of zero, mark the point, and label it $^{+}1$.
You keep the same unit length and label the points $^{+}2$, $^{+}3$, $^{+}4$, and so on.




1. Where does the point for $^{+}5$ go? 1 unit to the right of 4
2. How much farther can the line be extended to the right? It goes on forever.
3. How far would the line extend if the number $^{+}43$ were marked on it? 43 units to the right of 0
4. In which direction from 0 are the whole numbers marked on the line? Right
5. Is $^{+}2 < {}^{+}3$? Yes Is 2 placed on the left or the right of $^{+}3$ on the number line?
6. Is $0 < {}^{+}1$? Yes Does 0 lie to the left or the right of $^{+}1$?
7. If one whole number is smaller than another whole number, is the smaller one placed to the left or the right of the larger one on the number line?

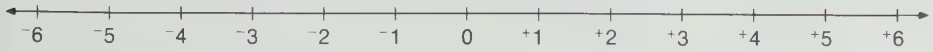



To show negative numbers, you use the same unit length you used for whole numbers. Measure one unit length to the *left* of 0, mark the point, and label it $^{-}1$.

Continue moving to the left. Mark and label the points for $^{-}2$, $^{-}3$, $^{-}4$, and so on.

294

Here is a number line that shows both positive and negative numbers.




The whole numbers, used for counting, are $^{+}1$, $^{+}2$, $^{+}3$, $^{+}4$, and so on.
These numbers are called *positive integers*.
They are always shown to the right of 0 on the number line.

The negative numbers, shown to the left of 0 on the number line, are called *negative integers*.

Zero is also an integer, but it is neither positive nor negative.

The positive integers, the negative integers, and zero all taken together are called the *integers*.

1. Can the number line for the integers be extended to the left? How far? Yes; It goes on forever

2. Is the negative integer $^{-}21$ to the right or the left of zero? Left

3. Is $^{-}7 < {}^{-}5$? Is $^{-}7$ placed to the left or the right of $^{-}5$ on the number line? Yes

4. How far from 0 is $^{-}5$? In which direction? 5 units; Left
   - **a** How far from 0 is $^{+}5$? In which direction? 5 units; Right
   - **b** How far is it between $^{-}5$ and $^{+}5$? 10 units

5. How far from 0 to $^{-}4$? In which direction? 4 units; Left
   - **a** How far from 0 to $^{+}1$? In which direction? 1 unit; Right
   - **b** How far from $^{-}4$ to $^{+}1$? 5 units
   - **c** How far from $^{-}4$ to $^{+}4$? 8 units

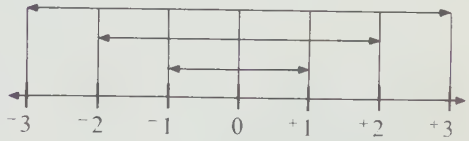


**goal** Examining zero as an integer

**page 295** The technical name for the positive and negative numbers and zero is introduced—the INTEGERS.

Focus on the importance of zero. Here the function of zero is to separate the positive integers from the negative integers. Zero **is** something! It is an integer. But it is neither a positive nor a negative number.

You may want to extend the learning on this page by taking a look at opposites. Numbers that are the same distance from 0 but in opposite directions are called **opposites**. Zero is its own opposite, believe it or not.




Ask questions such as these:
What's the opposite of $^{+}2$? the opposite of $^{-}1$? the opposite of $^{-}2$? Then go on. What's the opposite of a loss of 35¢? the opposite of a profit of 50¢?
You can all have a good time with this idea.

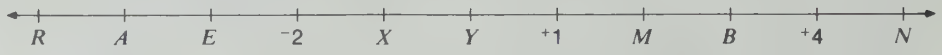
**lesson** Pages 296, 297

**goal** Examining applications of integers

**page 296** Zeroing in on zero will help in naming the points in problem 1 with a number.

The youngsters may intuitively want to determine how many days it would take Jill (problem 2) to make up the money she lost, or how much money Ellen (problem 3) will have when her brother gives her what he owes. That's fine. But no math sentences nor actual computation should be attempted just yet.

Some of the points on this number line are labelled with integers.
Others are labelled with letters.




**1**

**a** Name the integer represented by each of the following letters.

| A | E | N | X | B | M | R | Y |
|---|---|---|---|---|---|---|---|
| −4 | −3 | −5 | 1 | +3 | +2 | −5 | 0 |

**b** Which of these are positive integers? M, B, N

**c** Which are negative integers? R, A, E, X

**d** Are any of these integers neither positive nor negative? Which? Yes Y (zero)

**2**

Negative numbers can be used to show the loss of money.
For example, Jill lost 50¢ of her paper-route money.
This can be written as −50. Positive numbers show a profit.
If Jill makes 25¢ in one day, it can be written simply as +25.

**3**

Bill hasn't any money. In fact he owes his sister $3.00.
This debt can be shown by the integer −3. Ellen doesn't owe any money and she has $2.00. Her money can be shown by the integer +2.
What number shows a debt of $5.00? a sum of $8.00?

−5 +8 (or just 8)

Show these profits and losses as positive or negative numbers:

**a** a loss of 10¢ 10¢ **b** a profit of 10¢ +10¢ **c** a loss of 40¢ −40¢

**d** a profit of 40¢ +40¢ **e** a loss of 99¢ −99¢ **f** a loss of 75¢ −75¢

296

On the number line, the lesser of two numbers is always to the left of the greater.
For example, $^+3 < {}^+4$, so $^+3$ is to the left of $^+4$.
The greater of two numbers is always to the right.
For example, $^+6 < {}^+7$, so $^+7$ is to the right of $^+6$.

1. Use the number line as a guide. Write $>$ or $<$ to make each of these statements true.

**a** $^+1 \,?\, {}^+2$ **b** $^-1 \,?\, {}^+1$ **c** $0 \,?\, {}^-1$ **d** $^-1 \,?\, {}^+2$ **e** $^-1 \,?\, {}^-2$ **f** $^+2 \,?\, {}^-2$

2. Study these number sentences. Which are true? Which are false?

**a** $^-1 < {}^+3$ True **b** $^-1 < {}^+1$ True **c** $^-1 < 0$ True **d** $^-2 < {}^+3$ True **e** $^-2 < {}^+1$ True
**f** $^-2 < 0$ True **g** $^-3 < {}^+3$ True **h** $^-3 < {}^+1$ True **i** $^-3 < 0$ True

## PROGRESS CHECK

Skill: Comparing integers

True or false?

(1.) Every negative integer is less than $^+1$. True
(2.) No negative integer is less than 0. False
(3.) $^-1$ is the greatest negative integer. True
(4.) $^+10{,}000{,}000$ is the greatest positive integer. False
(5.) Every negative integer is less than any positive integer. True

Arrange each set of integers in order from least to greatest. Skill: Ordering integers

(6.) $0, {}^-3, {}^+2$ $-3, 0, {}^+2$
(7.) $^-1, {}^+2, {}^+1$ $-1, {}^+1, {}^+2$
(8.) $^+7, {}^-4, {}^-1, {}^+3$ $-4, -1, {}^+3, {}^+7$
(9.) $^-2, {}^-4, {}^+2, 0$ $-4, -2, 0, {}^+2$

297

See activity 1, page 304a.

See activity 2, page 304a.

**goal** Practice in comparing two integers: **Progress Check**—comparing and ordering integers

**page 297** Try expanding the generalization at the top of the page by asking whether the statement is true when comparing two negative integers. Then compare several examples.

Watch for pupils who completely disregard the negative sign and those who do not understand that **every** negative integer is less than zero, and therefore less than every positive integer. Examine problem 2 (at the top of the page) for patterns to cure the latter problem.
After completing the Progress Check, challenge your pupils to change every false statement in the first five problems to a true statement. Oral statements are sufficient.

Many children have a much better grasp of numerical quantity when they think in terms of money. Use this technique with pupils who have trouble with problems 6 through 9.

Bill hasn't any money. In fact, he owes his brother \$3. Represent how much money Bill has with an integer.
Jim hasn't any money, but he doesn't owe anyone. Represent the amount of money Jim has with an integer. Jake doesn't owe any money. In fact, he has \$2. Represent how much Jake has with an integer.

Now who has the least amount of money? the greatest amount? Continue in this manner until these pupils are comfortable with the order of integers.

**lesson** Page 298

**goal** Applying integers to graphing

**things** graph paper
rulers

**page 298** Examine the vertical scale on the graph. Only even numbers are shown. Where are the odd numbers located? Every graph needs a title—someone must have an idea.

Have your supersleuths check the newspapers to find how local temperatures are reported. Are any other temperatures given? Unfortunately, the high temperature for a given locale is not always reported uniformly. Don't discuss this before the youngsters start gathering information for their graphs. Compare the completed graphs. Should there be any discrepancies, ask what sources were used for gathering the information. Decide which source is probably the most reliable.

Examine pupil-made graphs for completeness.

- Is there a title?
- Are the vertical and horizontal scales labeled?
- Are the points indicating the high temperature for each day connected with straight lines?

Extend this supersleuth activity by examining the data shown in the graph. Find the highest temperature, the lowest temperature, the range of temperatures, and the average temperature. Does the graph show the temperature for a given time of any day? Did the temperature mostly rise or fall during that week?



In Nome, Alaska, the high temperatures for one week were as follows:

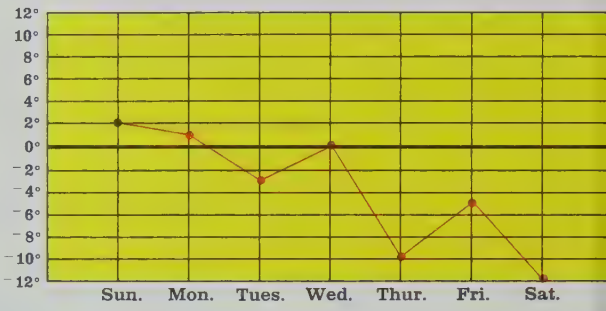
Sunday 2° Monday 1° Tuesday ⁻3° Wednesday 0°

Thursday ⁻10° Friday ⁻5° Saturday ⁻12°

1. Use this information to make a graph like this one.

   On the line for each day, mark a point to represent the temperature. Sunday is shown as an example. When all the temperatures are marked, connect the points with straight lines.

   Have you ever seen a graph like this on TV or in books? Does it make it easy to see what happened to the temperature during the week? Probably Discuss.




2. Did the temperature in Nome mostly rise or fall during the week?

3. What day had the highest temperature? the lowest? Saturday
   Sunday

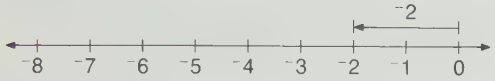
During the coming week, find the daily high temperatures for your community. Then make a graph of the temperatures like this one. Make sure you know where to find the information.
Information can be gathered from daily weather reports in newspapers or on radio and TV.

You already know how to show addition of positive integers on the number line. Now let's use the number line to show the addition of negative integers.

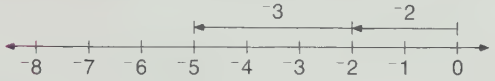
A sunken raft is 2 feet below sea level. A sunken chest is 3 feet deeper. How deep is the chest? $^{-}2 + {}^{-}3 = \ ?$

To show this addition on the number line, follow these steps.

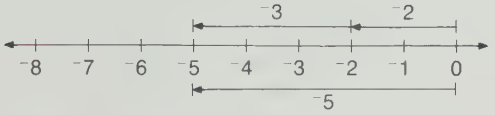
**Step 1:** Draw an arrow from zero for the first addend, $^{-}2$. Since the first addend is a negative integer, draw the arrow 2 units to the left from zero. Remember, negative integers are always to the left of zero.




**Step 2:** Draw a second arrow to show the second addend, $^{-}3$. It starts where the first arrow ends.




**Step 3:** Draw an arrow below the number line to show the sum. The starting point of this arrow is zero. It ends in line with the end of the second arrow.
This shows that $^{-}2 + {}^{-}3 = {}^{-}5$.






**lesson** Pages 299, 300, 301, 302

**goal** Exploring addition of two negative integers

**memo** Addition involving negative integers is explored on pages 299 through 302. Pupils are not expected to master this skill—merely to investigate.

**things** rulers

**page 299** The necessary steps are clearly stated on the page. Number-line diagrams are a must. After discussing the page, let the pupils make up a math sentence involving the addition of two negative integers. Then have them make a number-line diagram for their sentence. Emphasize that the first addend must start from zero and the arrow that represents a negative integer must point to the left. The arrow for the second addend must start from the endpoint of the first addend. The arrow below the number line that shows the sum must also start from zero and extend to the endpoint of the second arrow.

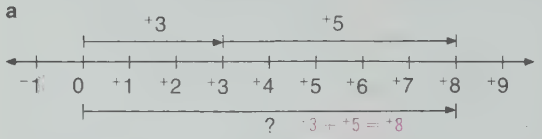
**goal** Applications—addition of two positive or two negative integers

**things** rulers

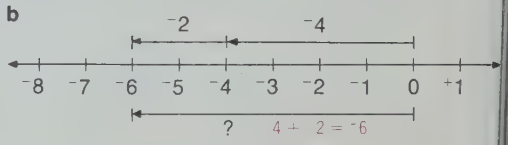
**page 300** The first problem checks the pupil's ability to read the number line. This reading skill is prerequisite to making number lines for problem 2. The abilities of your pupils will determine whether or not problems 3 and 4 should be completed independently.

**1.** Using the number-line diagrams, write and complete an equation for each of the following.

**a**




**b**



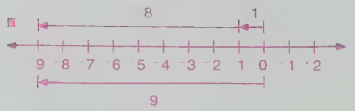

**2.** Draw a number-line diagram for each of the following. Then complete the sentence. See art below.

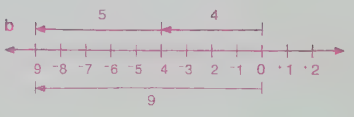
**a** $-1 + -8 = \underline{?}$ $-9$ **b** $-4 + -5 = \underline{?}$ $-9$ **c** $+6 + +3 = \underline{?}$ $+9$ **d** $-3 + -9 = \underline{?}$

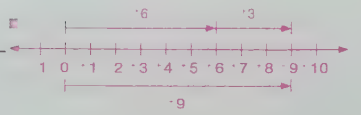
**3.** Clancy owes $2 to his brother and $4 to his father.
Write negative integers to show those debts.
Add them to find his total debt. Use a number line if you wish.
$-2 + -4 = -6$

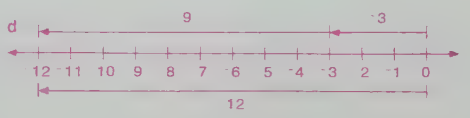
**4.** **a** Digger is official treasure burier for Blackbeard the Pirate.
He dug a huge hole 3 feet deep on Wednesday.
He recorded it in his book as $-3$.
On Thursday he dug down 4 more feet.
What numeral did he write to show the distance he dug that day? $-4$
How deep was the hole then? $-7$
Write an addition equation to show this. $-3 + -4 = -7$

**b** On Friday Digger finished the hole by digging down another 2 feet.
What did he record in his book to show the distance he dug? $-2$
What integer did he write to show the total depth of the hole? $-9$
Write an addition equation to show this. $-7 + -2 = -9$



**goal** Exploring addition of a positive and a negative integer

**things** rulers

**page 301** Two types of sentences are possible when adding one positive and one negative integer. The sentences are examined simultaneously to point out the contrast in the positive/negative values on the number-line models. Emphasize the direction for positive and for negative integer arrows. Focus on the arrow showing the sum. Work for understanding rather than mastery.

Try at least one more pair of sentences. If you have some sharpies, reverse the order of the positive/negative values from those used in the text.

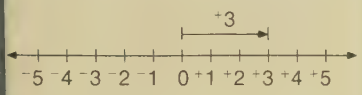
Try $^{-}2 + {}^{+}4 =$ ? $\quad {}^{+}2 + {}^{-}4 =$ ?

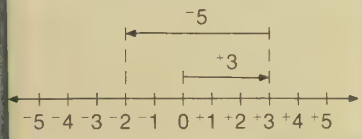
And just for fun do this one too.

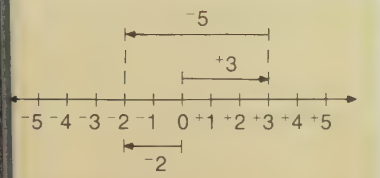
$^{-}3 + {}^{+}3 =$ ? $\quad {}^{+}3 + {}^{-}3 =$ ?

1

The temperature is $^{+}3°$.
It gets 5° colder.
Then what is the temperature?
$^{+}3 + {}^{-}5 =$ ?




$^{+}3 + {}^{-}5 = {}^{-}2$

Suppose one of the addends is positive and the other is negative. Here are two problems. The problems are different, but you use the same **Think** process on both.

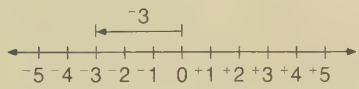
The first arrow always starts with zero. If it is positive, it goes to the right. If it is negative, it goes to the left.

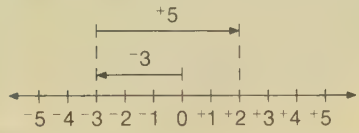
The second arrow always starts where the first arrow ended. Once again—positive goes right; negative goes left.

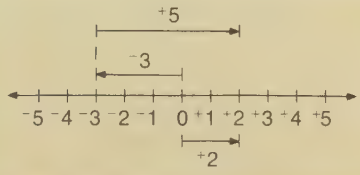
The sum is shown with an arrow below the number line. The arrow always starts at zero. It ends at the same point as the second arrow.

2

The temperature is $^{-}3°$.
It gets 5° warmer.
Then what is the temperature?
$^{-}3 + {}^{+}5 =$ ?




$^{-}3 + {}^{+}5 = {}^{+}2$

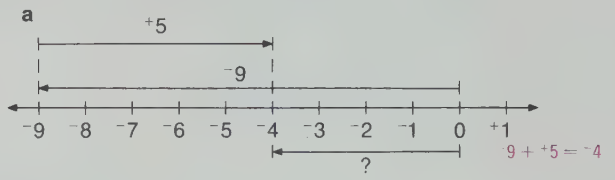
301

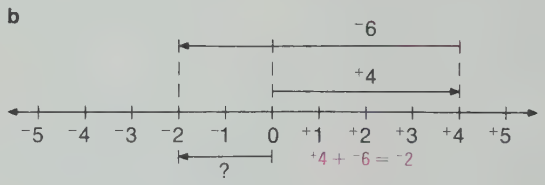
**goal** Applications—addition of a positive and a negative integer

**things** rulers

**page 302** Use problem 1 to check the pupil's ability to read the diagrams. Pupils who have trouble here should not attempt problem 2 independently. Use your discretion in assigning problems 3 and 4.

**1.** Write an equation for each of these number-line diagrams.




**2.** Draw a number-line diagram for each of the following. Then complete the sentence.

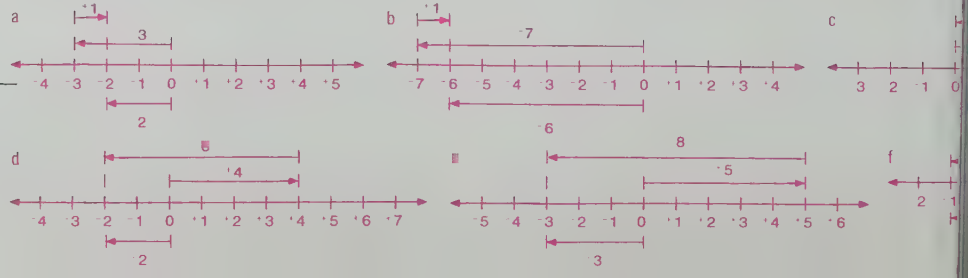
**a** $^{-}3 + {}^{+}1 = \underline{?}\ {}^{-}2$ **b** $^{-}7 + {}^{+}1 = \underline{?}\ {}^{-}6$ **c** $^{+}3 + {}^{-}3 = \underline{?}\ 0$

**d** $^{+}3 + {}^{-}6 = \underline{?}\ {}^{-}3$ **e** $^{+}5 + {}^{-}8 = \underline{?}\ {}^{-}3$ **f** $^{-}1 + 0 = \underline{?}\ {}^{-}1$

**3.** Bob owed a friend $12, so he put a slip of paper in his bank that said he was in debt $12. He paid back $8. What should the slip of paper say now? Write an equation to show your answer. $^{-}4$ $^{-}12 + {}^{+}8 = {}^{-}4$

***4.** A computer controls the inventory at Wilson's Warehouse. The records said there were 8 typewriters on hand. The next day 15 were shipped out. Now the records said there were $^{-}7$ on hand. But there were still 3 more on the shelf. Something was wrong. What number of typewriters should have been on the records to start with? 18





egative integer stand for a drop in price
a positive integer stand for a rise in price.
ce of meat drops 4¢/kg one week. It rises
he next week. Is the final price above or
he initial price? How much? +5¢

is on a diet to reduce. He loses 5 kg in
He gains back 3 kg in April. How much
s mass change in the two months? Is this
positive or negative? Loses 2 kg

nanufacturing plants label parts that are too
." Parts that are too small are labelled "−."
t is +2 cm, it is 2 cm too long.
t is −3 cm, it is 3 cm too short.
t is supposed to be 9 cm long but a
says it is −2, how long is the part? 7 cm

es's bank account
00 in it. He wrote a cheque
.00. What balance should
e in his account? −$4.00

ppose he puts $5.00 into
ount. What balance
he have in his
t this time? +1.00 (or $1.00)

| | | | | |
|---|---|---|---|---|
| #56 | TO Super Mart | DATE | AMOUNT | 7.00 |
| | FOR Groceries | 3-26 | BALANCE | 6.00 |
| #57 | TO Speedy Service | DATE | AMOUNT | 10.00 |
| | FOR Cash | 3-28 | BALANCE | ? |



**lesson** Page 303

**goal** Applications involving integers

**page 303** Use your discretion in assigning these situation problems. They may be handled best as a group project. Encourage number-line diagrams or illustrations for youngsters who need this help. Some may be able to **think** the solutions but unable to write a math sentence to fit the situation. Math sentences are not necessary.

Help your pupils interpret the cheque record in problem 4. Mr. Jones had what is known as an overdrawn account. What does that mean? After he made the $5 deposit, was the account still overdrawn? Without the deposit, what would have happened to the cheque?

**lesson** Page 304

**goal** **Checkout**—ordering and applying integers to problem situations

**memo** Use only questions 1 and 2 as independent work.

**page 304** A crude sketch of the building in problem 1 indicating the floors above and below ground level, may prove helpful in designing the elevator control panel. Watch for pupils who include a zero floor. This just isn't done in a building.

Stress **thinking** the answer for problem 2. Some youngsters may not be able to write a math sentence for this problem. That's O.K.—but everyone should be able to reason the answer.

Complete the last problem as a group. The computations are not difficult when the chart and how it functions are understood. You may want to simulate a test paper on the chalkboard to show Bill's scores.

Additional practice with integers is not appropriate at this time. But please take advantage of every opportunity you can find to use integers. Test scores, temperature, and newspaper stories usually present some raw material for such discussions.

# CHECKOUT

304

Skill: Ordering integers

1. Some new buildings are being built with many floors below ground level. Pretend there is a new building with 5 floors below ground level and 8 floors above ground level. Design an elevator control panel that will have a button that clearly shows each of the floors. Think carefully. There are some problems you'll have to solve. See below.*

Skill: Using an integer to show money needed

2. You owe your dad $6.00. He says you cannot buy the blouse you want, which costs $4.00, until you pay him back. How much money do you have to have in order to buy the blouse? $10.00

Skill: Writing integers to describe increase/decrease situations

3. The teacher wanted to discourage students from guessing the answers to a true-false test. The test was marked by giving 1 point for each right answer. Then 1 point was deducted for each question left blank and 2 points were deducted for each wrong answer. What were the final scores for each person on this true-false test?

| | Student | Number left blank | Number wrong | Number right |
|---|---|---|---|---|
| a | Bill | 0 | 2 | 8 |
| b | Jim | 5 | 0 | 5 |
| c | Jan | 2 | 2 | 6 |
| d | Jay | 2 | 3 | 5 |
| e | Helen | 4 | 0 | 6 |
| f | Tim | 0 | 5 | 5 |
| g | Sam | 3 | 3 | 4 |
| h | Lil | 6 | 0 | 4 |

*Designs may vary, but most include numbers for 5 floors below ground le and 8 floors above ground level. (No zero, but *G* for ground or *L* for lobby is O.K.)

See activity 3, page 304a.

See activity 4, page 304a.

# RESOURCES

## another form of evaluation

**for progress check** – page 297

True or false?

1. Every positive integer is greater than $^-1$. True
2. No negative integer is greater than 0. True
3. $^+1$ is the smallest positive integer. True
4. $^-100\,000\,000$ is the smallest negative integer. False
5. Every positive integer is greater than any negative integer. True

Arrange each set of integers in order from least to greatest.

6. $^+2, ^-1, 0$ $^-1, 0, ^+2$
7. $^+3, ^-2, ^-3$ $^-3, ^-2, ^+3$
8. $^+5, ^-3, ^+2, ^-1$ $^-3, ^-1, ^+2, ^+5$
9. $^-1, ^-6, ^+1, 0$ $^-6, ^-1, 0, ^+1$

**for checkout** – page 304

1. During one week in October, temperatures in Eis City, Kaltland, were as follows:
   Sunday $^-4°$, Monday $^-2°$, Tuesday 1°, Wednesday $^-3°$, Thursday 0°, Friday 4°, and Saturday 2°.
   a) Write these temperatures in order from lowest to highest $^-4°, ^-3°, ^-2°, 0°, 1°, 2°, 4°$
   b) What day was the coldest? the warmest? Sunday Friday
2. Doris is planning a budget for next month. She has saved \$9 from her earnings, but she owes \$5 for repairs to her bicycle. How much money will she have left to spend on other things next month? \$4
3. The chart that follows is a record of price difference for the same quantity of some items of food during one month.
   Use a negative integer for a drop in price, and a positive integer for a rise in price.
   For each item, record the total price difference during the month.

| Item | Drop in price | Rise in price | Total price difference |
|---|---|---|---|
| A. | 6 | 9 | $^+3$ |
| B. | 9 | 4 | $^-5$ |
| C. | 8 | 15 | $^+7$ |
| D. | 3 | 9 | $^+6$ |
| E. | 25 | 4 | $^-21$ |
| F. | 4 | 19 | $^+15$ |
| G. | 11 | 8 | $^-3$ |
| H. | 2 | 19 | $^+17$ |

## activities

**1. things** small cards

Prepare a card for each integer from $^-25$ through $^+25$. The same number of cards is dealt facedown to each player. Each player makes a stack of his cards – keeping his cards facedown. Play begins as each player turns over the top card on his stack. The player turning over the largest number takes all the faceup cards. Play continues until all of the cards have been played. The player taking the most cards wins the game.

Variations:

1. The player turning over the least number takes all the faceup cards.
2. Make two cards for each integer from $^-10$ through $^+10$. Players must predetermine a rule for two players turning over the same card.
3. Use a regular deck of playing cards. Remove the face cards. Aces count as ones. Let the black suits be positive integers and the red suits be negative integers.

**2. things** small cards

Prepare a card for each integer from $^-25$ through $^+25$. The cards are shuffled and each player (up to four) is dealt 4 cards facedown. The remaining cards are placed facedown in a stack. The top card is turned over to begin the discard stack. In turn, each player draws a card from either stack and discards a card. The game continues until a player is able to lay down cards showing four consecutive integers.

For example:

| $^-1$ | 0 | $^+1$ | $^+2$ |
|---|---|---|---|

Variation: Increase the number of cards for the negative integers and play with the negative integers only.

**3.** Group pupils in two teams. An able peer serves as the challenger. He thinks of an integer and records it on a slip of paper. Teams alternate, asking the challenger a question that can be answered by yes or no. The team that guesses the integer wins. Possible questions include "Is the integer less than zero?" and "Is the integer greater than 1 and less than 5?"

**4. things** 4-by-4 array

Pair pupils. Each pair will need to make a 4-by-4 array of squares. To begin play, one player writes the name of an integer in any square. The other player writes another integer in another square. Play alternates until one player succeeds in placing the name of an integer in the last empty square in a row, column, or diagonal so that the sum of the four integers in that row, column, or diagonal is zero.

A 3-by-3 array may be used to simplify the game. A 5-by-5 array will make the game more challenging.

## additional learning aids

**concepts**—chapter objectives 1, 2, 3, 4

**SRA products**

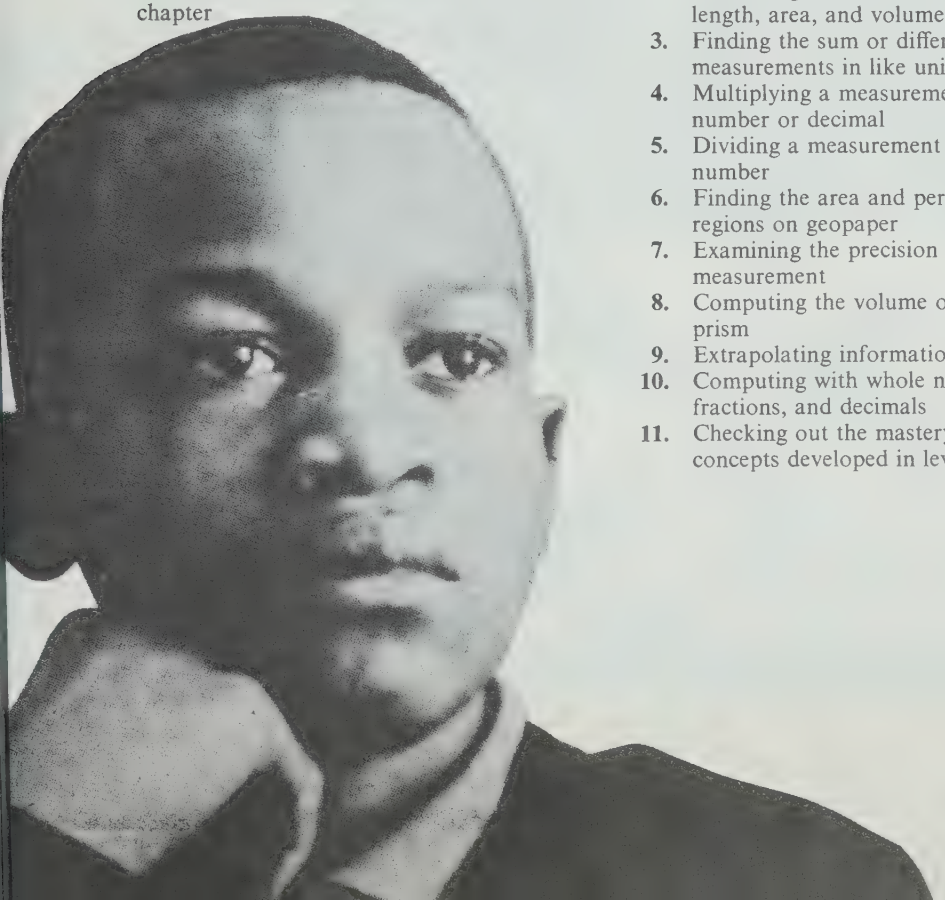
*Mathematics Learning System,*
*Activity Masters, level B*, SRA (1974)
Spirit masters: W–37
*Diagnosis: an instructional aid—*
*Mathematics Level B*, SRA (1972)
Probe: M–17
*Skill through Patterns, level 6*, SRA (1974)
Spirit master: 59
*Visual Approach to Mathematics, Rational Numbers*, SRA (1967)
Visual: 17

**other learning aids** (described on page 336i)—
Chip Trading Kit

# 13 MEASUREMENT COMPUTATION

**before this chapter the learner has –**

Developed the concepts and practiced the skills reviewed and checked out in this chapter



**in chapter 13 the learner is –**

1. Reviewing metric units of measure
2. Examining units of measure used for length, area, and volume
3. Finding the sum or difference for two measurements in like units
4. Multiplying a measurement by a whole number or decimal
5. Dividing a measurement by a whole number
6. Finding the area and perimeter of regions on geopaper
7. Examining the precision of a measurement
8. Computing the volume of a rectangular prism
9. Extrapolating information from a chart
10. Computing with whole numbers, fractions, and decimals
11. Checking out the mastery skills and concepts developed in level 6

**in the next level the learner will –**

1. Maintain the skills mastered
2. Apply and extend the concepts and skills mastered to new learning

# Notes & Things

Reviews do get tiresome for everyone unless it's not obviously the same old stuff. This review chapter puts all computational skills into a new framework of measurement. The old stuff of adding, subtracting, multiplying, and dividing is disguised with applications to the world around the pupils. Only the concept of volume is really brand new. You should therefore find the pupils operating with confidence while exploring measurement. There are lots of research projects, but not the kind that depends on an encyclopedia. The supermarket, newspapers, and interviews with older people will be the main source of information. The pupils will come into contact with ideas that their parents may be discussing and the arithmetic skills should take on new meaning. The skills reviewed here are used by people every day, not just by students who have to do a math assignment.

Most countries in the world are now using, or are in the process of adopting, the International System of Units. In carrying out research projects on consumer products, pupils will find labels using both systems. But the inconsistencies will continue to disappear as the country moves towards complete use of SI units. In many areas, in scientific work and in some commercial products, SI units are already the standard. Pupils will not find it difficult to work with these units.

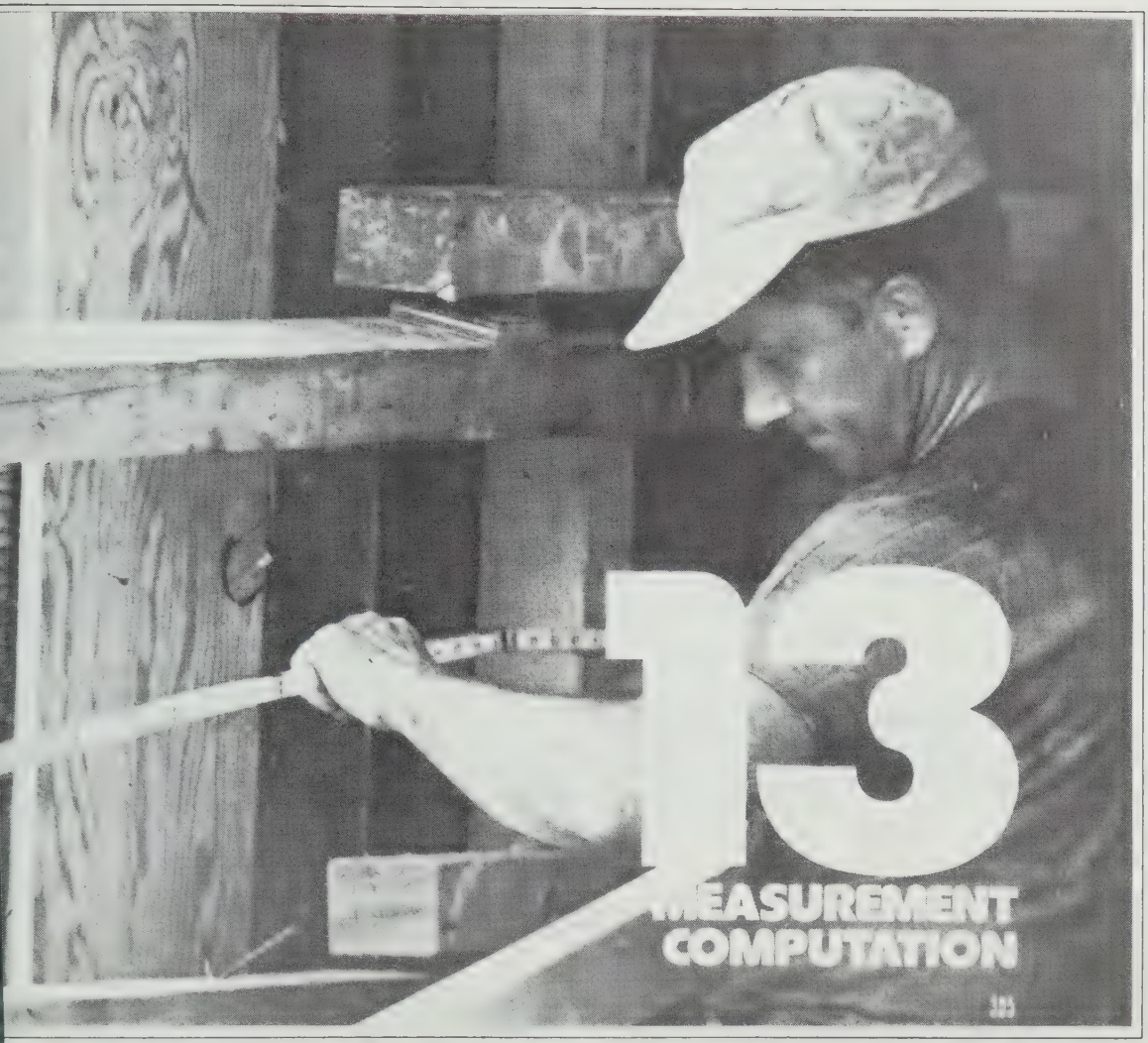
The metric units of length, mass, and capacity are reviewed in this chapter. The pupil should know that the prefix indicates the size and the root serves as a clue to the type of measure. From the beginning the children should learn the internationally accepted spelling of metric terms. Most countries have adopted the **re** spellings for **metre, litre**, and their derivatives, and these should be used without exception.

The comprehensive Checkout test will let you determine which pupils have actually achieved the goals that they had and the objectives that you had.

## things

scale for measuring mass
supermarket labels
sugar cubes
small empty boxes
newspaper stock-market pages

For the extra activities you will want to have these things available:
glue or transparent tape
envelopes




**goal** Think about and explore ideas through a picture clue

**page 305** You don't have to be a carpenter to use measurement. Start a wide-open discussion. What would the world be like without measurement?

- What might houses look like?
- How would you buy milk?
- How would you arrange the time to meet a friend at the movies?
- Could you buy your favorite record?
- Would the record work on your record player?
- How would you get clothes to fit?

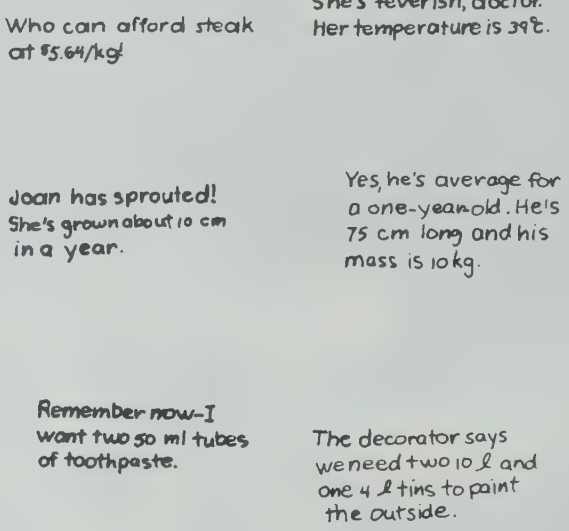
Why not start everyone on a personal measurement diary? Keep a record for 1 week. Write down each time that measurement is used in a situation in or out of school. Surely some sort of prize should be given to the person who has the longest list. Maybe the winner would like to plan a day of measurement activities for the group. It's always fun to be the teacher for a day.

**goal** **Survey**—ability to compute with decimals, measurements, and capacity

**page 306** The cartoon items provide a variety of measurement units — mass and money in one; height (length) and time in another; age, length, and mass in another; capacity and temperature in others. Discussion arising from these items may take some surprising directions.

Problems 1 to 5 will help you identify pupils who may be ready to skip some of the pages in this chapter. However, many unusual and interesting measurement situations are presented. You will want to skim the chapter and select some pages for enrichment with pupils who have mastered the computational skills reviewed.

**You can't escape measurement. It's all over the place. Everybody gets involved with it sooner or later. Your goal is to explore measurement in the real world and review your computational skills at the same time.**


Who can afford steak at $5.64/kg!
She's feverish, doctor. Her temperature is 39°C.
Joan has sprouted! She's grown about 10 cm in a year.
Yes, he's average for a one-year-old. He's 75 cm long and his mass is 10 kg.
Remember now—I want two 50 ml tubes of toothpaste.
The decorator says we need two 10 ℓ and one 4 ℓ tins to paint the outside.

1. They paid $2.69/kg for 4.5 kg of meat. How much did it cost? $12.11
2. Jan usually cycles to and fro from school. The journey takes 20 min. Because it was raining on Wednesday she took the bus. How much time did she spend in cycling to school that week? 2 h 40 min
3. They bought 4 bags of rice. Each bag had a mass of 0.750 kg. What was the total mass of the 4 bags? 3 kg
4. The Bereznickis drink 3 ℓ of milk per day. A litre costs 58¢. How much is their weekly milk bill? $12.18
5. Our living room is 21 $m^2$. How much will it cost to cover the room with carpet costing $12.95/$m^2$? $271.95

306

You have done a lot of work with numbers over the years. You should be able to put your knowledge to work for you. This chapter is about applications of numbers. All computation will be connected with a measurement situation.

How many types of measurement are there? Who knows? How many do you know? Find out. Make a chart. Put headings like these at the top of each column.

| LENGTH | MASS | CAPACITY | OTHERS |
|---|---|---|---|
| | | | |

Now put all the units of measure you know in their proper column. What does it mean if a measure is called a *standard* measure?

Money is a standard unit of measure within any given country. There is no international system of currency. The coins of Canada are different from the coins of Australia. The coins of Mexico are different too. Here is a list of the coins that are minted (made) regularly in our country.

| DENOMINATION | STANDARD MASS (grams) | DIAMETER (millimetres) |
|---|---|---|
| 1-cent piece | 3.20 | 19.05 |
| 5-cent piece | 4.55 | 21.23 |
| dime | 2.10 | 17.91 |
| quarter | 5.03 | 23.82 |
| half-dollar | 7.78 | 26.45 |

1. Why must like coins have the same mass and diameter? Would a Canadian coin work in an Australian vending machine?
   If it were the same dimension as an Australian coin.
2. Does the largest coin also have the greatest value? Yes
3. About how much mass would a bag of 100 quarters have? 503 g
4. About how many nickels would there be in a bag that has a mass of 25 g? 5 or 6
5. How many dollars would 100 quarters be worth? $25



**lesson** Page 307

**goal** Review of units of measure; applications involving multiplication and division with decimals.

**page 307** Completing the chart on the left side of the page is intended to help the pupil review his knowledge of measurement. It will show you the gaps in the pupils' knowledge that you will have to deal with.

Encourage pupils to compute problems 3 to 5 mentally. There will be enough written practice later.

**lesson** Pages 308, 309

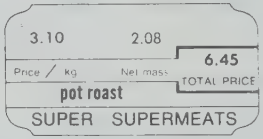
**goal** Exploring price based on mass; practice in multiplying decimals

**things** supermarket price labels

**page 308** Are labels similar to the one illustrated available in your area? Then compare actual examples to the illustration. Make sure the youngsters understand that although no dollar sign appears on the label, the heading identifies a number as a price. Many adults, unfamiliar with reading decimals, are confused when this notation is used to record mass. Students can help their parents in this area. Examine the label on this page. Is 2.08 kg nearer to 2 kg or to 3 kg ?

Problems 2 and 3 provide practice in multiplying with decimals.

Money is needed to buy food. Food comes in packages. But you pay for the amount that is inside the package. You pick up a package of meat and you see this label:




The scale automatically computes the price and mass of the meat. Some cuts of meat cost more than others. Pot roast costs \$3.10/kg . It is cheaper than porterhouse steak which costs \$6.55/kg, but more expensive than rolled roast which costs \$2.99/kg .

1. Why do different cuts of meat vary in price?

Discuss

2. Using the label above, how much would you pay for a pot roast with a mass of 3.08 kg? 3.64 kg? 2.56 kg?

\$9.55 \$11.28 \$7.94

3. If the pot roast costs exactly \$3.00/kg, what would be the cost of 2 kg? 3.5 kg? 4.75 kg?

\$6.00 \$10.50 \$14.25

he machine in the meat market is letting labels slip before the total price is printed. he repair man has been called. But you have to get some prices on packages of meat o that people can buy them.

**1.**

| 2.60 | 1.00 | ? |
|---|---|---|
| Price / kg | Net mass | TOTAL PRICE |
| pork chops | | |
| SUPER SUPERMEATS | | |

$2.60

**2.**

| 2.99 | 4.27 | ? |
|---|---|---|
| Price / kg | Net Mass | TOTAL PRICE |
| rolled roast | | |
| SUPER SUPERMEATS | | |

$12.77

**3.**

| 6.55 | 1.07 | ? |
|---|---|---|
| Price / kg | Net mass | TOTAL PRICE |
| porterhouse steak | | |
| SUPER SUPERMEATS | | |

Accept $7.00 or $7.01

**4.**

| 3.42 | 3.39 | ? |
|---|---|---|
| Price / kg | Net mass | TOTAL PRICE |
| half leg of lamb | | |
| SUPER SUPERMEATS | | |

$11.59

**5.**

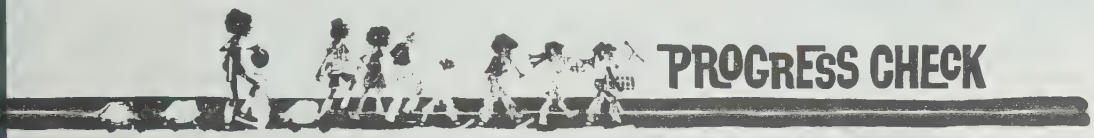
| 2.26 | 0.89 | ? |
|---|---|---|
| Price / kg | Net mass | TOTAL PRICE |
| Italian sausage | | |
| SUPER SUPERMEATS | | |

$2.01

**6.**

| 2.54 | 1.19 | ? |
|---|---|---|
| Price / kg | Net mass | TOTAL PRICE |
| meatloaf mix | | |
| SUPER SUPERMEATS | | |

$3.02



## PROGRESS CHECK

The store had a special sale on lamb chops and chicken. Compute the cost.

| Lamb chops per kilogram $2.25 | | Chicken per kilogram $2.15 | |
|---|---|---|---|
| kg 1.00 (1.) ? cost | $2.25 | kg 1.00 (7.) ? cost | $2.15 |
| 1.10 (2.) ? | $2.48 | 1.30 (8.) ? | $2.80 |
| 1.50 (3.) ? | $3.38 | 1.80 (9.) ? | $3.87 |
| 1.65 (4.) ? | $3.71 | 2.23 (10.) ? | $4.79 |
| 1.79 (5.) ? | $4.03 | 2.55 (11.) ? | $5.48 |
| 1.90 (6.) ? | $4.28 | 2.63 (12.) ? | $5.65 |

309

See activity 1, page 336b.

See activity 2, page 336b.

**goal** Practice in multiplying decimals; **Progress Check**—multiplying decimals

**page 309** More labels. Consider why decimals rather than common fractions are used to record mass. The youngsters may have some good ideas. Then think about why hundredths are used rather than tenths. (More exact measure)

Multiplication with decimals has not been practised recently. You may want to use problems 1 to 6 to review the algorithm and placement of the decimal point **or** to survey the pupil's ability. Have the youngsters round both the mass and the unit price indicated, then estimate the total price. This will help in placing the decimal correctly in the computed answer. Exact prices will be computed in ten-thousandths and need to be rounded up to hundredths. Consider the difference in price between rounding up and rounding down. How much would the storekeeper lose if he sold 1 000 kg? 10 000 kg? Work with those who need additional help before assigning the Progress Check.

Use your discretion in assigning the Progress Check. Perhaps girls can find the costs for lamb chops while boys work on the costs for chicken. Possible trouble spots to look for:

- Failure to master multiplication facts
- Failure to master multiplication algorithm, including renaming
- Confusion on placing the decimal in the product
- Failure to understand how to round the product to hundredths

**goal** Exploring measure of capacity; prefixes and symbols

**page 310** It may help children here to recall the units for measuring distance and mass. The litre and other units of capacity then fit nicely into the pattern that is already familiar. Some pupils may find it helpful to make charts like the one given here for the units measuring distance and mass. Through such charts the more alert pupils may be able to see the relationships between the units for distance, mass, and capacity.

Draw attention to the script $\ell$ symbol for litre. Ask the pupils why they think this form is used.

Independent workers may complete the problems on their own. Others may need some help. The Supersleuth section is for those who understand how to change the position of the decimal point.

Remember the litre? It's the commonest unit for measuring capacity.
To measure larger or smaller amounts, we affix prefixes to the litre.

Before you answer the next two questions, check the prefixes that can be affixed to the litre.

smaller
- 0.001 $\ell$ is a millilitre.
- 0.01 $\ell$ is a centilitre.
- 0.1 $\ell$ is a decilitre.

☆ 1 $\ell$

larger
- 10 $\ell$ are a decalitre.
- 100 $\ell$ are a hectolitre.
- 1000 $\ell$ are a kilolitre.

1. **a** How many decilitres in a litre? 10
   **b** How many centilitres in a litre? 100
   **c** How many millilitres in a litre? 1000

2. **a** How many litres in a decalitre? 10
   **b** How many litres in a hectolitre? 100
   **c** How many litres in a kilolitre? 1000

The symbol for litre is a script $\ell$ . When prefixes are affixed to the symbol, it is no longer necessary to use a script $\ell$ .
millilitre symbol = ml .
centilitre symbol = cl .
decilitre symbol = dl .
decalitre symbol = dal .
hectolitre symbol = hl .
kilolitre symbol = kl .

Using the chart, replace the question marks with the correct symbol.

3. **a** 40 $\ell$ can equal 4 ? dal
   **b** 600 $\ell$ can equal 6 ? hl
   **c** 9000 $\ell$ can equal 9 ? kl
   **d** 0.8 $\ell$ can equal 8 ? dl
   **e** 0.58 $\ell$ can equal 58 ? cl
   **f** 0.006 $\ell$ can equal 6 ? ml

Think carefully.

| ml | cl | dl | $\ell$ |
|---|---|---|---|
| 80 | 8 | 0.8 | 0.08 |
| 46 80 | 468 | 46.8 | 4.68 |
| 500 | 50 | 5 | 0.5 |
| 2000 | 200 | 20 | 2 |

| $\ell$ | dal | hl | kl |
|---|---|---|---|
| 50 | 5 | 0.5 | 0.05 |
| 500 | 50 | 5 | 0.5 |
| 5000 | 500 | 50 | 5 |
| 50 000 | 5000 | 500 | 50 |

310

What's a kilojoule?
It is a standard unit of measure that tells the energy-producing value of food. Usually people need to eat so that the kilojoules taken in through food equal the kilojoules used up through activities. (Extra kilojoules are turned into fat and stored in the body.)

Here's the number of kilojoules thought to be needed by one group of people.

| | Age | Mass (kilograms) | Height (centimetres) | Kilojoules needed daily |
|---|---|---|---|---|
| **For boys** | 10-12 | 35 | 140 | 10 500 |
| | 12-14 | 43 | 150 | 11 340 |
| | 14-18 | 60 | 170 | 12 600 |
| **For girls** | 10-12 | 35 | 142 | 9500 |
| | 12-14 | 44 | 155 | 9660 |
| | 14-16 | 52 | 158 | 10 080 |
| | 16-18 | 54 | 160 | 9660 |

1. Why do you think the listing for boys is different from that for girls? The bone structure of males is different from that of females. Girls grow faster than boys at certain ages.

2. If a boy is taller than the chart says for the age and mass, do you think the kilojoules needed would be more? Why? Maybe Kilojoule needs are determined by energy used in activities.

3. If a girl's mass is more than the chart says for the age and height, do you think the kilojoules needed would be more? Why? Probably not She probably has enough stored up in body fat.

4. Do you know anyone who is on a diet? What does it mean to be on a diet? Why should you see a doctor before dieting? A diet is an attempt to regulate mass (either to increase it or to decrease it). One should always see a doctor before dieting.

311

**lesson** Pages 311, 312, 313

**goal** Examining the kilojoule as a unit of measure

**page 311** For young people still in their growing years, recommendations for mass and kilojoules are based on sex, height, and age. Charts for adults base their recommendations on sex, height, and body structure.

Examine the chart. Have each youngster find where he fits and how many kilojoules per day this chart says he needs to maintain his mass. If fewer kilojoules are eaten for several days, what may happen? People who are above normal mass have stored up extra kilojoules. What must people do to use up some of the stored kilojoules?

All kilojoules recommendations are only estimations. Individual activity and metabolism (rate at which the body burns kilojoules) affect these recommendations.

Problems 2 and 3 should lead to some interesting comments. Height does affect recommended kilojoule intake. The pupils should see that a kilojoule intake higher than the chart recommends means a gain in mass.

When discussing problem 4, be sure to examine the possibility of an individual's being diabetic or allergic to certain foods. Such individuals will be on diets different from those designed to control mass.

**goal** Extrapolating information from a chart

**page 312** Special crash diets are created continuously. These diets may be followed for a short period of time. The pupils are encouraged here to develop a well-balanced diet—a life style of healthful eating. Consider how something sweet that is not cake or candy can be included in the diet. Examine especially the listing of sandwiches. Consider how they combine the four groups of food.

Selecting the actual foods for the diet can make an interesting homework assignment.

Pretend you have a friend who is heavier than he should be. The doctor put him on a diet. He can have no more than 7560 kJ each day. Plan one day's meals. Pick foods from the list below. You'll need food from each of these four groups to keep him healthy.

Answers will vary, depending on the individual choice.

Check to make sure each of the four groups of food are represented.

## 1 Source of protein

meat, poultry, fish, eggs, dried beans, or peas

| | Kilojoules |
|---|---|
| 110 g roast beef | 1260 |
| half small chicken | 1260 |
| 85 g ham | 1424 |
| 85 g lamb chop | 1530 |
| 85 g tuna | 710 |
| 1 egg | 323 |

## 2 Source of calcium

all foods from milk

| | Kilojoules |
|---|---|
| 1 cup milk | 697 |
| chocolate milkshake | 2184 |
| malted milk | 2360 |
| cocoa | 987 |
| 1 tablespoon butter | 420 |
| 1 cup cottage cheese | 903 |

## 3 Source of vitamins and minerals

fruits and vegetables

| | Kilojoules |
|---|---|
| apple | 319 |
| banana | 370 |
| 1 cup orange juice | 454 |
| 1 cup fried potatoes | 1932 |
| 1 cup mashed potatoes | 668 |
| 1 cup peas | 466 |

## 4 Source of starch

food made from grain such as cereal or bread

| | Kilojoules |
|---|---|
| 1 doughnut | 525 |
| 1 cup rice cereal | 504 |
| 1 cup cereal | 462 |
| 1 cup oatmeal | 600 |
| 1 hard roll | 672 |
| 1 slice bread | 252 |

Here are some sandwiches:

| | Kilojoules |
|---|---|
| 1 hamburger | 1113 |
| ham and cheese | 1512 |
| peanut butter/jelly | 1134 |
| bacon/tomato | 1218 |
| liverwurst | 1470 |
| corned beef | 1701 |

And you can't forget about the extras

| | Kilojoules |
|---|---|
| 1 olive | 59 |
| 1 pickle | 8 |
| 1 tablespoon sauce | 84 |
| 25 g fudge | 625 |
| 1 iced cake | 609 |
| 1 piece apple pie | 983 |

312

# PROGRESS CHECK

Skill: Identifying larger or smaller units of measure by using prefixes

1. The base unit of length is the metre. Without looking back, can you name the length that is 1000 m? 1 km

2. What is the length that is 0.01 m? 1 cm

3. Use metric prefixes and make a list of three units of length smaller than 1 m. Decimetre, centimetre, millimetre

4. Now list three units of length greater than 1 m. Decametre, hectometre, kilometre

We use energy even when we are sleeping. The average person uses 5.5 kJ/min lying in bed resting or sleeping; 33.0 kJ/min bicycle riding; 47.0 kJ/min swimming; and 22.0 kJ/min walking.

1. Maybe you are a champion sleeper. How many kilojoules do you use in an eight-hour sleep? 2640 kJ

2. What's the farthest you ever rode a bike? About how long did it take you? How many kilojoules did you use? Answers will vary

3. All athletes must practise. A champion swimmer practises at least two hours each day. How many kilojoules does he use practising? 5640 kJ

4. A good walker can complete a 4 km race in less than 25 min. How many kilojoules would he use in one race? About 550 kJ

**things** cards or slips of paper

Write the various metric prefixes and their numerical values on cards or slips of paper. Mix the cards. The pupil matches each prefix with its numerical value. For example:

milli .001

Then he arranges the cards in order of size.

**lesson** Page 313

**goal** Progress Check - Reviewing units of length; practice with kilojoules

**page 313** In this Progress Check the pupil reviews his knowledge of the metric prefixes used in naming units for measuring length. Have pupils who make errors prepare a chart showing the three basic types of metric measure (length, mass, capacity) and the units of each type in order from greatest to least. Have the pupil underline the prefixes.

The Supersleuth exercise provides practice in multiplying, and at the same time gives pupils a chance to become more familiar with kilojoules in real-life situations.

**lesson** Pages 314, 315, 316

**goal** Finding equivalent measurements of length

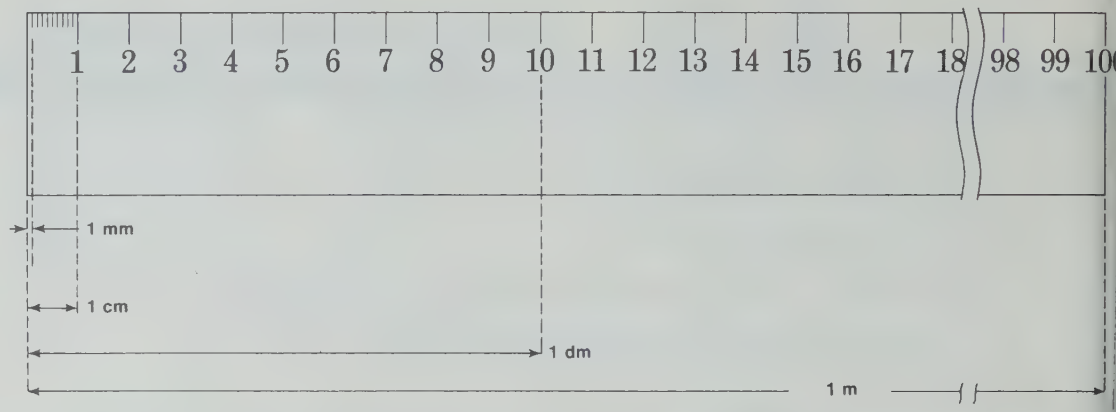
**page 314** Use the broken metrestick illustration to review the metric prefixes one more time.

The use of decimal notation simplifies changing from one unit to another a great deal.

You may want to examine the following pattern together before the youngsters begin renaming in problem 4.

| | |
|---|---|
| $1 \times 5 = 5$ | $5 \div 1 = 5$ |
| $10 \times 5 = 50$ | $5 \div 10 = 0.5$ |
| $100 \times 5 = 500$ | $5 \div 100 = 0.05$ |
| $1000 \times 5 = 5000$ | $5 \div 1000 = 0.005$ |

When changing to a larger metric unit, which operation is used? (Division) When changing to a smaller unit, which operation is used? (Multiplication) The smaller the unit, the more will be needed; the larger the unit, the fewer will be needed.




If the stick weren't broken, it would show the length of 1 m . That is the base unit of length.

0.1 of 1 m is a decimetre.
You don't see that length used often.
It is easier to say 0.1 m .

0.01 of 1 m is a centimetre.
You will see that unit used a lot.

0.001 of 1 m is a millimetre.
That is a very short length. You have heard this unit used. Maybe you have 8 mm film in your classroom.

1. How many millimetres in 1 cm? (10) in 1 m? (1000)
2. How many centimetres in 1 dm? (10) in 1 m? (100)
3. How many decimetres in 1 m? (10)
4. Decimal fractions are used when referring to measurement. For example 1.5 cm would be the same length as 1 cm plus 5 mm, or the same as 15 mm . Name the following lengths in another way.

There are many possible ways. Examples:

**a** 5 cm (50 mm, 0.5 dm) **b** 5 dm (50 cm, 0.5 m) **c** 0.5 cm (5 mm, 0.05 dm)
**d** 0.5 m (5 dm, 50 cm) **e** 0.05 m (0.5 dm, 5 cm) **f** 0.005 m (0.05 dm, 0.5 cm)

314

1. Suppose you have a rule marked with centimetres only.

   Measure each of the following line segments.

   **a** ⊢—— Does this measure 1 cm? Yes

   **b** ⊢———⊣ Does this measure more than 1 cm but less than 2 cm? Which is closer? 2 cm

   **c** ⊢———⊣ Does this measure 2 cm? Yes

   **d** ⊢———⊣ Is this between 2 and 3 cm? Which is it closer to? 2 cm

   Yes

2. Now suppose you found a rule marked with centimetres and millimetres. Measure each of the segments again. Record the measurements.

   a 1 cm; 10 mm b 1.7 cm; 17 mm c 2 cm; 20 mm d 2.3 cm; 23 mm

3. Which set of measurements is more accurate? millimetre

   Could there be an even more accurate measurement taken?
   (You might have to use a magnifying glass to do it.) Yes, a smaller metric unit

   There is no such thing as a precise measurement. A measurement will be only as accurate as the measuring device you are using. That is why you will sometimes see the symbol ≈ used. The symbol stands for "*approximately equal to.*"

4. Measure each of these line segments to the nearest decimetre, to the nearest centimetre, and to the nearest millimetre.

   1 dm **a** ⊢——— 10 cm ——— 100 mm

   1.7 dm **b** ⊢——— 17 cm ——— 170 mm

   1.34 dm **c** ⊢——— 13.4 cm ——— 134 mm

   98 dm **d** ⊢——— 9.8 cm ——— 98 mm

315

**goal** Examining the precision of measurement

**things** centimetre rulers (rules) also marked with millimetres

**page 315** All measurement is only an approximation. The precision of a measurement depends on two things:

- The size of unit of measure used in measuring
- The accuracy of the measuring device used

Talk about the possibility of the wood used for the ruler being cut inaccurately or warped, and call attention to the width of the mark printed on the ruler itself. Some printed marks on wood rulers actually measure 1 mm. The smaller the unit of measure, the more accurate the measure will be.

Make sure the words **precise** and **accurate** are understood or the symbol ≈ and the phrase APPROXIMATELY EQUAL TO will be meaningless.

**goal** Examining and practice in adding measurements

**page 316** When adding or subtracting decimals, annexing a zero is fine. But when computing with measurements, this cannot be done. Annexing a zero to a measurement would indicate that a more precise measurement was made. When two measurements differ in precision (number of decimal places), the more precise measurement must be rounded to the less precise unit of measure (fewer decimal places) **before** computation.

Once again, rounding is a prerequisite skill to be checked and practised, if necessary, before beginning actual computation.

Emphasize that in work with measurements rounding precedes computation.

## ADD

$$\begin{array}{r} 1.345 \\ +1.46 \\ \hline \end{array}$$

2.805

## NOW ADD THIS

$$\begin{array}{r} 1.345\ \text{m} \\ +1.46\ \text{m} \\ \hline \end{array}$$

2.81 m

Your answer should not be the same.
You must add like units of measure.
1.345 is a measurement to the nearest millimetre.
1.46 is to the nearest centimetre. You can't just annex a zero.
You must round 1.345 to the nearest centimetre. 1.345 rounds to 1.35

$$\begin{array}{rr} & +1.46 \\ \text{NOW ADD} & ? \\ & 2.81 \end{array}$$

## PRACTICE

1. Round these millimetres to the nearest centimetre.

   **a** 9.585 9.59 **b** 7.323 7.32 **c** 105.782 105.78 **d** 52.677 52.68

2. Round these centimetres to the nearest metre.

   **a** 3.64 4 **b** 10.574 11 **c** 25.51 26 **d** 2.546 3

All the next problems will be in pairs.
One is not a measurement. The other is a measurement.
Compute both. Watch out! There are both addition and subtraction problems.

| | a | b | a | b |
|---|---|---|---|---|
| 3. | 4.68 + 3.759 = 8.439 | 4.68 m + 3.759 m = 8.44 m | 4. 10.255 − 4.68 = 5.575 | 10.255 m − 4.68 m = 5.58 m |
| 5. | 15.246 + 9.75 = 24.996 | 15.246 m + 9.75 m = 25.00 m | 6. 73.2 − 26.27 = 46.93 | 73.2 m − 26.27 m = 46.9 m |
| 7. | 35.28 + 64.7 = 99.98 | 35.28 m + 64.7 m = 100.0 m | 8. 10.627 − 9.62 = 1.007 | 10.627 m − 9.62 m = 1.01 m |

316

new-born baby: 55 cm long
ribbon: 65 cm long

string: 125 cm long
carpet: 2 m long

Numbers attached to length don't mean a thing unless you can "see" the length in your mind. You should be able to draw a line segment about 1 cm long.

This line segment is 1 cm long.

This area is 1 $cm^2$. Area is a measure of a square unit. A square unit has two dimensions. It has both length and width.

Can you "see" the size of a square metre ($m^2$) in your mind? Your classroom can be measured in square metres by measuring its length and width.

Can you imagine the size of the field inside an athletic track? If you can, you can "see" the size of a hectare (ha). A hectare is 10 000 $m^2$.

Now imagine how much land is needed for 30 city blocks. That's a lot of land and is about the size of a square kilometre ($km^2$).

The square centimetre symbol is $cm^2$.
The square metre symbol is $m^2$.

The hectare symbol is ha .
The square kilometre symbol is $km^2$.

**TRY THESE**

Look at the symbols for area measurement and write the ones that would be used to describe the following areas:

**a** floor tiles $cm^2$ **b** farmland ha **c** countries $km^2$ **d** footprints $cm^2$

**e** carpet $m^2$ **f** oceans $km^2$ **g** parks ha **h** doors $cm^2$

**i** the moon $km^2$ **j** a bookcase $cm^2$ **k** graph paper $cm^2$ **l** houses $m^2$

317

**goal** Exploring square measure and area; symbols for units of square measure

**page 317** The work on this page should help pupils to visualize area of 1 $cm^2$, 1 $m^2$, 1 ha, and 1 $km^2$. Let the pupils all draw squares with sides 1 cm long. Assign pairs of children to measure off a square metre on the floor, and to draw one on the board. Help them develop a concept of the size of a square kilometre by giving familiar local references that name the corners of an area approximately a square kilometre. Similarly specify an area that is approximately one hectare, such as the field inside an Olympic track. Such exact references will help the youngsters until they are thoroughly familiar with the units for measuring area.

The exercises may be done together, by the whole class or in groups, or independently by those pupils who have grasped the concept. The concept is a difficult one, and the pupils should not attempt independent work until they have mastered it.

**goal** Finding area and perimeter

**page 318** Non-scale drawings may prove helpful in solving problems 1 and 2. Make sure the length and the width of each figure are labelled correctly.

Encourage your pupils to compute the area of each rectangle in problem 3 by multiplication. No one can count squares to find area for the rest of his life—computation is a far better method. Watch for the labelled answers. Area measure must be recorded in **square** units. Make sure that all pupils are writing the symbol for square units correctly, with the 2 raised above the level of the numbers.

There's no chance of getting into a rut on this page. Perimeter requires units of length. The answers in problem 4 are **not** recorded in square units. Emphasize computing the perimeter of each figure. (Answers may be verified by counting. This will reassure the less capable student.)

1. How many square centimetres in a tile 1 cm long and 1 cm wide? $1\ cm^2$

   **a** How many square centimetres in a tile 100 cm long and 100 cm wide? $10\ 000\ cm^2$

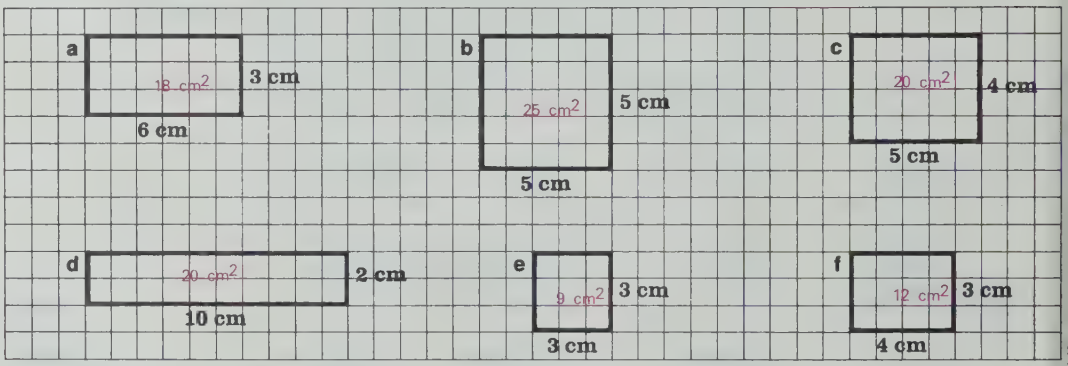
   **b** Then how many square centimetres in a square metre? $10\ 000\ cm^2$

2. How many square metres in a piece of land 1 m long and 1 m wide? $1 m^2$

   **a** How many square metres in a piece of land 1000 m long and 1000 m wide? $1\ 000\ 000\ m^2$

   **b** There are 1000 m in 1 km . How many square metres in a square kilometre? $1\ 000\ 000\ m^2$

3. Find the area. The graph paper will help you check your answers.
   ☐ Let this represent 1 $cm^2$. (It's not really one though.)

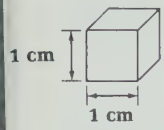



4. Go back. Find the perimeter of each of the figures above.

a 18 cm b 20 cm c 18 cm d 24 cm e 12 cm f 14 cm

318

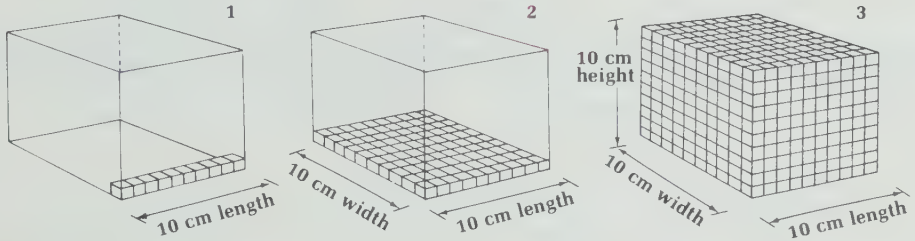
This is a picture of a cubic centimetre.




This is only a picture. A real cubic unit is 1 unit wide, 1 unit long, 1 unit thick. It has three dimensions. It takes up space. It has volume. Volume measurements are given in cubic units.

One unit of volume measurement is the cubic centimetre ($cm^3$).
Another unit of volume measurement is the cubic metre ($m^3$).

**1.** A block measures 10 cm high, 10 cm wide, and 10 cm long. How many cubic centimetres does it have? 1000 $cm^3$




**a.** How many cubic centimetres does the row have? 10

**b.** How many rows does the layer have? How many cubic centimetres in the layer? 10, 100

**c.** How many layers does the block have? How many cubic centimetres does it have? 10, 1000

The volume of this block is $10 \times 10 \times 10$ $cm^3$, or 1000 $cm^3$. Each side is a decimetre long. Can you think of another unit to describe the volume of the block? 1 cubic decimetre ($dm^3$)



**lesson** Pages 319, 320, 321

**goal** Examining units of volume measure

**things** centimetre cubes

**page 319** Hands-on experience with the real thing will help develop understanding of cubic units of measure. Pupils who do not understand the distinctions between measurements of length, of area, and of volume, should draw a line 1 cm long, and a square 1 cm, and make a paper model of a cube, dimensions 1 cm. Sugar cubes are close to a cubic centimetre.

If further practice is needed, let them make a strip of paper 2 cm long, a square with sides 2 cm long, and a cube with sides 2 cm long.

**goal** Introduction to computing volume

**page 320** The youngsters have been computing volume without the benefit of the sentence given in problem 1.

In problems 3 and 4, explore whether changing the order in which the measures are multiplied changes the volume. Check carefully that all answers are labelled with a cubic unit of measure. Encourage mental computation. **Answers only** need be recorded if these problems are assigned as written work.

Again, check that the symbols for cubic units are written correctly.

Finding the volume of blocks is easy.

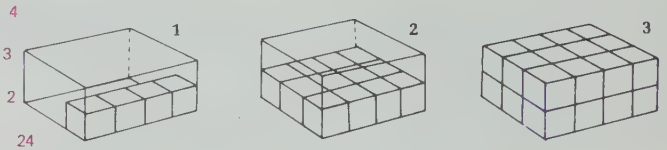
1. Length × width × height is the volume.

   **a** What's the length? 4

   **b** What's the width? 3

   **c** What's the height? 2

   **d** What's the volume? 24



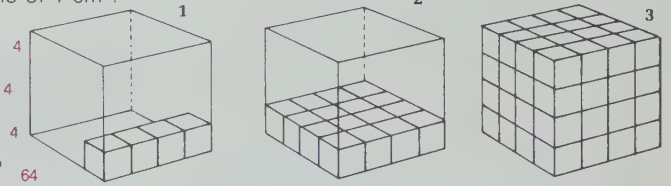

2. Each cube has a volume of 1 $cm^3$.

   **a** What's the length? 4

   **b** What's the width? 4

   **c** What's the height? 4

   **d** What's the volume? 64

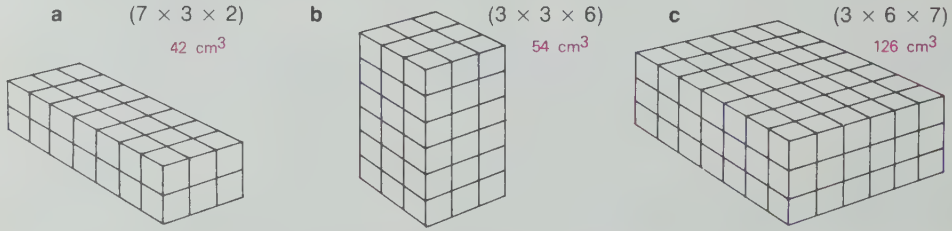



3. Each unit cube measures 1 cm on each side. Find the volume of each block.

   **a** $(7 \times 3 \times 2)$ 42 $cm^3$

   **b** $(3 \times 3 \times 6)$ 54 $cm^3$

   **c** $(3 \times 6 \times 7)$ 126 $cm^3$



4. If each cube in problem 3 measured 1 m on each side, what would be the volume of each block? a 42 $m^3$ b 54 $m^3$ c 126 $m^3$

320

1. What's the volume of a box that measures —

**a** 2 cm long, 3 cm wide, and 2 cm high? 12 $cm^3$

**b** 5 m long, 3 m high, and 2 m wide? 30 $m^3$

**c** 3 cm high, 4 cm long, and 10 cm wide? 120 $cm^3$

**d** 12 cm wide, 5 cm long, and 1 cm high? 60 $cm^3$

**e** 10 m by 5 m by 2 m? 100 $m^3$

**f** 10 cm by 4 cm by 3 cm? 120 $cm^3$

# PROGRESS CHECK

What is the volume of a box that measures —

(1.) 1 m by 1 m by 1 m? 1 $m^3$

(2.) 3 cm by 2 cm by 1 cm? 6 $cm^3$

(3.) 1 cm by 2 cm by 3 cm? 6 $cm^3$

(4.) 6 cm by 4 cm by 4 cm? 96 $cm^3$

(5.) 10 cm by 4 cm by 4 cm? 160 $cm^3$

(6.) 10 cm by 10 cm by 10 cm? 1000 $cm^3$

A power shovel was used to dig a hole 8.5 m wide, 1.5 m deep and 10 m long. How many cubic metres of earth were removed from the hole? 127.5 $m^3$

321

**goal** Practice in computing volume; **Progress Check**—computing volume

**things** sugar cubes
small empty boxes

**page 321** Mental computation for problem 1 should be encouraged. Again, watch for properly labelled answers.

The label of cubic unit of measure is one thing to watch for in the Progress Check. And if someone makes errors because he doesn't know how to multiply, you have every right to be discouraged. Back to the flash cards or drill program.

More experience with counting rows, columns, and layers is in order with pupils who are having trouble with the concept of volume. Use sugar cubes or let pupils work in pairs to make cube models from paper, with the dimensions given in problem 1, or in problems 2 to 6 of the Progress Check.

Supersleuth is only for those who have fully grasped the concept and computation of volume. Let's not confuse the others as yet with decimals and zeros.

**lesson** Pages 322, 323

**goal** Extrapolating information from a chart

**memo** A five-page Checkout begins on page 324. You may want to use pages 322 and 323 as a change of pace between parts of the Checkout.

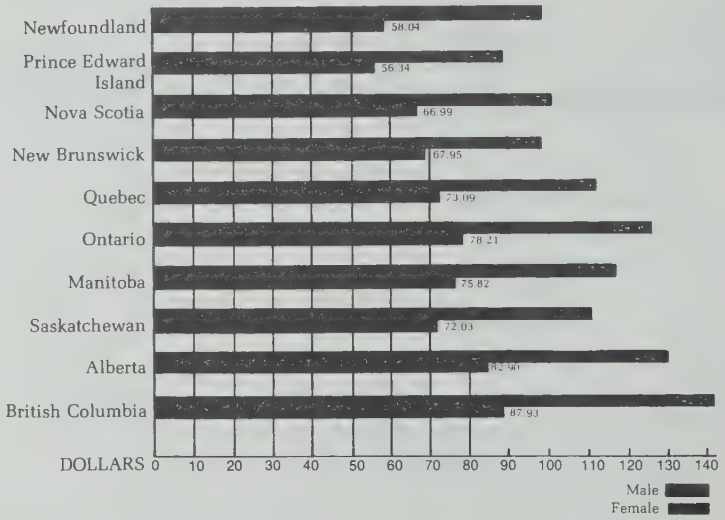
**page 322** There's lots to talk about on this page. In which provinces do women have the best pay? What reasons might there be for this? Why is there such a difference between the pay for men and for women? Do you think this is right?

In exercise 4 pupils will be most interested in finding information about their own province, and perhaps in comparing it to their neighbouring provinces. This is as much a social studies lesson as a mathematics lesson, and will give scope to the researchers in the class.

One dollar is a unit of measure too. But its value is not always the same. Its purchasing power is greater at one time than another. And it has been shown that the same number of dollars are not always paid to people for the same job.

Here is a chart that shows the average weekly pay men and women received in a recent year.

THE WAGE GAP: AVERAGE WEEKLY EARNINGS OF RETAIL-TRADE EMPLOYEES, 1970




1. What is the difference between the pay for men and the pay for women in each province?

2. Which province has the least inequality? New Brunsw

3. Which province has the greatest inequality? British

4. Find this year's almanac. Get the same type of information. Has progress been made towards equal pay? Discuss

5. Would a person get to take home all the money earned?

*1. Newfoundland: $40.19
Prince Edward Island: $32.20
Nova Scotia: $33.75
New Brunswick: $29.71
Quebec: $38.70
Ontario: $46.75
Manitoba: $39.96
Saskatchewan: $38.10
Alberta: $46.33
British Columbia: $54.23

No. Taxes, pension, insurance, hospital insurance, unemployment insurance, annuities, contributions are some deductions.

322

**Newspapers have pages that look like these:**

Stock Exchange

| 1973 High | 1973 Low | Stocks and Div. In Dollars | Sls. P/E | Sls. 100s | High | Low | Last | Net Chng |
|---|---|---|---|---|---|---|---|---|
| | | **A—B—C—D** | | | | | | |
| 80⅞ | 61 | AbbtLb 1.20 | 21 | 31 | 66½ | 66 | 66 | −1 |
| 49¾ | 40½ | ACF Ind 2.40 | 10 | 2 | 42¾ | 42¾ | 42¾ | − ¼ |
| 17⅛ | 12¼ | AcmeCiv .88 | 7 | 1 | 13 | 13 | 13 | + ⅛ |
| 26 | 16½ | AcmeMkt 1 | | | | | | |

Stock Exchange

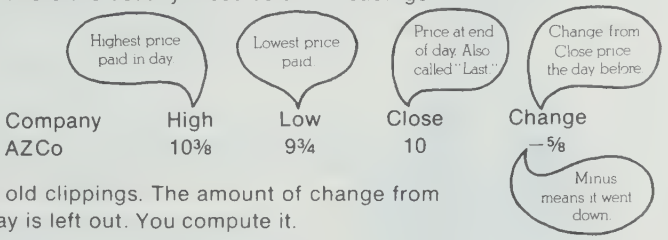
| Day's Sales | Wednesday | Year Ago | Year to Date 1973 | Year to Date 1972 |
|---|---|---|---|---|
| 1 933 620 | 1 879 730 | 3 176 443 | 488 239 340 | 816 669 530 |

*Most Active Stocks*

| Company | Volume | Last | Net Chng. |
|---|---|---|---|
| **Mansfd TR** | **115 000** | 5¼ | — ¾ |
| **Kaiser Ind** | **86 100** | 7¼ | + ½ |

These are stock-market pages. They list a lot of companies. People can own parts of these companies by buying shares in them. A share in one company might cost less than a dollar. A share in another company might cost hundreds of dollars. The price depends on how much people will pay. Most stocks are bought and sold through stock exchanges such as the Montreal, Vancouver, and Toronto exchanges.

The value of a share is given in dollars and eighths of a dollar. There are usually these column headings:




Here are two very old clippings. The amount of change from Monday to Tuesday is left out. You compute it.

**STOCK EXCHANGE**

MONDAY, OCTOBER 28, 1929.

| Day's Sales. | Saturday. | A Year Ago. |
|---|---|---|
| 9,212,800 | 2,087,660 | 3,770,570 |

| Stock | High | Low | Close | Ch'ge |
|---|---|---|---|---|
| Abitibi Pow & Paper | 48½ | 43½ | 44¼ | − 4¾ |
| Adams Millis | 27⅝ | 27½ | 27½ | − ¼ |
| Ahumada Lead | 1¼ | 1 | 1⅛ | |
| Air Reduction | 189¼ | 135 | 145 | −49⅞ |
| Air Way El Ap | 28½ | 28½ | 28½ | − 5⅜ |
| Alleghany Corp | 36 | 29 | 29⅛ | − 7½ |

**STOCK EXCHANGE**

TUESDAY, OCTOBER 29, 1929.

| Day's Sales. | Monday. | A Year Ago. |
|---|---|---|
| 16,410,030 | 9,212,800 | 3,483,770 |

| Stock | High | Low | Close | Ch'ge | |
|---|---|---|---|---|---|
| Abitibi Pow & Paper | 40 | 38 | 38 | **a** | —6¼ |
| Adams Millis | 26 | 24¾ | 24¾ | **b** | —2¾ |
| Ahumada Lead | 1 | ¾ | ¾ | **c** | —⅜ |
| Air Reduction | 128 | 100¼ | 120 | **d** | —25 |
| Air Way El Ap | 25 | 25 | 25 | **e** | —3½ |
| Alleghany Corp | 24¾ | 18⅛ | 20⅜ | **f** | —8½ |

Some very interesting history is connected with these clippings. If you have grandparents, they can tell you the story. You can find it in reference books, too.

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**things** newspaper stock-market page

Remove the **change** column from a stockmarket report. Have the youngsters actually compute the amount of change. This computation emphasizes subtraction with mixed numbers.

See activity 9, page 336c.

**goal** Application of subtraction of fractions

**things** newspaper stock-market pages

**page 331** Some youngsters are interested in the stock market at a very early age. To others this will be a completely new idea. Expect almost any type of reaction. An examination of stock-market pages from a local newspaper will make the whole discussion more exciting and relevant. You may be surprised to find you have a young stockbroker in class.

Finding out what happened in 1929 is the pupils' project. Please don't tell.

**lesson** Pages 324, 325, 326, 327, 328

**goal** **Checkout**—place value and numeration; ordering whole numbers, fractions and decimals

**memo** The remainder of this book is a Checkout of skills reviewed or developed in level 6. You will want to spread these pages over several days, giving your pupils an opportunity to measure how much they know rather than how long they can endure before reaching the point of fatigue.

This Checkout is divided into four parts:

| Part | Page(s) | Skill |
|---|---|---|
| I | 324 | Numeration |
| II | 325 | Operations with whole numbers |
| III | 326-327 | Operations with fractions |
| IV | 328 | Applications |

How best to use these pages with your pupils is your decision.

**page 324** Use this chart to guide you in providing additional help.

| Trouble with problem(s) | Go to | |
|---|---|---|
| | Level | Page(s) |
| 1–3 | 4 | 3–5 |
| 4 | 6 | 88–90 |
| 5 | 5 | 12–13 |
| 6 | 5 | 81–82 |
| 7a | 6 | 74–78 |
| 7b | 5 | 81–82 |

# CHECKOUT

Now it's time to measure yourself and your skills. This is a long Checkout. Don't try to do it all at once.

## Part I–What do you know about numbers?

Skill: Place value and numeration

1. How many different digits are used in our number system?

2. Using each digit only once, write the largest whole number that can be written. 9 876 543 210

3. Write all the different 3-digit whole numbers that can be written with the digits 7, 8, and 9.
789, 798, 879, 897, 978, 987

4. Write the place value of each digit in this number.
9 182 736 450 9: billions 8: ten millions 3: ten thousands
1: hundred millions 2: millions 6: thousands
7: hundred thousands 4: hundreds

Skill: Ordering numbers

5. Write the numbers in order from least to greatest.
100 100 100 001 100 010 101 010 101 100
100 001 100 010 100 100 101 010 101 100

6. Are the following names for the same number? No
$\frac{1}{9}$, $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{5}$

   a Write them in order from least to greatest. $\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}$

   b If the numerator is 1, which digit would be used in the denominator to show the fraction of greatest value?

7. Write these numbers in order from least to greatest.

   a 0.1, 0.001, 1.0, 0.01 1.1, 1.11, 1.01

   b $\frac{9}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{2}{2}$, $\frac{5}{2}$, $\frac{1}{2}$, $\frac{4}{2}$

   a 0.001, 0.01, 0.1, 1.0, 1.01, 1.1, 1.11
   b $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$

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# Part II-What do you know about operations on whole numbers?

Skill: Properties of a math system

1. Pick any two whole numbers. Add them.
   Can you find a pair that does not have a whole-number answer? No

2. Pick any whole number. Add zero.
   Is the answer always the same as the first number you picked? Yes

3. Pick any two whole numbers. Add them.
   Change the order of the two numbers. Add again.
   Do you get the same answer? Yes

4. Pick any three whole numbers. Add them.
   Write the three numbers, but group them differently.
   Add again. Do you get the same answer? Yes

5. Go back to problems 1 through 4, but multiply this time.
   Do you get the same results? (If you say yes, you are WRONG!) No
   Which problem has a different result? Problem 2

6. Go back to problems 1 through 4. Subtract this time.
   Which problems have different results when you subtract? Problems 1, 3, 4

Skill: Whole-number computation

Complete these.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 7. | 23 756<br>+16 438<br>40 194 | 502<br>× 79<br>39 658 | 27 R 729<br>27)729 | 6520<br>−2001<br>519 | 100 R10<br>14)1410 |
| 8. | 785<br>6215<br>+ 15<br>7015 | 6704<br>−1916<br>4788 | 107 R40<br>50)5390 | 1628<br>× 25<br>40 700 | 6000<br>−4201<br>1799 |

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**goal** **Checkout**—operating with whole numbers

**page 325** If reading is a problem, do help out. Problems 1 through 6 contain no curve balls, but they do require some good thinking.

Use this chart to guide you in providing additional help.

| Trouble with problem(s) | Go to page(s) |
|---|---|
| 1–6 | These ideas have been explored in levels 1 through 6. Try a few examples for each problem. |
| 7a, 8a | 14–15, 150 |
| 7b, 8d | 38–41, 155 |
| 7c, 7e, 8c | 45–48 |
| 7d, 8b, 8e | 19–24 |

**goal** **Checkout**—adding and subtracting fractions

**page 326** Talk about the chart at the top of the page. It won't serve a useful purpose unless everyone knows how to read it. Only you know how much of a review your pupils need on finding a common denominator. Declare time-out for pupils who are having trouble. Use this chart to guide you in providing additional help.

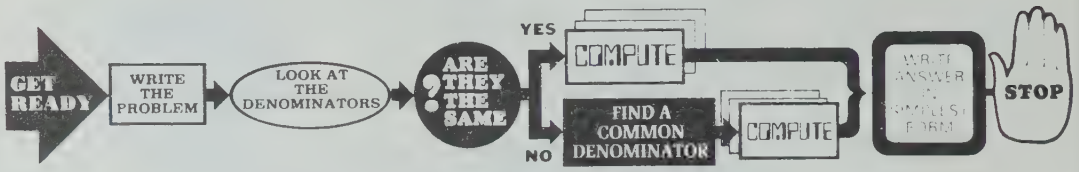
| Trouble with problem(s) | Go to page(s) |
|---|---|
| 1 | 177–178* |
| 2, 3 | 244, problem 2 |
| 4, 5 | 242, 245–246 |

*Level 5, pages 225–226, will also help

Praise pupils who were having trouble before and now show they have mastered these skills.

## Part III—What do you know about operations with fractions?

There is a step-by-step thinking process you use for both addition and subtraction of fractions.




Review how you can find a common denominator.

4 is not a factor of 6. What number has both 4 and 6 as factors?
You can multiply $4 \times 6$. 24 will work as a common denominator.
Or you can look again. You know 4 and 6 are factors of 12 also.
12 is the least common multiple you can use as a denominator.

Skill: Finding a common denominator

Find a common denominator for each pair of fractions.

1. **a** $\frac{1}{2}, \frac{1}{8}$ $\frac{4}{8}, \frac{1}{8}$ **b** $\frac{1}{4}, \frac{1}{3}$ $\frac{3}{12}, \frac{4}{12}$ **c** $\frac{1}{10}, \frac{1}{4}$ $\frac{2}{20}, \frac{5}{20}$ **d** $\frac{1}{6}, \frac{1}{4}$ $\frac{2}{12}, \frac{3}{12}$

Now compute. Watch the operation symbols.
Write the simplest name for each answer.

Skills: Adding and subtracting mixed numbers with like denominators

2. **a** $3\frac{5}{9} + 2\frac{2}{9}$ $5\frac{7}{9}$ **b** $2\frac{3}{5} + 1\frac{2}{5}$ $4$ **c** $1\frac{6}{7} + 1\frac{5}{7}$ $3\frac{4}{7}$ **d** $3\frac{7}{8} + 1\frac{5}{8}$ $5\frac{1}{2}$

3. **a** $3\frac{5}{9} - 2\frac{2}{9}$ $1\frac{1}{3}$ **b** $2\frac{3}{5} - 1\frac{2}{5}$ $1\frac{1}{5}$ **c** $1\frac{6}{7} - 1\frac{5}{7}$ $\frac{1}{7}$ **d** $3\frac{7}{8} - 1\frac{5}{8}$ $2\frac{1}{4}$

Skills: Adding and subtracting mixed numbers with unlike denominators

4. **a** $\frac{2}{5} + \frac{3}{10}$ $\frac{7}{10}$ **b** $3\frac{1}{3} + 2\frac{1}{6}$ $5\frac{1}{2}$ **c** $6\frac{3}{4} + 4\frac{1}{2}$ $11\frac{1}{4}$ **d** $5\frac{1}{6} + 1\frac{1}{2}$ $6\frac{2}{3}$

5. **a** $\frac{2}{5} - \frac{3}{10}$ $\frac{1}{10}$ **b** $3\frac{1}{3} - 2\frac{1}{6}$ $1\frac{1}{6}$ **c** $6\frac{3}{4} - 4\frac{1}{2}$ $2\frac{1}{4}$ **d** $5\frac{1}{6} - 1\frac{1}{2}$ $3\frac{2}{3}$

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Skill: Dividing fractions with unlike denominators

You use common denominators in division too. Show your skills.

**6.** **a** $\frac{1}{3} \div \frac{1}{6}$ 2 **b** $\frac{5}{8} \div \frac{1}{4}$ $2\frac{1}{2}$ **c** $\frac{3}{4} \div \frac{1}{3}$ $2\frac{1}{4}$ **d** $\frac{2}{5} \div \frac{1}{2}$ $\frac{4}{5}$

Skill: Multiplying fractions

Renaming the product is the hardest part of multiplication.
But you have had lots of practice. Prove you can do it.
Multiply. Write the simplest name for each product.

**7.** **a** $2 \times \frac{1}{2}$ 1 **b** $\frac{2}{3} \times 3$ 2 **c** $\frac{1}{4} \times \frac{3}{5}$ $\frac{3}{20}$ **d** $\frac{1}{3} \times \frac{3}{4}$ $\frac{1}{4}$

**8.** **a** $1\frac{1}{2} \times \frac{1}{4}$ $\frac{3}{8}$ **b** $\frac{4}{5} \times 1\frac{1}{4}$ 1 **c** $2\frac{1}{5} \times 1\frac{1}{2}$ $3\frac{3}{10}$ **d** $1\frac{2}{3} \times \frac{3}{5}$ 1

Skill: Computing with decimals

Operating with decimals is a joy.
Place value is the only thing you have to watch out for.

Add these.

**9.** **a** 2.26 + 1.82 = 4.08 **b** 62.7 + 1.3 = 64.0 **c** 27.608 + 3.521 + 681.671 = 712.800 **d** 9 + 0.01 + 6.7 = 15.71

Now subtract.

**10.** **a** 6.70 − 3.21 = 3.49 **b** 5.02 − 4.13 = 0.89 **c** 21.7 − 9.18 = 12.52 **d** 20 − 0.85 = 19.15

Multiply.

**11.** **a** 6.01 × 0.2 = 1.202 **b** 5.1 × 35 = 178.5 **c** 0.04 × 0.2 = 0.008 **d** 0.15 × 0.15 = 0.0225

And now divide.

**12.** **a** $5\overline{)6.25}$ 1.25 **b** $0.7\overline{)3.57}$ 5.1 **c** $0.10\overline{)500}$ 5000 **d** $8.2\overline{)21.32}$ 2.6



**goal** **Checkout**—dividing and multiplying fractions; computing with decimals

**page 327** Division of fractions requires a common denominator; multiplication does not. If this can cause a problem for some students, then do not assign problem 6 along with problems 7 and 8.

Computation with decimals parallels computation with whole numbers. The only problem is the placement of the decimal. Praise youngsters whose only error is in placing the decimal point for correct computation; then work on the placement skill.

| Trouble with problem(s) | Go to page(s) |
|---|---|
| 6 | 261–265 |
| 7a, b | 181–184 |
| 7c, d | 185–186 |
| 8 | 187 |
| 9a, b, c | 109–113, 316 |
| 9d | 114, 316 |
| 10a, b | 110–113, 316 |
| 10c, d | 114, 316 |
| 11a, b, c | 116–123 |
| 11d | 309 |
| 12 | 130–135 |

**goal** **Checkout** – multiplying decimals; finding perimeter and area; stating the probability of an event

**page 328** Two types of skills are necessary on this page:
- Using computational skills
- Solving problem situations

| Trouble with problem(s) | Go to page(s) |
|---|---|
| 1–3 | 29, 124 |
| 4 | 146–148, 154, 190, 318 |
| 5 | 155, 160 |
| 6 | 205 |

## Part IV–How well can you use what you know?

Skill: Multiplying decimals

Provincial tax on fuel varies.
Complete the following tax tables.

**1.**

| litres | tax | | |
|---|---|---|---|
| 1 | $0.05 | | |
| 2 | **a** | .10 | |
| 5 | **b** | .25 | |
| 10 | **c** | .50 | |
| 15 | **d** | .75 | |

**2.**

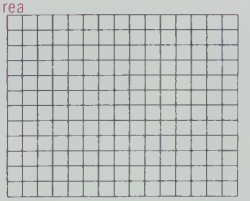
| litres | tax | |
|---|---|---|
| 1 | $0.0658 | |
| 2 | **a** | .1316 |
| 5 | **b** | .329 |
| 10 | **c** | .658 |
| 15 | **d** | .987 |

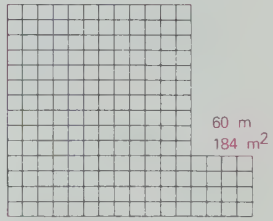
**3.**

| litres | tax | |
|---|---|---|
| 1 | $0.085 | |
| 2 | **a** | .17 |
| 5 | **b** | $.42\frac{1}{2}$ |
| 10 | **c** | .85 |
| 15 | **d** | $1.27\frac{1}{2}$ |

Skill: Finding perimeter and area

**4.** Find the perimeter of each figure. Then find the area. Let each square represent $1 m^2$.




Skill: Multiplying decimals

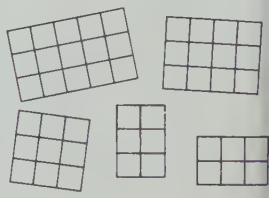
**5.** Suppose the figures above are really scale models of large offices. Each of the floors needs to be covered with carpet. The cost of carpet is $\$10.50/m^2$. What is the total cost if you cover both offices? $1890; $1932

Skill: Stating the probability of an event happening

**6.** Suppose the shapes on the right are in a paper bag.

You have this one.

Someone is to pick one of the shapes out of the bag for you. What is the probability that you would get a shape congruent to the one you already have? $\frac{0}{5}$ or 0



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# RESOURCES

## another form of evaluation

**for progress check**—page 309

The shop had a special on fresh fruit.
Compute the cost.

Grapes—price per kilogram: $0.69

| kilograms | | cost |
|---|---|---|
| 1.00 | 1. | $0.69 |
| 1.20 | 2. | $0.83 |
| 1.35 | 3. | $0.93 |
| 1.46 | 4. | $1.01 |
| 1.60 | 5. | $1.10 |
| 1.78 | 6. | $1.23 |

Tomatoes—price per kilogram: $0.49

| kilograms | | cost |
|---|---|---|
| 1.00 | 7. | $0.49 |
| 1.10 | 8. | $0.54 |
| 1.95 | 9. | $0.96 |
| 2.17 | 10. | $1.06 |
| 2.44 | 11. | $1.20 |
| 2.70 | 12. | $1.32 |

**for progress check**—page 313

Use page 313 again.

**for progress check**—page 321

What is the volume of a box that measures—

1. 1 m by 1 m by 1 m? 1 $m^3$
2. 2 m by 2 m by 2 m? 8 $m^3$
3. 12 cm by 6 cm by 2 cm? 144 $cm^3$
4. 10 cm by 5 cm by 5 cm? 250 $cm^3$
5. 15 cm by 4 cm by 10 cm? 600 $cm^3$
6. 2 m by 2 m by 1.5 m? 6.0 $m^3$

**for checkout**—pages 324-328

Part I—What do you know about numbers?

1. Name all the different digits used in our number system. 0 1 2 3 4 5 6 7 8 9
2. Using each digit only once, write the smallest whole number that can be written. 1 023 456 789
3. Write all the different 4-digit numbers that can be written with the digits 3, 4, 5, and 6. 3456, 3465, 3546, 3564, 3645, 3654 4356, 4365, 4536, 4563, 4635, 4653, 5346, 5364, 5436, 5463, 5634, 5643, 6345, 6354, 6435, 6453, 6534, 6543
4. Write the place value of each digit in this number: 8 307 654 921 8: billions, 3: hundred-millions, 0: ten-millions, 7: millions, 6: hundred-thousands, 5: ten-thousands, 4: thousands, 9: hundreds, 2: tens, 1: ones
5. Write the numbers in order from least to greatest.
   200 020; 202 000; 220 000; 200 002; 200 200; 200 002; 200 020; 200 200; 202 000; 220 000;
6. Are the following names for the same number? No
   $\frac{3}{4}$ $\frac{3}{6}$ $\frac{3}{11}$ $\frac{3}{9}$ $\frac{3}{5}$ $\frac{3}{10}$ $\frac{3}{8}$ $\frac{3}{7}$
   a) Write them in order from least to greatest. $\frac{3}{11} \cdot \frac{3}{10} \cdot \frac{3}{9} \cdot \frac{3}{8} \cdot \frac{3}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} \cdot \frac{3}{4}$
   b) Write the fraction of largest value that you can with a 3 in the numerator. $\frac{3}{1}$
7. Write these numbers in order from least to greatest. 0.002, 0.02, 0.2, 2.0, 2.02, 2.2, 2.22
   a) 0.002 0.02 2.22 0.2 2.0 2.2 2.02
   b) $\frac{2}{4}$ $\frac{7}{4}$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{8}{4}$ $\frac{1}{4}$ $\frac{4}{4}$ $\frac{1}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} \cdot \frac{4}{4} \cdot \frac{5}{4} \cdot \frac{7}{4} \cdot \frac{8}{4}$

Part II—What do you know about operations on whole numbers?

True or false?

1. If you add, multiply, or subtract any two whole numbers, you will always get a whole number. False
2. If you add zero to any whole number, you will always get zero. False
3. If you add any two whole numbers, change their order, and add again, you will always get the same sum. True
4. If you add any three whole numbers, you will get different sums, depending upon how you group the numbers. False
5. If you multiply any whole number by zero, you will always get zero. True
6. If you multiply any two whole numbers, change their order, and multiply again, you will get the same product. True
7. If you multiply any three whole numbers, you will get the same product no matter how you group the numbers. True
8. If you subtract zero from any whole number, your answer will always be the same as the whole number you chose. True
9. If you subtract any two whole numbers, change their order, and subtract again, you will always get the same answer. False
10. If you subtract any three whole numbers, you will get different answers depending upon how you group the numbers. True

Compute these.

| | (a) | (b) | (c) | (d) | (e) |
|---|---|---|---|---|---|
| 11. | 35 648 + 27 326 = 62 974 | 609 × 74 = 45 066 | 24)888 = 37 | 4056 − 3709 = 347 | 25)2536 = 101 R11 |
| 12. | 459 + 3741 + 87 = 4287 | 8054 − 6897 = 1157 | 60)6570 = 109 R30 | 1753 × 42 = 73 626 | 7000 − 4035 = 2965 |

Part III—What do you know about operations with fractions?

Remember the step-by-step thinking process you use for addition and subtraction of fractions.

- Write the problem.

- Look at the denominators. If they're the same, you're in good shape. If they're not the same, you must find a common denominator.
- Compute.
- Make sure your answer is in simplest form.

Practice. Find a common denominator for each pair of fractions.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 1. | $\frac{1}{3}, \frac{1}{6}$ | $\frac{2}{6}, \frac{1}{6}$ | $\frac{1}{2}, \frac{1}{3}$ | $\frac{3}{6}, \frac{2}{6}$ $\frac{1}{9}, \frac{1}{6}$ | $\frac{2}{18}, \frac{3}{18}$ | $\frac{1}{3}, \frac{1}{8}$ | $\frac{8}{24}, \frac{3}{24}$ |

Now compute. Watch the operation symbols. Write the simplest name for each answer.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 2. | $4\frac{2}{7} + 3\frac{3}{7}$ | $1\frac{5}{6} + 3\frac{1}{6}$ | $3\frac{8}{9} + 2\frac{2}{9}$ | $1\frac{7}{10} + 1\frac{9}{10}$ |
| | $7\frac{5}{7}$ | 5 | $6\frac{1}{9}$ | $3\frac{3}{5}$ |
| 3. | $3\frac{4}{5} - 1\frac{2}{5}$ | $4\frac{5}{6} - 2\frac{1}{6}$ | $5\frac{6}{7} - 2\frac{3}{7}$ | $4\frac{7}{9} - 1\frac{4}{9}$ |
| | $2\frac{2}{5}$ | $2\frac{2}{3}$ | $3\frac{3}{7}$ | $3\frac{1}{3}$ |
| 4. | $\frac{1}{4} + \frac{3}{8}$ | $2\frac{1}{4} + 4\frac{1}{12}$ | $3\frac{1}{3} + 5\frac{5}{6}$ | $4\frac{1}{10} + 1\frac{1}{2}$ |
| | $\frac{5}{8}$ | $6\frac{1}{3}$ | $9\frac{1}{6}$ | $5\frac{3}{5}$ |
| 5. | $\frac{3}{4} - \frac{5}{8}$ | $4\frac{1}{3} - 2\frac{1}{12}$ | $5\frac{5}{6} - 3\frac{1}{3}$ | $4\frac{1}{10} - 1\frac{1}{2}$ |
| | $\frac{1}{8}$ | $2\frac{1}{4}$ | $2\frac{1}{2}$ | $2\frac{3}{5}$ |

You use common denominators in division too. Show your skills.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 6. | $\frac{1}{4} \div \frac{1}{8}$ 2 | $\frac{5}{6} \div \frac{1}{3}$ $2\frac{1}{2}$ | $\frac{4}{5} \div \frac{1}{4}$ $3\frac{1}{5}$ | $\frac{2}{7} \div \frac{1}{2}$ $\frac{4}{7}$ |

Renaming the product is the hardest part of multiplication. But you have had lots of practice. Prove you can do it. Multiply. Write the simplest name for each product.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 7. | $3 \times \frac{1}{3}$ 1 | $\frac{3}{4} \times 4$ 3 | $\frac{1}{5} \times \frac{2}{3}$ $\frac{2}{15}$ | $\frac{1}{4} \times \frac{4}{5}$ $\frac{1}{5}$ |
| 8. | $1\frac{1}{3} \times \frac{1}{7}$ $\frac{4}{21}$ | $\frac{6}{7} \times 1\frac{1}{6}$ 1 | $2\frac{1}{2} \times 1\frac{1}{6}$ $2\frac{11}{12}$ | $2\frac{1}{4} \times \frac{4}{9}$ 1 |

Operating with decimals is a job. Place value is the only thing you have to watch out for. Add these.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 9. | 3.42 | 74.1 | 74.323 | 7+0.04+8.9 |
| | +2.65 | + 3.9 | 8.601 | 15.94 |
| | 6.07 | 78.0 | +432.576 | |
| | | | 515.500 | |

Now subtract.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 10. | 9.80 | 7.04 | 47.6 − 9.27 | 30−0.64 |
| | −4.63 | −6.58 | 38.33 | 29.36 |
| | 5.17 | 0.46 | | |

Multiply.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 11. | 5.03 | 6.3 | 0.03 | 0.17 |
| | × 0.3 | × 32 | × 0.03 | × 0.17 |
| | 1.509 | 201.6 | 0.009 | 0.0289 |

And now divide.

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| 12. | 1.84 | 7.1 | 2000 | 3.6 |
| | 4)7.36 | 0.6)4.26 | 0.20)400 | 7.4)26.64 |

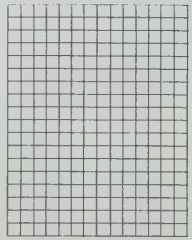
Part IV—How well can you use what you know?

Complete these tables for tax on fuel.

| 1. Litres | Tax | 2. Litres | Tax | 3. Litres | Tax |
|---|---|---|---|---|---|
| 1 | $0.07 | 1 | $0.0742 | 1 | $0.095 |
| 2 | a) $0.14 | 2 | a) $0.1482 | 2 | a) $0.19 |
| 5 | b) $0.35 | 5 | b) $0.371 | 5 | b) $0.475 |
| 10 | c) $0.70 | 10 | c) $0.742 | 10 | c) $0.95 |
| 15 | d) $1.05 | 15 | d) $1.113 | 15 | d) $1.425 |

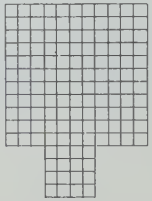
4. Find the perimeter of each figure. Then find the area. Let each square represent 1 $m^2$.

a)



Perimeter: 62 m
Area: 234 $m^2$

b)



Perimeter: 52 m
Area: 137 $m^2$

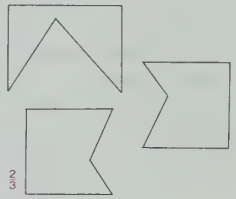
5. Suppose the figures above are scale models of rooms. Each room needs to be covered with a rug. The cost of a rug is $7.90/$m^2$. What would you pay for each of the rugs?

a) $1848.60 b) $1082.30

6. Suppose the shapes on the right are in a paper bag.

You have this one.

Someone is to pick one of the shapes out of the bag for you. What is the probability that you would get a shape congruent to the one you already have? $\frac{2}{3}$



## activities

**1.** Individual activity (Provide the pupil with the following directions.)

Place the decimal point in each numeral so that each statement makes sense.

1. The length of my mathematics book is 2 5 0 cm. 25.0
2. The classroom is 3 7 5 m high. 3.75
3. A ticket to the game costs $3 5 0. 3.50
4. My father has a mass of 8 7 5 kg. 87.5
5. The length of my pencil is 1 9 1 cm. 19.1
6. The decimal equivalent for $1\frac{1}{4}$ is 1 2 5. 1.25
7. The average amount of snowfall per year for Calgary, Alberta, is 1 5 3 9 cm. 153.9
8. Allen's time walking in the 4 km race was 2 3 5 min. 23.5

**2.** The product for multiplying 4.3 by 7.8 can be found on a slide rule. **But**, a slide rule gives only the digits in the product—not including the zeros on the right. The person using the slide rule must locate the decimal point properly.

Here are some problems and a slide-rule reading for each product. Your job—place the decimal point in the slide-rule reading so that the product is reasonable.

| | Problem | Slide-rule Reading |
|---|---|---|
| 1. | $4.3 \times 2.7$ | 1 1 6 (11.6) |
| 2. | $3.76 \times 18.2$ | 6 8 4 (68.4) |
| 3. | $11.5 \times 4.33$ | 4 9 8 (49.8) |
| 4. | $4.2 \times 19$ | 7 9 8 (79.8) |
| 5. | $0.51 \times 8.07$ | 4 1 2 (4.12) |
| 6. | $1.5 \times 4.3$ | 6 4 5 (6.45) |
| 7. | $7.5 \times 0.15$ | 1 1 2 5 (1.125) |
| 8. | $2.4 \times 35.7$ | 8 5 7 (85.7) |
| 9. | $20 \times 3.3$ | 6 6 (66.) |

**3.** Memorization of all the equivalent measures is a great deal to expect of any child. If he doesn't know them, he certainly should know all the places that such a table can quickly be found—dictionary, almanac, cookbook, and so on. A math book isn't always available. Go on a "where to find it" hunt. Make sure everyone knows how to use the tables that are found.

Looking for the way measures are marked on packages will continue to make the pupil aware of the importance of measurements and how required markings serve to protect the consumer.

**4.** Hunting for little-known units of measure can be fun. Specialized units of measure are used in certain regions of the country. **Barrel**, for example, is a standard unit of measure for fruits and vegetables. Would you believe that a barrel of cranberries is the same as $86\frac{45}{64}$ dry quarts, but all other barrels of fruit are the same as 105 dry quarts? You can't win with generalizations. A **bale** is not in any table of measures, but there are many people whose income is based on bales of cotton or hay.

**5. things** empty boxes; rulers; envelopes

Grab every empty box in sight. Put them in a corner of the room with rulers and a sign: FIND MY VOLUME. Attach an envelope to each box. Have each pupil write his name and answer on a slip of paper and then put the paper in an envelope. Wait a couple of days. Check the responses and crown the Champion Volume Computer. Somehow Champion Capacity Computer sounds like more fun, but make sure the kids know the relationship of volume and capacity.

**6.** Which would you need to find—*volume*, *area*, or *perimeter*—in order to solve the following problems?

1. How much water is needed to fill a swimming pool? (Volume)
2. How much tile is needed for a floor? (Area)
3. How much dirt will a truck hold? (Volume)
4. How much fencing is needed to fence a playground? (Perimeter)
5. What size belt do I need? (Perimeter)
6. How large is the canvas that covers a baseball diamond? (Area)
7. How much gas will a tank hold? (Volume)
8. How much concrete is needed for a new sidewalk? (Volume)
9. How much material is needed to make a picture frame? (Perimeter)
10. How many shingles are needed to roof a house? (Area)

**7.** Set M provides practice in renaming for multiplication, set D in renaming for division. Using a table of measures is O.K.

**Set M**

1. 5 hr. 125 min. = 7 hr. ___?___ min. (5)
2. 8 lb. 52 oz. = 11 lb. ___?___ oz. (4)
3. 9 ft. 30 in. = ___?___ ft. 6 in. (11)
4. 9 m 210 cm = ___?___ m 10 cm (11)
5. 5 yd. 10 ft. = 8 yd. ___?___ ft. (1)
6. 10 qt. 5 pt. = ___?___ qt. ___?___ pt. (12 qt. 1 pt.)

**Set D**

1. 2 hr. 15 min. = ___?___ min. (135)
2. 8 lb. 4 oz. = ___?___ oz. (132)
3. 5 ft. = ___?___ in. (60)
4. 3 m 20 cm = ___?___ cm (320)
5. 6 yd. 2 ft. = ___?___ ft. (20)
6. 4 qt. 1 pt. = ___?___ pt. (9)

**8.** How many boxes 2 in. × 3 in. × 1 in. will fit inside a box that is 6 in. × 9 in. × 2 in.? (18) If you need a clue, answer these questions:

- What is the volume of the larger box?
- What is the volume of the smaller box?
- How many of the smaller boxes will it take to occupy the same amount of space as the larger box?

Here are some more challenges:

1. How many boxes 1 in. × 6 in. × 3 in. will fit inside a box that is 4 in. × 6 in. × 6 in.? (8)
2. How many boxes 2 in. × 3 in. × 2 in. will fit inside a cube 6 inches on each side? (18)

**9.** Independent activity. Have each pupil select a stock he would like to buy and follow its performance for a week. The changes can be recorded in a graph. The change in actual cash value, which is reported in mixed numbers, can also be computed. Did the person who bought a share on Monday make money or lose money during the week?

Pupils who are interested can extend the period of time.

## additional learning aids

**operation**—chapter objectives 10, 11

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: P–15, 16, 17, 18, 19
*Computational Skills Development Kit*, SRA (1965)
Whole Number cards: all
Fraction cards: all
Decimal cards: all
*Cross-Number Puzzles* (*Fractions*), SRA (1965)
Cards: all
*Cross-Number Puzzles* (*Whole Numbers*), SRA (1965)
Cards: all
*Math Applications Kit*, SRA (1971)
Social Studies card: 47
Everyday Things cards: 13, 15, 20, 21, 24, 39
*Mathematics Involvement Program*, SRA (1971)
Cards: 85, 96, 296
*Skill Modes in Mathematics*, SRA (1974)
Level I, Molecules: A, B, C, D, E, F, I, J
Level II, Molecules: A, C, D, E, F, G, I, J, K, L, N, O, P
Level III, Molecules: D, M, Q, S, T, U, V, W, X
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 57, 58, 60, 71, 72

**measurement**—chapter objectives 1, 2, 3, 4, 5, 6, 7, 8

**SRA products**

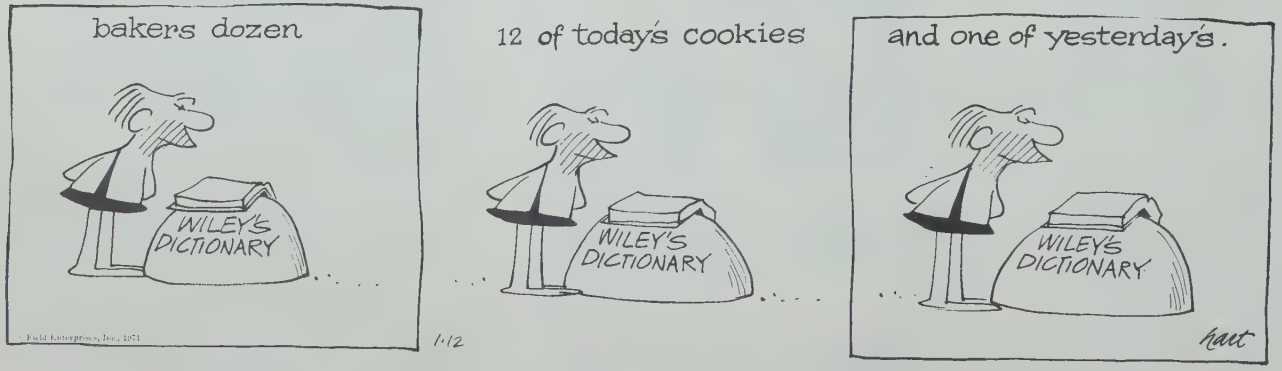
*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: M–4, 6, 8, 9, 10, 11, 13; P–4, 13, 14
*Math Applications Kit*, SRA (1971)
Appeteasers card: 7
Science cards: 21, 27, 28, 31, 32, 33, 36, 42, 43, 46, 48, 49
Sports and Games cards: 11, 22, 27, 29, 40, 41
Occupations cards: 7, 20, 29, 30, 38, 41
Social Studies card: 47
Everyday Things cards: 3, 9, 10, 11, 14, 26, 41, 42
*Mathematics Involvement Program*, SRA (1971)
Cards: 325, 356
*Skill through Patterns, level 6*, SRA (1974)
Spirit masters: 40, 41, 52, 53

**other learning aids** (described on page 336i)—The Fatal Foot, The Ghastly Gallon, The Perilous Pound, Polygons

**measurement of events**—chapter objective 9

**SRA products**

*Mathematics Learning System, Activity Masters, level B*, SRA (1974)
Spirit masters: S–4, 5, 7
*Math Applications Kit*, SRA (1971)
Occupations cards: 32, 33
Social Studies card: 17
Everyday Things card: 12

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my name_____ my teacher's name_____

| | I can do it every time. | I will be able to do it soon. |
|---|---|---|
| I can round a number like 1 256 439 821 to any place you name. | | |
| I can tell the value of each digit in a number as great as 327 046 921 845. | | |
| I can estimate the answer and find the exact answer to problems like these:<br>a) $39\overline{)726}$ b) $21\overline{)476}$ c) $93\overline{)4621}$ d) $52\overline{)9282}$ | | |
| I can also estimate the answer to problems like these:<br>a) 359 + 782 b) 4209 − 1975 c) 32 × 5 d) 631 × 12 | | |
| I can find the answer to a word problem like the one below and then write a math sentence to show that my answer is correct.<br>Mrs. Davis drove 12 km back and forth to work 6 days a week. How far did she drive in 28 days? How far in 310 days? | | |
| I can write these numbers in order from least to greatest:<br>a) 2.704 2.71 2.695 2.701 | | |
| b) $\frac{3}{10}$ $\frac{31}{100}$ $\frac{307}{1000}$ | | |
| If you ask me to write a decimal like 1 and 4 tenths, 23 hundredths, or 6 thousandths, I can do it and tell the value of each digit. | | |
| I can write decimals like these as fractions:<br>a) 0.03 b) 0.4 c) 0.23 d) 0.047 | | |
| I can find answers to problems like these:<br>a) 1.2 + 0.8 b) 0.36 + 0.76 c) 1.05 − 0.76 d) 3. − 0.42 | | |
| e) 0.3 × 0.4 f) 35.04 × 5 g) 2.8 × 0.2 h) 0.23 × 0.7 | | |
| i) $2\frac{1}{2} \times 2$ j) $1\frac{1}{3} \times 1\frac{1}{3}$ k) $1\frac{1}{4} \times 2\frac{4}{5}$ l) $4\frac{1}{3} \times 6\frac{3}{4}$ | | |
| m) $5 \div \frac{3}{4}$ n) $3 \div 1$ o) $\frac{1}{4} \div \frac{1}{3}$ p) $6 \div 1\frac{1}{2}$ | | |

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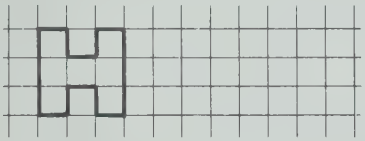
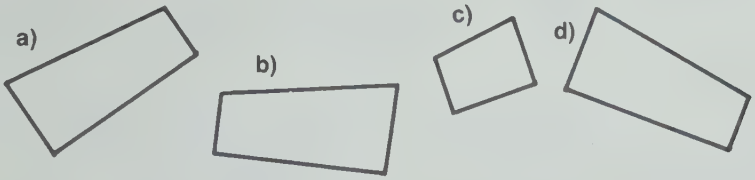
I can tell whether two figures have the same size and shape by tracing one and matching the tracing to the other figure. . . . . _____ _____

I can make a figure that is the same size and shape as this one.



. . . . . _____ _____

I can tell which of these figures are the same size and shape.



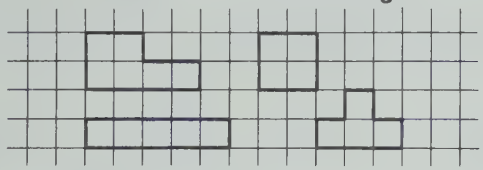

_____ _____

I can tell which of these units are smaller than a litre and which are larger than a litre:
millilitre decalitre centilitre hectolitre decilitre kilolitre . _____ _____

I can tell units smaller than a metre and those larger than a metre. . . . . _____ _____

I can tell units smaller than a gram and those larger than a gram. . . . . _____ _____

I can find the area of each of these figures:



. . . . . _____ _____

I can roll a die 50 times and show on a chart or bar graph how many times each number lands faceup. . . . . _____ _____

I can look at the spinner for a game and tell you the probability of landing on each section. . . . . _____ _____

**my name**_____

**date**_____

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individual progress chart for _____
(pupil's name)

recorded by _____ – sixth-level teacher

| evaluation of major learning goals | date of 1st try | date of 2d try | comments |
|---|---|---|---|
| Knows how to estimate and find the sum for two or three 3-digit numbers and two 4-digit numbers (p. 7, 13, 15) | | | |
| Knows how to subtract a 2- or 3-digit number from a 3-digit number (p. 24) | | | |
| Can round to the nearest ten or hundred (p. 27*) | | | |
| Can estimate sums and differences (p. 27*) | | | |
| Knows how to add and subtract two 3-digit numbers (p. 27*) | | | |
| Knows how to multiply a 2- or 3-digit number by a 2-digit number or a multiple of 100 (p. 39, 41) | | | |
| Knows how to divide a 3- or 4-digit number by a 1- or 2-digit divisor (p. 46, 48) | | | |
| Knows how to subtract two 4-digit numbers (p. 52*) | | | |
| Can multiply a 2- or 3-digit number by a 2-digit number (p. 52*) | | | |
| Knows how to divide by a 1- or 2-digit divisor (p. 52*) | | | |
| Can identify congruent figures (p. 64, 72*) | | | |
| Can tell the value of each digit of a decimal (p. 83) | | | |
| Knows how to estimate and find the exact sum of two decimals (p. 87) | | | |
| Can write a decimal when a number is given in word form (p. 100*) | | | |
| Can write a decimal for a fraction (p. 83, 100*) | | | |
| Can write a fraction for a decimal (p. 83, 100*, 126) | | | |
| Can round a decimal to the nearest hundredth, tenth, and whole number (p. 87, 100*, 126) | | | |
| Can write a standard numeral for an exponent (p.96, 100) | | | |
| Can order a set of decimals (p. 87, 106*, 126) | | | |
| Knows how to add and subtract two decimals with renaming (p. 115, 126, 144*) | | | |
| Knows how to multiply two decimals (p. 126, 144*) | | | |
| Knows how to divide by a decimal divisor (p. 135, 144*) | | | |
| Can find area and perimeter of figures on graph paper (p. 165*) | | | |
| Knows how to add 2- and 3-digit numbers (p. 165*) | | | |
| Knows how to multiply 2- and 3-digit numbers (p. 165) | | | |
| Knows how to add and subtract fractions with like or unlike denominators (p. 179) | | | |
| Knows how to multiply a whole number and a fraction, two fractions, two mixed numbers (p. 189, 192*) | | | |
| Can show data in a tally chart and make a bar graph (p. 216*) | | | |
| Can write probability of equally likely outcomes (p. 216*) | | | |
| Can identify and measure angles (p. 238*) | | | |
| Knows how to add and subtract fractions and mixed numbers with unlike denominators and renaming (p. 247, 268*) | | | |
| Knows how to divide by a fraction divisor (p. 265, 268*) | | | |
| Can identify congruent shapes (p. 288*) | | | |
| Can order a set of integers (p. 297) | | | |
| Can solve simple word problems involving integers (p. 304*) | | | |
| Can tell whether a unit of measure with a prefix is larger or smaller than related unit (p.313) | | | |
| Can find volume of a rectangular prism (p. 325) | | | |
| Knows how to operate with measurements (p. 329) | | | |

*Checkout

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# Other Learning Aids

## whole-number notation

**Abacus board** (Creative Publications) Counting board useful for teaching place value
**Chip Trading** (Scott Scientific) Game to develop an understanding of place value
**Place Value I and II** (Creative Publications) Self-correcting cards to provide practice in reading numbers through hundred millions

## whole-number operations

**Dial-A-Matic Adding Machine** (Sigma Scientific) A simple calculator for practice in addition and subtraction
**Dividing Machine** (Developmental Learning Materials) Self-checking machine to be used for practice with the division facts
**Good Time Mathematics** (Holt, Rinehart & Winston) A multimedia program to give activity-based learning experiences
**I Win** (Scott, Foresman) [sets 1, 2, and 3] Card game for practice in basic operations
**Japanese Abacus** (Creative Publications) An abacus for place value and basic operations
**Mathfacts Games** (Milton Bradley) Self-checking games for multiplication and division facts
**Napier's Rods** (Sigma Scientific) Rods for practice in multiplication
**Numble** (Sigma Scientific) Crossword-type number game for basic operations
**Orbiting the Earth** (Scott, Foresman) Game to provide practice in all four operations
**Prime-O** (Creative Publications) Card game to provide practice in prime factorization
**Rally with Remainders** (Math Shop) A self-correcting game providing division practice
**Sequence** (Math Shop) Puzzle-type game that uses addition combinations to find patterns
**Ting** (SEE) A jigsaw puzzle for multiplication
**Triscore** (Creative Publications) Games for practice in the basic operations
**Veri-Tech Senior** (ETA) [addition, subtraction, and multiplication books] A self-checking device that provides practice with operations
**Winning Touch** (Ideal) Game for reinforcement of multiplication facts

## fractional-number notation

**Decimal Fraction Dominoes** (Mind/Matter Corp.) Game for practice in recognizing relationships of fractions and decimals
**Decimal/Fraction Matching Cards** (SEE) Cards to aid in learning about decimals and fractions
**Experiments in Fractions** (Math Shop) Activities for notation and operations
**The Fat Fraction Game** (ETA) Card game for drill in simplifying fractions
**Fraction Bars Student Activity Book** (Creative Publications) Games and activities to teach fractions and their operations
**Fraction Dominoes** (SEE) Game involving matching a fractional numeral with its model
**Fraction Line Set** (Sigma Scientific) Activity to help visualize operations by computing with fraction strips
**Fraction Tally** (Math Shop) Game for practice in addition and subtraction
**Fractional Number Cards** (Math Shop) Cards used with a geoboard for finding equivalent fractions
**Tripletts** (Math Shop) A rummy-type card game for identifying equivalent fractions

## fractional-number operations

**Action Fraction Games** (Constructive Playthings) Game to develop concepts and skills
**Fraction Multifax** (Math Shop) Game to reinforce multiplication of fractional numbers
**Mathimagination** (Math Shop) [book D—Fractions] Puzzles to reinforce operations

## geometry

**Geoboard Activity Cards** (Creative Publications) [intermediate set] Activities for the geoboard
**Geoboard Kit** (Cuisenaire) Plastic geoboards and activity cards that show geometric concepts
**Geoboards and Motion Geometry** (Scott, Foresman) Resource book dealing with congruence, coordinates, transformations, and area
**Great Shapes** (Cuisenaire) Game to develop principles of patterns, symmetry, and so on
**Learn to Fold—Fold to Learn** (Lyons & Carnahan) Workbook of paper-folding activities to demonstrate geometric figures and symmetry
**Mira** (Creative Publications) An aid for investigating properties of plane geometry
**Mira Math for Elementary School** (Creative Publications) Activities for the Mira
**Mirror Magic** (Lyons & Carnahan) Workbook activities for exploring the concept of symmetry
**Paper and Pencil Geometry** (Lyons & Carnahan) Activities to develop geometric concepts
**Polygons** (Math Shop) Cards for understanding of basic properties of geometric figures
**Rotation, Translation and Reflection Kit** (Invicta) Activity cards dealing with motion geometry
**Shape Tracers** (Math Shop) Set of basic geometric shapes
**Tangrams** (Creative Publications) Puzzle to aid in the discovery of geometric properties
**Tangramath** (Creative Publications) Book of tangram shapes to assist in learning concepts of shapes, congruence, similarity, and area

## measurement

**The Fatal Foot** (Math Shop) Game for drill in addition of linear measures
**Geometric Ruler** (Math Shop) Folding rule to demonstrate perimeter-area relationships
**The Ghastly Gallon** (Math Shop) Game for practice in operations with liquid measures
**Introducing the Metric System with Activities** (Math Shop) Activities to develop basic understanding of the metric system
**Learning about Measurement** (Lyons & Carnahan) A workbook of activities using the metric and customary systems
**Metric Place Value Chart** (Ideal) Chart for meaning of metric system of measures
**The Perilous Pound** (Math Shop) Game for practice in regrouping weight measures

## statistics and probability

**Block Graph** (ESA) Demonstration set for introducing bar graphs, averages, and so on
**Histogram Board** (ESA) Board for making bar diagrams—to be used with Stern Unit Cubes
**Making and Using Graphs and Nomographs** (Lyons & Carnahan) A workbook for skills in making and reading graphs
**Probability Maze** (ESA) A board to illustrate probability and statistics
**Probability Set** (Invicta) Apparatus to demonstrate simple probability
**Towards Probability** (Cuisenaire) Book of easy experiments dealing with probability

## problem solving and applications

**Foo** (Cuisenaire) Card game for order of operation in open math sentences
**Heads Up** (Creative Publications) Game providing practice with equations
**True or False** (ESA) Game for deciding true or false statements

# Bibliography

## the study of numbers

### whole-number concepts

#### children's books


**Adler, Irving and Ruth.** *Numbers Old and New.* New York: John Day, 1960. (4–5)

———. *Numerals: New Dresses for Old Numbers.* New York: John Day, 1964 (4–5)

**Alexander, Arthur.** *The Magic of Words.* Englewood Cliffs, N. J.: Prentice-Hall, 1962. (5)

**Bendick, Jeanne, and Levin, Marcia.** *Take a Number.* New York: McGraw-Hill, 1961. (4–6)

**Carona, Philip B.** *True Book of Numbers.* Chicago: Childrens Press, 1964. (4)

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**Computer Series 1: An Introduction to Computers:* "History of Computing Devices." Color w-cassettes. BFA Educ. Media. (4–6)

*Macmillan Math Film Loops:* "Exponents," "General Order Relations," "Place Value." Super-8mm cartridges. Macmillan. (4–6)

*The Magic of a Counter.* 16mm, sound, color. BFA Educ. Media. (4–6)

*Modern Mathematics: Number Sentences.* 16 mm, sound, color. BFA Educ. Media. (4–6)

**Using Modern Mathematics,* Group 4: "Number Line—Whole Numbers," "Numeration: Base Ten." Color w/captions. Singer/SVE. (4–6)

**Using Modern Mathematics,* Group 5: "Numeration: Base Five." Color w/captions. Singer/SVE. (6)


### whole-number operations

#### children's books


**Grant, Eldon.** *Twenty White Horses, A Book of Division.* New York: Holt, Rinehart & Winston, 1964. (4–5)

**Jonas, Arthur.** *More New Ways in Math.* Englewood Cliffs, N. J.: Prentice-Hall, 1964. (5)

**Pink, Heinz-Guenther.** *Multiplication Hula.* Honolulu: Institute of Simplified Mathematics, 1972. (5)

**Whitney, David C.** *The Easy Book of Division.* New York: Watts, 1970. (4–6)

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———. *Let's Find Out About Subtraction.* New York: Watts, 1968. (4)


#### films, filmstrips* and slides


*Harbrace Mathematics Instructional Slides:* "Addition and Subtraction of Whole Numbers," "Multiplication and Division of Whole Numbers." Cartridges of 140 slides, color w/captions. Harcourt Brace Jovanovich. (1–6)

*Macmillan Math Film Loops:* "Addition," "Division," "Inverse Operations (Doing and Undoing)," "Models: When to Add or Subtract," "Models: When to Multiply or Divide," "Multiplication," "Subtraction." Super-8mm cartridges. Macmillan. (4–6)

**Stumbling Blocks in Arithmetic:* "Regrouping in Subtraction," "The Two-Place Divisor," "The Two-Place Multiplier." Color w/records or cassettes. Pathescope. (4–6)

**Using Modern Mathematics,* Group 4: "Division Facts—Sets," "Multiplication Facts—Sets." Color w/captions. Singer/SVE. (4–6)


### fractional-number concepts

#### children's books


**Dennis, J. Richard.** *Fractions Are Parts of Things.* New York: Thomas Y. Crowell, 1971. (5)

**Whitney, David C.** *The Easy Book of Fractions.* New York: Watts, 1970. (4–6)


#### films, filmstrips* and slides


*Elementary Mathematics for Students:* "Between the Whole Numbers," "Equivalent Fractions," "Comparing Rational Numbers." 16mm, sound, color. Developed by NCTM, distributed by Silver Burdett. (4–6)

**Fractions: A New Approach,* Group 1: "Equivalent Fractions," "Fractions Equal to, or Greater Than One," "Simplifying Fractions," "What Are Fractions?" Group 2: "Order of Fractional Numbers," "The Properties of Operation, Part 1," "The Properties of Operation, Part 2." Color w/records or cassettes. Singer/SVE. (4–6)

*Harbrace Mathematics Instructional Slides:* "Rational Numbers." Cartridge of 140 slides, color w/captions. Harcourt Brace Jovanovich. (1–6)

*Macmillan Math Film Loops:* "The Concept of Fractional Numbers," "Fractional Parts." Super-8mm cartridges. Macmillan. (4–6)

**Stumbling Blocks in Fractions:* "Equivalent Fractions," "The Language of Fractions." Color w/cassettes. Pathescope. (4–6)

**Using Modern Mathematics,* Group 4: "Fractions," "Number Line—Fractions." Group 5: "Fraction Numerals: Concepts." Color w/captions. Singer/SVE. (4–6)


### fractional-number operations

#### films, filmstrips* and slides


*Elementary Mathematics for Students:* "Adding with Fractions," "Adding with Mixed Numerals," "Dividing with Decimals," "Equivalence Classes in Addition," "Multiplying with Decimals," "The Remainder in Division," "Subtracting with Fractions," "Subtracting with Mixed Numerals." 16mm, sound, color. Developed by NCTM, distributed by Silver Burdett. (4–6)

**Fractions: A New Approach,* Group 1: "Addition and Subtraction of Fractional Numbers," "Addition and Subtraction of Mixed Numerals." Group 2: "Division of Fractions and Mixed Numerals." Color w/records or cassettes. Singer/SVE. (4–6)

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**Using Modern Mathematics,* Group 5: "Addition and Subtraction of Fractions," "Multiplication of Fractions." Color w/captions. Singer/SVE (4–6)

# geometry

## children's books

**Adler, Ruth and Irving.** *Directions and Angles.* New York: John Day, 1969. (4–6)
**Bendick, Jeanne, and Levin, Marcia.** *Take Shapes, Lines and Letters.* New York: McGraw-Hill, 1962. (4–5)
**Diggins, Julia E.** *String, Straightedge and Shadow:* The Store of Geometry, New York: Viking, 1965. (4–6)
**Hogben, Lancelot.** *The Wonderful World of Mathematics,* rev. ed. New York: Doubleday, 1968. (4–6)
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**Murray, William D., and Rigney, Francis J.** *Paper Folding for Beginners.* New York: Dover, 1960. (5)
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———. *Symmetry.* New York: Doubleday, 1968. (5–6)
**Russell, Solveig Paulson.** *Lines and Shapes.* New York: Walck, 1965. (4–5)
**Sitomer, Mindel and Harry.** *Lines, Segments, Polygons.* New York: Thomas Y. Crowell, 1971. (4)
———. *What Is Symmetry?* New York: Thomas Y. Crowell, 1970. (4–5)

## films, filmstrips* and slides

*Harbrace Mathematics Instructional Slides:* "Geometry, Measurement, and Graphing." Cartridge of 140 slides, color w/captions. Harcourt Brace Jovanovich. (1–6)
*Macmillan Math Film Loops:* "Circles," "Congruence (Same Size, Same Shape)," "Similarity (Same Shape)," "Symmetry," "Topology (Inside, Outside, On)." Super-8mm cartridges. Macmillan. (4–6)
**Using Modern Mathematics,* Group 4: "Geometry: Sets, Rays, Angles, Figures." Color w/captions. Singer/SVE. (5)
**Using Modern Mathematics,* Group 5: "Geometry: Perimeters, Areas, Space Figures." Color w/captions. Singer/SVE. (6)

# measurement

## children's books

**Adler, Irving.** *The Giant Golden Book of Mathematics.* New York: Golden Press, 1960. (4–6)
**Asimov, Isaac.** *Realm of Measure.* Boston: Houghton Mifflin, 1960. (4–5)
**Bendick, Jeanne.** *How Much and How Many: The Story of Weights and Measures.* New York: McGraw-Hill, 1947. (4–6)
———. *Measuring.* New York: Watts, 1971. (4–6)
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**Simon, Leonard.** *Stretching Numbers.* New York: Holt, Rinehart & Winston, 1964. (4)
**Weyl, Peter K.** *Men, Ants, and Elephants: Size in the Animal World.* New York: Viking, 1959. (4–6)

## films, filmstrips* and slides

*Elementary Mathematics for Students:* "The Biggest Rectangle," "Hidden Treasure." 16mm, color, sound. Developed by NCTM, distributed by Silver Burdett. (4–6)
*Macmillan Math Film Loops:* "Area," "Measuring Lengths," "Using a Protractor," "Volume." Super-8mm cartridges. Macmillan. (4–6)
**The Metric System.* Set of six, color w/cassettes or records. Pathescope. (4–6)
**Using Modern Mathematics,* Group 5: "Using Measures." Color w/captions. Singer/SVE (4–6)

# statistics and probability

## children's books

**Linn, Charles F.** *Probability.* New York: Thomas Y. Crowell, 1972. (4–6)
**Lowenstein, Dyno.** *First Book of Graphs.* New York: Watts, 1969. (5–6)
**Razzell, Arthur G., and Watts, K. G.** *Probability—The Science of Chance.* New York: Doubleday, 1967. (5–6)

## films, filmstrips* and slides

*Predicting Through Sampling.* 16 mm, sound, color. BFA Educ. Media. (4–6)
*The Probabilities of Zero and One.* 16mm, sound, color. BFA Educ. Media. (4–6)
*Probability: An Introduction.* 16mm, sound, color. BFA Educ. Media. (4–6)
**Using Modern Mathematics,* Group 5: "Graphs: Pictographs, Bar, Line, Number Pairs, Maps." Color w/captions. Singer/SVE. (4–6)

# problem solving

## children's books

**Barr, George.** *Entertaining with Number Tricks.* New York: McGraw-Hill, 1971. (4–6)
**Brooke, Maxey.** *One Hundred and Fifty Puzzles in Crypt-Arithmetic.* New York: Dover, 1972. (5–6)
**Charosh, Mannis.** *Mathematical Games for One or Two.* New York: Thomas Y. Crowell, 1972. (4)
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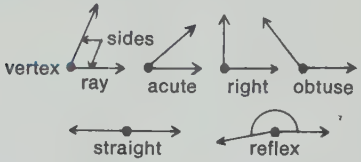
**Kohn, Bernice.** *Secret Codes and Ciphers.* Englewood Cliffs, N. J.: Prentice-Hall, 1968. (6)
**Linn, Charles F.** *Estimation.* New York: Thomas Y. Crowell , 1972. (4–6)

# GLOSSARY

**angle** Two rays with a common endpoint



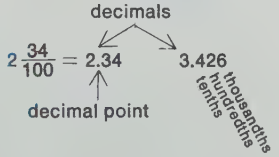

**area** The amount of surface inside a plane figure

Area is 8 square units.

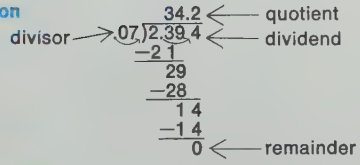
**congruent** Two figures are congruent if they have the same size and shape.

**curve** A figure drawn without lifting the pencil

**decimal** Another name for a fraction




**division**




**exponent**

$1000 = 10 \times 10 \times 10 = 10^3$ ← exponent—read "10 to the third power"

**factoring** Writing a number as a product of factors

$28 = 4 \times 7$ $\quad 24 = 2 \times 3 \times 4$

**fraction** A number that tells how much

$\frac{6}{7}$ ← numerator ← denominator $\quad \frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

equivalent fractions

$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$ $\quad \frac{1}{5} = \frac{2}{10}$

simplest name

$\frac{7}{10} = \frac{7}{10}$ ← common denominators

**greatest common factor**

factors of 24: 1, 2, 3, 4, ⑥, 8, 12, 24
factors of 18: 1, 2, 3, ⑥, 9, 18
6 is the greatest common factor of 18 and 24.

**integer** Any of the whole numbers 0, +1, +2, +3, and so on, or the negative numbers −1, −2, −3, −4, and so on

**math sentence**

true: $3 + 5 = 8$ $\quad$ open: $3 + \square = 7$
false: $4 < 3 \times 0$ $\quad$ open: $9 - a = b$

**mixed number** Another name for some fractions

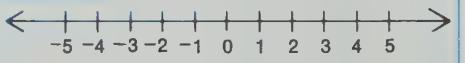
$2.3 = 2\frac{3}{10}$ $\quad 6\frac{7}{6} = 7\frac{1}{6}$, $2\frac{3}{5} = \frac{13}{5}$

mixed numbers

**multiplication**

.6 ← factor
× .7 ← factor
.42 ← product

**number line**



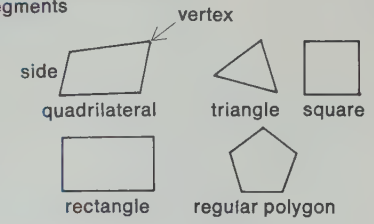

**percent** Parts of one hundred

$.25 = \frac{25}{100} = 25\%$—read "25 percent"

**perimeter** The distance around a polygon or other simple closed curve

Perimeter is 15 cm.

**polygon** A simple closed curve made with line segments




All sides and angles are congruent in a regular polygon.

**prime number** A whole number greater than 1 whose only pair of factors is 1 and itself. 2, 3, 5, 7, 11 are some prime numbers.

**probability** A number from 0 to 1 that tells how likely something is to happen

The outcome of a spin is A or B.
Select the *event* A.

The probability of A is $\frac{1}{4}$.

The *odds* in favor of A are 1 to 3.

**reflection**

**rotation** The turning of a figure

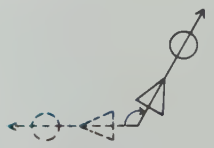


Figure has rotated 120° clockwise.

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